Let $N \subset I \times \mathbb{R}^m$ be a countable set;

$$I = (t_1, t_2), \quad t_1 < t_2.$$

Fix $x_1, x_2 \in \mathbb{R}^m$.

Theorem 1. *There exists a function*

$$w \in C^\infty(\overline{I}, \mathbb{R}^m), \quad w(t_i) = x_i, \quad i = 1, 2$$

such that the graph

$$\{(t, w(t)) \mid t \in I\}$$

does not intersect N.

Proof. Introduce the following set

$$F = \{u \in C^\infty(\overline{I}, \mathbb{R}^m) \mid u(t_i) = x_i, \quad i = 1, 2\}$$

and its subset

$$F_{(\tau,\zeta)} = \{u \in F \mid u(\tau) = \zeta\}, \quad \tau \in I.$$

Being equipped with a collection of the standard $C^k(\overline{I}, \mathbb{R}^m)-$normes $(k \in \mathbb{N})$ the space $C^\infty(\overline{I}, \mathbb{R}^m)$ becomes a Fréchet space.

The set F is a complete metric space as a closed subset of the Fréchet space $C^\infty(\overline{I}, \mathbb{R}^m)$.

Lemma 1. *The set $F_{(\tau,\zeta)}$ is closed and does not have interior points in F.*

Indeed, the closeness is evident.

To check the second assertion take any function $v \in F_{(\tau,\zeta)}$ and a scalar function

$$\varphi \in C^\infty(I), \quad \operatorname{supp} \varphi \subset I, \quad \varphi(\tau) = 1.$$

Construct a sequence

$$v_j(t) = v(t) + \frac{1}{j}\varphi(t) \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

It is clear that $v_j \in F, \quad v_j \notin F_{(\tau,\zeta)}$ and $v_j \to v$ as $j \to \infty$. This proves the lemma.

By the Baire category theorem

$$F \backslash \Big(\bigcup_{(\tau,\zeta) \in N} F_{(\tau,\zeta)} \Big) \neq \emptyset.$$

Corollary 1. *Any two points $A, B \in \mathbb{R}^n \backslash \mathbb{Q}^n, \quad n > 1$ can be connected with a C^∞-smooth curve that does not intersect $\mathbb{Q}^n$.*