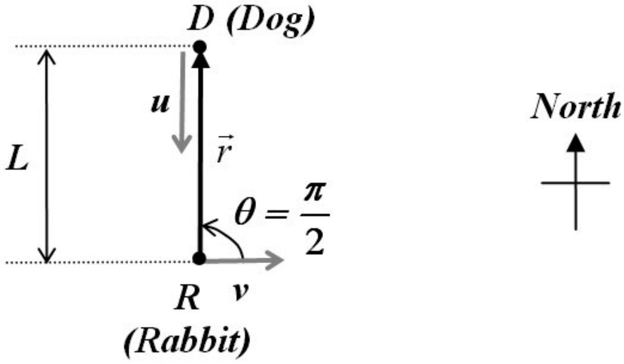
PHYS1414 General Physics I
Assignment One
Due Date: 5:00p.m., October 24, 2011

1. A dog is at a distance L due north of a rabbit, it observes the rabbit running in a vast field. The positions of them at the instant are shown in the figure. When the dog sees the rabbit, it starts to pursue the rabbit and its motion always points to the rabbit. Given that the rabbit keeps running due east with a constant speed v and the dog's speed is a constant u, where $v < u$. Find the time that the dog catches the rabbit according to the method stated below.



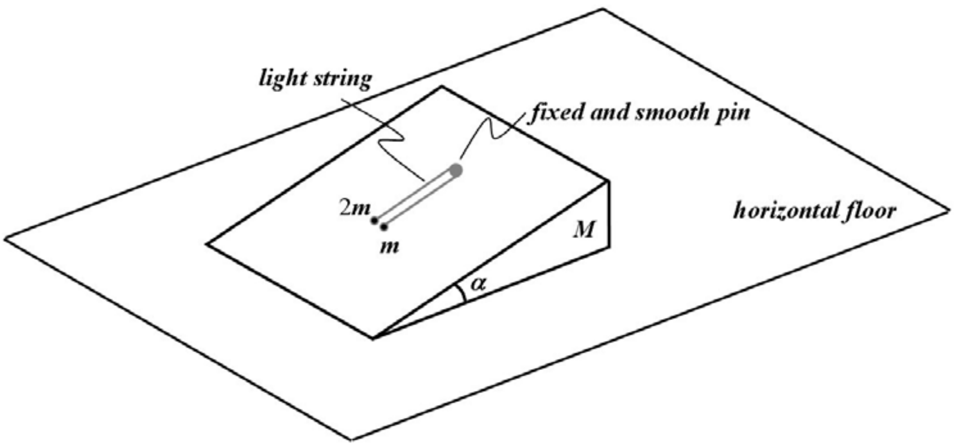
At the moment that the dog sees the rabbit

   (a) Consider the rabbit as a moving origin of a polar coordinates (r, θ) and label $\vec{r}$ as the position vector of dog relative to rabbit. Write down the velocity vectors relative to the rabbit along $\hat{e}_r$ and $\hat{e}_\theta$ respectively, where $\hat{e}_r$ and $\hat{e}_\theta$ are the unit vectors of the polar coordinates.

   (b) Show that
$$r = \frac{L(\cot\frac{\theta}{2})^{\frac{u}{v}}}{\sin\theta}.$$

   (c) Use the results of (a) and (b) to find τ, where τ is the time for the dog to catch the rabbit.
   [**Hint**: Consider the relation $\tau = \int_0^\tau \mathrm{d}t$ and $\mathrm{d}t = \mathrm{d}\theta/\dot{\theta}$.]

2. A wedge has smooth surfaces, of mass M and inclination angle α, is free to slide on a horizontal floor. Two particles of mass m and $2m$ respectively, are connected by a light string hanging over a fixed and smooth pin on the inclined surface of the wedge. Initially, all objects are at rest and they are released and observed by an inertial observer A on the floor. The acceleration of the wedge is noted by a and the acceleration of particle m relative to the wedge is a_1, where a and a_1 are the magnitudes of the quantities. Write down the equation of motion of the wedge along the floor as observed by A. This equation is labelled as (*) and will be used for the latter parts. Obtain the acceleration of the wedge a by the following methods.



Method I: Consider the motions of particles relative to observer A

(a) Write down the equations of motion of m parallel and perpendicular to the floor respectively, as observed by A.

(b) Write down the equations of motion of $2m$ parallel and perpendicular to the floor respectively, as observed by A.

(c) Use the results of (a), (b) and equation (*) to show the relation $(3m + M)\, a = ma_1 \cos\alpha$.

(d) Hence, show that the acceleration of wedge a has the form

$$a = \frac{mg \sin\alpha \cos\alpha}{pm + qM + rm \sin^2\alpha},$$

where p, q and r are constants. Find p, q and r.

Method II: Consider the motions of particles relative to observer B on the pin of the wedge

(a) Write down the equations of motion of m along and normal to the inclined surface of the wedge respectively, as observed by B.

(b) Write down the equations of motion of $2m$ along and normal to the inclined surface of the wedge respectively, as observed by B.

(c) Use the above equations due to method II and equation (*) to find a.

3. A particle is released from rest and it moves under the gravity in a viscous medium that exerts a retarding force on it. Data from experiments showed that the terminal speed is v_0 and the retarding force is proportional to the square of its instantaneous speed v. Please formulate the following.

[**Hint**: You may consider the downward motion as positive and take the retarding force $f = -mkv^2$, where m is the mass of the particle and k is a positive constant.]

(a) Show that the speed of the particle at time t is given by $v = v_0 \tanh\left(\frac{gt}{v_0}\right)$.

(b) Show also that the time taken for the particle to fall a distance h is $\frac{v_0}{g}\cosh^{-1}\left(e^{gh/v_0^2}\right)$.

[Hint: $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ and $\int \tanh x \, \mathrm{dx} = \ln\cosh u$, where $\cosh x = \frac{e^x + e^{-x}}{2}$.]

4. A bug moves on a curved path such that its location at time t is described by polar coordinates

$$\begin{cases} r = \dfrac{bt}{c^2}(2c - t) \\[2ex] \theta = \dfrac{t}{c} \end{cases}$$

where b amd c are positive constants and $0 \le t \le 2c$.

(a) Find the time that the bug has its minimum speed. Find also the minimum speed.

(b) Find the acceleration of the bug when it achieves its minimum speed.

5. A uniform sphere of radius R is cut into two portions. The smaller portion S has height h measured from its flat surface, where $h < R$.

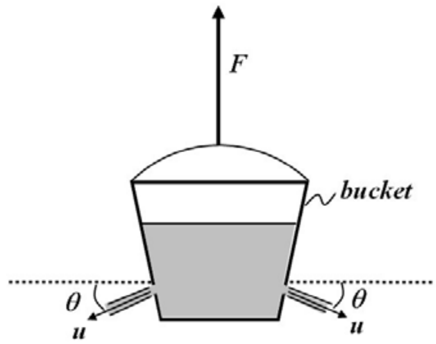
(a) Locate the centre of mass of S from its flat surface.

(b) Use the result in (a) to locate the centre of mass of a hollow H which has the same shape and same size as S. The hollow has only a thin curved surface and without any flat surface.

(c) Obtain the result in (b) by direct integration.

[**Hint**: Consider the hollow as a collection of many rings.]

6. Water is ejected from the two indentical holes in the bucket with constant speed u in the direction shown in the figure. The speed u is recorded relative to the bucket. The bucket has net mass m_0 and the radius of hole in it is r. It is set to move upward from rest by a force F when the mass of water inside it is m. If the bucket rises with a constant acceleration a, work on the following problems. The density of water is ρ.



(a) Find the magnitude of F.

(b) An upward propelling force is acting on the bucket when water ejects from the holes. Find the magnitude of such force when the bucket has water $m/2$ left in it. You may assume that the water level inside bucket is still above the holes and ejection of water takes place. Find also the magnitude of F at the instant.

END OF PAPER