Important Clarification

From the preservation of the dot product we have,

$$g_{\alpha\beta} a^{\alpha} b^{\beta} = g_{\alpha\beta} a^{/\alpha} b^{/\beta}$$

$$g^{k\alpha} g_{\alpha\beta} a^{\alpha} b^{\beta} = g^{k\alpha} g_{\alpha\beta} a^{/\alpha} b^{/\beta}$$

Both on the left and the right side of the above equation we have sixteen terms .But only one term one each side survives, since metric coefficients with unequal coefficients vanish

We choose k=1 as an illustration.

$$g^{1\alpha} g_{\alpha\beta} a^{\alpha} b^{\beta} = g^{1\alpha} g_{\alpha\beta} a^{/\alpha} b^{/\beta}$$

Only one term on each side survives. We have,

$$g^{11} g_{11} a^{1} b^{1} = g^{11} g_{11} a^{/1} b^{/1}$$

This is simply because of the fact that metric coefficients with unequal indices vanish.

Thus we have,

$$a^{1} b^{1} = a^{/1} b^{/1}$$

Since the norm itself is a vector product we have:

$$[a^{1}]^{2} = [a^{/1}]^{2}$$

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