

Parallel Transport Examined

Parallel Transport equations for a latitude [$\theta = Const$]. We hold $r = \text{constant}$

$$\frac{dA^\theta}{d\varphi} - \cos\theta \sin\theta A^\varphi = 0$$

$$\frac{dA^\varphi}{d\varphi} + \frac{1}{r} A^r + \cot\theta A^\theta = 0$$

$$\frac{dA^r}{d\varphi} - (r - 2M) A^\varphi = 0$$

$$\frac{dA^t}{d\varphi} = 0$$

Solutions:

$$A^\varphi = C_1 \cos k\varphi + C_2 \sin k\varphi$$

$$A^\theta = \frac{\sin\theta \cos\theta}{k} [C_1 \sin k\varphi - C_2 \cos k\varphi] + C_3$$

$$A^r = \frac{r - 2M}{k} [C_1 \sin k\varphi - C_2 \cos k\varphi] - C_3 r \cos^2 \theta$$

$$A^t = C_4$$

In the above calculations $k^2 = \frac{r - 2M}{r} \sin^2 \theta + \cos^2 \theta$

Observations:

- 1) Even if A^r is kept zero at the initial point of the round tour, it assumes non zero values in the path in a general manner. The vector seems to lift out of the tangent plane.
- 2) We cannot assign integral values to k and so we do not have periodic functions of 2π in the trigonometric functions. Nevertheless the lifting out of the vector from the tangent plane should be given attention
- 3) With A^t we have no problem

For transport along a meridian [$\varphi = \text{Const}$, $r = \text{const}$], the equations are:

$$\frac{dA^\theta}{d\theta} + \frac{1}{r} A^r = 0$$

$$\frac{dA^\varphi}{d\theta} + \cot\theta A^\varphi = 0$$

$$\frac{dA^r}{d\theta} - (r - 2M) A^\theta = 0$$

$$\frac{dA^t}{d\theta} = 0$$

Solutions:

$$A^\theta = C_1 \cos k\theta + C_2 \sin k\theta$$

$$A^r = \frac{r - 2M}{k} [C_1 \sin k\theta - C_2 \cos k\theta]$$

$$A^\varphi = C_3 \cot\theta$$

$$A^t = C_4$$

$$[k^2 = \frac{r - 2M}{r}]$$

In the third equation above by substituting $\theta = 0$ on either side we can reduce the number of constants. Therefore we have four arbitrary constants for the four equations.

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