Compute the velocity components $u(r,z)$ and $v(r,z)$ with the Method of Lines, i.e. discretize the r-variable with $n = 50$ steps. The result will be an ODE-system with $2n$ variables, $u_1(z), u_2(z), \ldots, u_n(z), \sigma(z), v_2(z), v_3(z), \ldots, v_n(z)$, where $\sigma(z) = \frac{1}{\rho}\frac{dp}{dz}$. From the boundary conditions $v_1(z) = 0$, hence there is no ODE for this component.

Formulate the Implicit Euler method with stepsize h_z for this ODE-system. For every step in the z-direction there will be a nonlinear system with $2n$ equations to be solved with Newtons method. If the unknowns are sorted as before, i.e. $u_1, u_2, \ldots, u_n, \sigma, v_2, v_3, \ldots, v_n$ the jacibian matrix will have a blockstructure of type

$$J = \begin{pmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{pmatrix}$$

where $\mathbf{J}_1$ is tridiagonal, $\mathbf{J}_2$ has nonzero elements only in the first column and the diagonal, $\mathbf{J}_3$ is diagonal and $\mathbf{J}_4$ is tridiagonal (perhaps with exception for the first row, depending on how $\frac{\partial v}{\partial r}$ is approximated at $r = 0$).

Close to the inlet the velocity variations are large, use a small step $h_z = 0.001$ up to $z = 0.005$. Continue with $h_z = 0.005$ to $z = 0.04$. Further up in the pipe the stepsize can be increased. Continue the computations of u, v and p up to the z-value where the u-velocity has reach its stationary distribution up to a given accuracy.

Present you results graphically.