A thin uniform metal disk lies on an infinite conducting plane. A uniform gravitational field is oriented normal to the plane. Initially the disk and plane are uncharged; charge is slowly added. The mass of the disk is M, and its area is A. What value of charge density is required to cause the disk to leave the plate?

Assuming no edge effects by the disk, and that only the component of electric field normal to the infinite plane is of concern, Gauss's Law says the electric field of any large charged plane,

$$\mathbf{E}_{above} - \mathbf{E}_{below} = \frac{\sigma}{\varepsilon_0} \tag{I.1}$$

$$\mathbf{E}_{above} = \mathbf{E}_{below} \tag{I.2}$$

This means the electric field of the plane with charge density σ is,

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0} \tag{I.3}$$

The force on the disk comes from summing the electric field [I.3] over all charge elements of the disk,

$$\mathbf{F} = \int \mathbf{E} \cdot dq = \int \frac{\sigma}{2\varepsilon_0} \cdot d(\sigma A) = \frac{\sigma^2}{2\varepsilon_0} \int dA = \frac{\sigma^2 A}{2\varepsilon_0} \tag{I.4}$$

This force must be equal to gravity, so,

$$Ma = \frac{\sigma^2 A}{2\varepsilon_0} \quad \rightarrow \quad \boxed{\sigma = \pm\sqrt{\frac{2\varepsilon_0 Ma}{A}}} \tag{I.5}$$

Consider virtual work. Do some virtual work on the plate, and see what happens.

Start with the potential of a uniformly-charged disk, EM 00, 145, pr 21?