CALCULATIONS ON PARALLEL TRANSPORT

We are to consider the parallel transportation of a vector along the 45 degree latitude in a round trip on a sphere of constant radius. The vector always lies on the surface of the sphere .

Our relevant equations are as follows:

$$\frac{dA^{\theta}}{d\varphi}+\Gamma^{\theta}{}_{\varphi\theta}A^{\theta}+\Gamma^{\theta}{}_{\varphi\varphi}A^{\varphi}=0$$

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Using the expressions for the Christoffel symbols we have,

$$\frac{dA^{\theta}}{d\varphi}-\sin\theta\cos\theta\,A^{\varphi}=0$$

$$\frac{dA^{\varphi}}{d\varphi}+\cot\theta\,A^{\theta}=0$$

For $\theta=45^{0}$, we have,

$$\frac{dA^{\theta}}{d\varphi}-\frac{1}{2}A^{\varphi}=0$$

$$\frac{dA^{\varphi}}{d\varphi} + A^{\theta} = 0$$

Solving them we have,

$$A^{\theta} = A\cos\frac{1}{\sqrt{2}}\varphi + B\sin\frac{1}{\sqrt{2}}\varphi$$

$$A^{\varphi} = C\cos\frac{1}{\sqrt{2}}\varphi + D\sin\frac{1}{\sqrt{2}}\varphi$$

Using the boundary conditions ,for $\varphi = 0$, $A^{\theta} = K_1$ and $A^{\varphi} = K_2$ we obtain,

$$A^{\theta} = K_1\cos\frac{1}{\sqrt{2}}\varphi$$

$$A^{\varphi} = K_2\cos\frac{1}{\sqrt{2}}\varphi$$

Therefore,

$$\frac{A^{\theta}}{A^{\varphi}} = \frac{K_1}{K_2}$$

Which remains constant