Lennart Edsberg

Nada, KTH

Applied Numerical Methods I, DN2221

Project 5
Fluid velocities in a cylindrical pipe

In a long narrow pipe with circular cross section a fluid is streaming and we want to investigate how the fluid velocity is varying in the pipe. Denote by u the velocity component in the z-direction and by v the velocity component of the r-direction. The flow is circular symmetric, i.e. in cylindrical coordinates the dependent variables depend only on r and z, not ϕ.

At the inlet $z = 0$, the velocity is $0.1\,[m/s]$ in the z-direction, hence $u(r,0) = u_0 = 0.1, \quad v(r,0) = 0, \quad 0 \leq r \leq a$. The cross section radius of the pipe is $a = 0.05\,[m]$. The density of the fluid is $\rho = 1000\,[kg/m^3]$ and the viscosity $\nu = 10^{-5}\,[m^2/s]$. In fluid dynamic problems the Reynolds number $Re = u_0 a/\nu$ is an important quantity. When $Re >> 1$, as is the case here, the Navier-Stokes equations ' can be simplified to

$$u\frac{\partial u}{\partial z} = \nu(\frac{\partial^2 u}{\partial r^2} + \frac{1}{2}\frac{\partial u}{\partial r}) - v\frac{\partial u}{\partial r} - \frac{1}{rho}\frac{dP}{dz} \tag{1}$$

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial(vr)}{\partial r} = 0 \tag{2}$$

The pressure p depends only on z. At $z = 0$ the pressure is assumed to be $p_0 = 10^4\,[pascal]$. At $r = 0$ the boundary conditions are

$$\frac{\partial u}{\partial r}(0,z) = 0, \quad v(0,z) = 0$$

At the wall $r = a$ both velocity components are zero, i.e.

$$u(a,z) = v(a,z) = 0$$

Far away from the inlet (several meters from the inlet) the velocity will have a stationary distribution: $u = 2u_0(1 - (r/a)^2), \quad v = 0$.

The differential equations (1) and (2) must be specially treated when $r = 0$. Show with the help of l'Hôpitals rule that they get the form

$$u\frac{\partial u}{\partial z} = 2\nu\frac{\partial^2 u}{\partial r^2} - \frac{1}{\rho}\frac{dp}{dz}, \quad \frac{\partial u}{\partial z} + 2\frac{\partial v}{\partial r} = 0$$