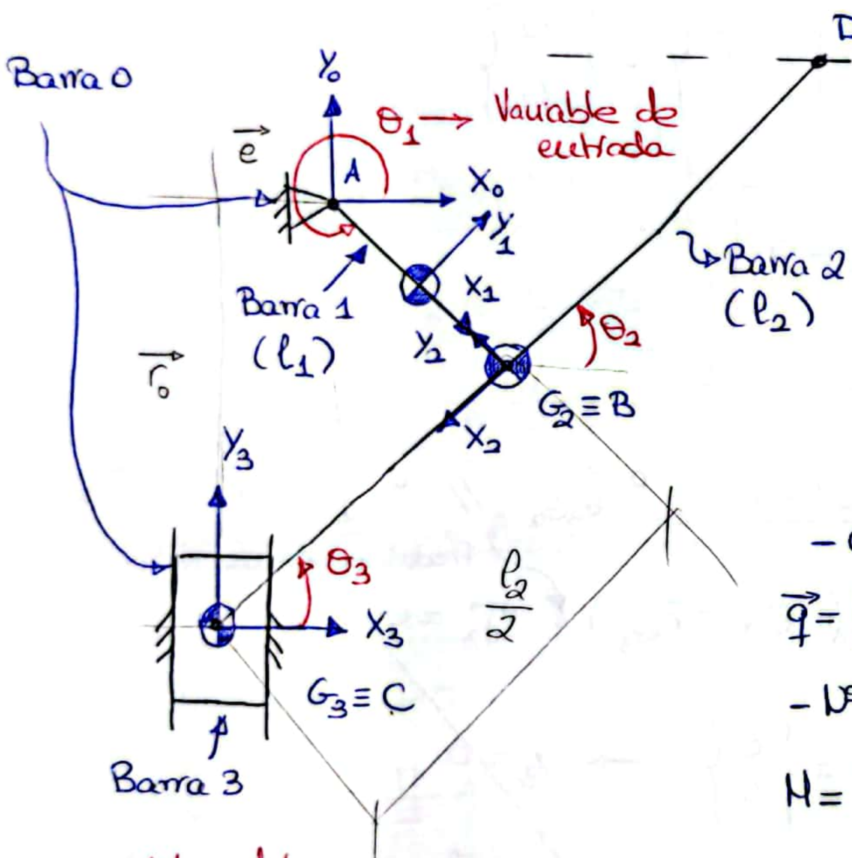
José Gargallo Tatay
21804770 B
Mecanismo Nº 3

Ejercicio Modelado Cinemático

a) Dibujar un diagrama cinemático [...] dimensiones y variables empleadas para modelarlo.



Sentido antihorario ⊕

3 Pares R
1 Par P
$\} J_1 = 4$

4 Barras → $n = 4$

$F = 3\cdot(n-1) - 2J_1 - J_2$

$$\boxed{F = 3\cdot(4-1) - 2\cdot 4 = 1 \text{ g.d.l}}$$

– Coordenadas generalizadas:

$$\vec{q} = [\theta_1 \; x_1 \; y_1 \; x_2 \; y_2 \; \theta_2 \; x_3 \; y_3 \; \theta_3]$$

– Nº Ecuaciones de restricción:

$$M = N - F = 9 - 1 = 8$$

b) Plantear las ecuaciones de restricción mediante coordenadas cartesianas.

– En A (Par R) → Barras 0–1: $\quad {}^0\cancel{\vec{r}_{A_0}} = {}^0\vec{r}_{AG_1} + {}^0\vec{r}_{G_1A}$

$$\{0\} = {}^0\vec{r}_{AG_1} + {}^0R_1 \, {}^1\vec{r}_{G_1A} \rightarrow \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} + \begin{bmatrix} & \\ & \end{bmatrix} \begin{Bmatrix} -\frac{\ell_1}{2} \\ 0 \end{Bmatrix}$$

$${}^0R_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix} \rightarrow \text{Matriz de rotación}$$

$$\boxed{\begin{aligned} \Phi_1 &: \; x_1 - \frac{\ell_1}{2} c\theta_1 = 0 \\ \Phi_2 &: \; y_1 - \frac{\ell_1}{2} s\theta_1 = 0 \end{aligned}}$$

→ En B (Par R) → Barras 1-2 : $\quad {}^0\vec{r}_{AG_1} + {}^0\vec{r}_{G_1B_1} = {}^0\vec{r}_{AG_2} + {}^0\vec{r}_{G_2B_2}$

$${}^0\vec{r}_{AG_1} + {}^0R_1\,{}^1\vec{r}_{G_1B_1} = {}^0\vec{r}_{AG_2} + {}^0R_2\,\underbrace{\cancel{{}^2\vec{r}_{G_2B_2}}}_{0}$$

$$\begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} + \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix} \begin{Bmatrix} \frac{l_1}{2} \\ 0 \end{Bmatrix} = \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix}$$

$$\boxed{\begin{aligned} \Phi_3 &: \quad x_1 - x_2 + \frac{l_1}{2} c\theta_1 = 0 \\ \Phi_4 &: \quad y_1 - y_2 + \frac{l_1}{2} s\theta_1 = 0 \end{aligned}}$$

→ En C (Par P) → Barras 0-3 : $\quad \vec{r}_0 \parallel {}^0\vec{j}_3 \quad$ y $\quad \theta_3 = \theta_2$

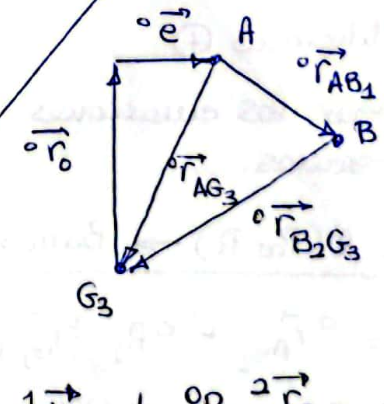
Producto escalar nulo

$${}^0\vec{r}_0 = {}^0\vec{e} + {}^0\vec{r}_{AG_3} \qquad ({}^0\vec{e} + {}^0\vec{r}_{AG_3}) \cdot {}^0\vec{j}_3 = 0$$

$$\begin{Bmatrix} 0 \\ r_0 \end{Bmatrix}$$

$$\left(\begin{Bmatrix} e \\ 0 \end{Bmatrix} + \begin{Bmatrix} x_3 \\ y_3 \end{Bmatrix} \right) \cdot \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \rightarrow y_3 = 0$$

$$\boxed{\begin{aligned} \Phi_5 &: \quad y_3 = 0 \\ \Phi_6 &: \quad \theta_3 - \theta_2 = 0 \end{aligned}}$$



Comprobación :

$${}^0\vec{r}_0 = {}^0\vec{e} + {}^0\vec{r}_{AB_1} + {}^0\vec{r}_{B_2G_3} = {}^0\vec{e} + {}^0R_1\,{}^1\vec{r}_{AB_1} + {}^0R_2\,{}^2\vec{r}_{B_2G_3} =$$

$$= \begin{Bmatrix} e \\ 0 \end{Bmatrix} + \begin{bmatrix} c\theta_1 & -s\theta_1 \\ s\theta_1 & c\theta_1 \end{bmatrix} \begin{Bmatrix} l_1 \\ 0 \end{Bmatrix} + \begin{bmatrix} c\theta_2 & -s\theta_2 \\ s\theta_2 & c\theta_2 \end{bmatrix} \begin{Bmatrix} \frac{l_2}{2} \\ 0 \end{Bmatrix} =$$

$$= \begin{Bmatrix} e + l_1 c\theta_1 + \frac{l_2}{2} c\theta_2 \\ l_1 s\theta_1 + \frac{l_2}{2} s\theta_2 \end{Bmatrix} \qquad {}^0\vec{r}_0 \cdot {}^0\vec{j}_3 = 0 \rightarrow l_1 s\theta_1 + \frac{l_2}{2} s\theta_2 = 0$$

$$\boxed{\begin{aligned} \Phi_5 &: \quad l_1 s\theta_1 + \frac{l_2}{2} s\theta_2 = 0 \\ \Phi_6 &: \quad \theta_3 - \theta_2 = 0 \end{aligned}}$$

→ <u>En C (Par R) ⇒ Barras 2-3:</u> $\quad {}^{0}\vec{r}_{AC_2} = {}^{0}\vec{r}_{AC_3}$

$$\underbrace{{}^{0}\vec{r}_{AG_2}} + \underbrace{{}^{0}\vec{r}_{G_2C_2}} = {}^{0}\vec{r}_{AG_3} + \cancel{{}^{0}\vec{r}_{G_3C_3}}^{\,0}$$

$${}^{0}R_2\,{}^{2}\vec{r}_{AB_2} + {}^{0}R_2\,\vec{r}_{G_2C_2} = {}^{0}\vec{r}_{AG_3}$$

$$\begin{Bmatrix} x_3 \\ y_3 \end{Bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 \\ s\theta_2 & c\theta_2 \end{bmatrix} \begin{Bmatrix} 0 \\ -\ell_1 \end{Bmatrix} + \begin{bmatrix} c\theta_2 & -s\theta_2 \\ s\theta_2 & c\theta_2 \end{bmatrix} \begin{Bmatrix} \frac{\ell_2}{2} \\ 0 \end{Bmatrix}$$

$$\boxed{\begin{aligned} \Phi_7 &: x_3 + \ell_1 c\theta_2 - \frac{\ell_2}{2} c\theta_2 = 0 \\ \Phi_8 &: y_3 + \ell_1 s\theta_2 - \frac{\ell_2}{2} s\theta_2 = 0 \end{aligned}}$$

→ <u>En C (Par P) → Barras 2-3:</u>

$$\begin{cases} \theta_3 = 90° = \frac{\pi}{2} \text{ rad} \\ x_3 = \text{cte.} = e \rightarrow x_3 - e = 0 \end{cases}$$

$y_3 \neq 0$

$$\boxed{\begin{aligned} \Phi_5 &: x_3 - e = 0 \\ \Phi_6 &: \theta_3 - \frac{\pi}{2} = 0 \end{aligned}}$$