CHAPTER 21 Electric Charge

Answers to Understanding the Concepts Questions

1. The only point where the net force on a test charge is zero is the midpoint between the two charges.

2. Yes, as Kepler's laws require that two objects attract each other with an inverse-square force, be it gravitational or electrostatic. Kepler's first law still holds true, namely, the two charges will move in an elliptical orbit around the center-of-mass of the system (provided that their total energy is negative — see the concept of electrostatic potential energy discussed in Chapter 24) . Since the force is aligned with the line connecting the two charges it provides no torque on either charge, so the angular momentum of each charge is still conserved and Kepler's second law (the law of area) remains valid. Finally, Kepler's third law, a direct consequence of the inverse-square nature of the force, is also true here: T^2 is proportional to r^3.

3. The balloon acquires a charge when rubbed on a sweater. It then induces an opposite charge on the wall through polarization, and the attraction between the charges on the balloon and those on the wall keeps the balloon there for a while.

4. The spark is caused by a transfer of the charge carried by the walker to the door knob. This can happen only if there is a build-up of charge on the walker. In the winter the air is drier; that is, there is less water vapor in the air. Since water molecules are efficient at picking up and carrying off charges on an object in their presence, it is more difficult for you to build up a significant charge in humid air. In the winter there is a better chance for a charge build-up, and therefore of a dramatic discharge.

5. In a short time interval Δt the objects cannot move very far, so we can consider their separation, r, essentially constant. Therefore, the electrostatic repulsion between them is also a constant: $F = kq_1q_2/r^2 \approx$ constant. The distance each can travel follows from $\Delta x_i = \frac{1}{2}a_i(\Delta t)^2 = \frac{1}{2}(F/m_i)t^2 \approx \frac{1}{2}(kq_1q_2/r^2m_i)(\Delta t)^2$, where $i = 1, 2$. So the factors that determine the distance each object can travel are q_1, q_2, r, Δt, and the mass the object in question. The ratio is $\Delta x_1 / \Delta x_2 = m_2 / m_1 = 3$, i.e., the object with three times the mass travels 1/3 of the distance of the other one.

6. The basic principle of the quantitative operation of the electroscope is outlined in Problem 21-5. The measurement of the angle made by a gold leaf can be translated into a measurement of the charge. In order to measure accurately the charge you carry you might want to stand on an insulating mat as you touch the metal top of the electroscope. That both controls the situation and ensures that your charge doesn't leak off elsewhere just as you are trying to measure it.

7. The ones farther from the nucleus are more involved in chemical reactions. This is because chemical reactions involve rearrangement of electronic orbitals, and electrons in an outer orbital are not as strongly tied to the nucleus so they are more likely to be able to rearrange themselves in a chemical reaction.

8. Let's call the quarks with charge $2e/3$ u-quarks, and those with charge $-e/3$ d-quarks (this is the standard nomenclature). Then the following compose all the possible combinations of three quarks, together with their charges: *uuu:* $2e$; *uud:* e; *udd:* 0; *ddd:* $-e$. The second combination has charge

corresponding to that of the proton, while the third combination has charge corresponding to that of the neutron. The other two combinations do not occur as stable particles.

9. Rubbing between the fingers and the packaging material results in charge transfer between the two, causing both to be oppositely charged. The opposite charges then attract each other, and for the small pieces of packaging materials (peanuts) with very little weight, this attraction is relatively strong enough to cause them to stick to the fingers. They are difficult to shake off as their masses are so small, and so the electrostatic attraction is often enough to withstand the violent acceleration of the shaking.

10. As an object is exposed to the air, it can get a fresh supply of electrons from humid air and neutralize the excess positive charge.

11. The net force on an object at the center of the circle is indeed zero. However, this point is not a stable equilibrium point. If the positive charge moves away from the center along the axis of the circular charge distribution, all the forces act to repel it, so that it will accelerate away from the center. The center is thus a point of unstable equilibrium, analogous to a ball resting on the top of a hill. The slightest displacement will cause it to move away from its starting point.

12. As charges move, a certain amount of negative may leave one contact but an equal amount of negative charge would enter the other one, thereby preserving the total amount of charges.

13. Let ball 1 have an initial charge of –4.8 (in units of 10^{-19} C); balls 2,3,4 are initially uncharged. If we assume that the balls are identical, then touching 1 and 2 will give each one of them a charge of –2.4 units, while balls 3 and 4 remain uncharged. If now ball 1 (or 2) is made to touch both 3 and 4 simultaneously, then the three balls each get one third of the available charge, that is –0.8 units.

14. Assuming that the electrostatic force between the two objects is not negligible compared with the their weights. You can suspend each object with a string and place them close together to see whether they move closer to, or away from, each other. The absolute sign of the charge on each object cannot be determined without further information. All we know can determine is whether their signs are the same or opposite.

15. A spark occurs as a result of the charge transfer between your hand and the car door. The fact that the rubber tires are good insulators only means that any excessive charges carried by the car cannot flow into the ground immediately, and that does not prevent the charge transfer from happening between the hand and the metal door knob, which is a conductor.

16. If the electrical charge of a fundamental particle such as the electron depended on its velocity, then we would have a chance to measure the tiny parameter κ only because of a departure from neutrality. Gravity is so weak that for all practical purposes two electrically neutral blocks of material do not exert forces on one another. Under the hypothesis, such objects would only be neutral because any surplus of charge due to a different motion of the electrons and the positive ions would be neutralized by ambient charges. However, we could put the two blocks in a vacuum. If at that point there is no force between the objects, then presumably the values of κ for the electrons and ions are such that at the given temperature the net charge of each object is zero. That is because a given temperature for the electrons corresponds, on average, to a certain value of v_e^2 for the electrons and, by equipartition, another value of v_i^2 for the ions determined by the relation $m_e v_e^2 = m_i v_i^2$. But now all we have to do is to keep the objects in the vacuum and raise the temperature. Each object will now acquire the same net charge, different from zero, and the repulsion should be detectable. We know that any such parameter κ must be very tiny, if it is not zero, because the existence of a temperature-dependent inverse square force has not been observed to the accuracy of our instruments.

17. No. The measurement of e may yield v, but v is only the velocity of the charged particle relative to a certain reference frame, not the absolute velocity, which is meaningless and impossible to measure.

18. No, because the data only show that for electrons and for protons the charge of each does not change. It is entirely possible that in electron-proton collisions at high temperatures (high energies) at some stage of the development of the quasar, some new net charge is produced from a neutral environment. This would correspond to charge nonconservation. As long as the amount of new charge is small, and as long as the processes producing these new charges do not affect the processes which cause the radiation that we observe, charge conservation would not be observed by the study of the colors of the quasar light.

19. If Earth and the Moon each has an equal number of protons and electrons and only the electronic charge is modified, then yes, in principle, Earth and the Moon would each carry a net charge and there would be a, net electrostatic force of the form $1/r^2$ between them. Whether that force would overpower the gravitational force depends on the amount of net charge on each of them. However, it would be more likely for each of them to be electrically neutral when they were formed, meaning that they would each end up with different number of protons and electrons.

20. This is exactly analogous to the motion of a mass inside Earth. We found in Chapter 12 that a mass inside Earth undergoes harmonic motion about the center. A point charge of one sign embedded in a spherically symmetric charge distribution of the opposite sign win also oscillate about the center of the sphere. The frequency can be found when the force inside the sphere can be calculated. This will be done in Chapter 23.

21. The force exerted on q_1 by q_2 doubles, as does that on q_3 by q_2.

22. The electrostatic force is largely responsible for the structures of atoms and molecules making up the star. These particles emit electromagnetic waves, both visible and invisible, than may be detected by us. If the electrostatic force over there were to have a different form than that on Earth, then the wavelengths of the electromagnetic waves emitted from that star would differ significantly from the corresponding values on Earth. This is not the case.

Solutions to Problems

1. The number of electron charge units in the excess positive charge is
   $N = Q/e = (1 \times 10^{-9}\ \text{C})/(1.602 \times 10^{-19}\ \text{C/electrons}) = 6.2 \times 10^{9}$ electrons,
   so the cork ball has $\boxed{6.2 \times 10^{9} \text{ fewer electrons}}$.

2. Because the uranium atom was initially neutral, the charge is positive:
   $q = +21\,|e| = +21(1.602 \times 10^{-19}\ \text{C}) = \boxed{+3.36 \times 10^{-18}\ \text{C}}$.
   The nucleus contains the positive charge of 92 protons:
   $Q = +92(1.602 \times 10^{-19}\ \text{C}) = \boxed{1.47 \times 10^{-17}\ \text{C}}$.

3. Each molecule of CO_2 contains 6 + 2(8) = 22 electrons. The charge in 1 g is
   $q = [(1\ \text{g})/(44\ \text{g/mol})](6.02 \times 10^{23}\ \text{molecules/mol})(22\ \text{electrons/molecule})(1.602 \times 10^{-19}\ \text{C/electron})$
   $= \boxed{4.82 \times 10^{4}\ \text{C}}$.

4. Because the spheres are identical, charge will be distributed equally when they are connected. After the initial connection, each sphere will have a charge
   $q = \frac{1}{3}Q$.
   The grounded sphere will lose its charge. When it is connected to the other sphere, the charge on that sphere will divide equally:
   $q' = \frac{1}{2}(\frac{1}{3}Q) = Q/6$. Thus the charges will be $\boxed{Q/3, Q/6, Q/6}$.

5. Each gold atom has 79 electrons, so removing one electron in 10^{13} means
   $79 \times 10^{-13} = \boxed{7.9 \times 10^{-12}}$ electrons per atom removed.

6. There are 79 protons per gold atom, and the number of gold atoms in the coin is $N = (28.4\ \text{g})/(197\ \text{g/mol})(6.022 \times 10^{23}/\text{mol}) = 8.682 \times 10^{22}$, so the total number of protons in the coin is
   $(79\ \text{protons/atom})(8.682 \times 10^{22}\ \text{atoms}) = \boxed{6.86 \times 10^{24}\ \text{protons}}$.

7. From symmetry considerations, each time two identical cork balls touch, the charge is shared evenly. At the first touch, the first cork ball (and the second cork ball) will have $\frac{1}{2}$ of the original charge:
   $q_1 = \frac{1}{2}q_0 = \frac{1}{2}(-4 \times 10^{-10}\ \text{C}) = \boxed{-2 \times 10^{-10}\ \text{C}}$, and the number of electrons gained is
   $N_1 = (2 \times 10^{-10}\ \text{C})/(1.602 \times 10^{-19}\ \text{C/electron}) = \boxed{1.25 \times 10^{9}\ \text{electrons}}$.
   At the second touch, the second cork ball (and the third cork ball) will have $\frac{1}{2}q_1$:
   $q_2 = \frac{1}{2}q_1 = \frac{1}{2}(-2 \times 10^{-10}\ \text{C}) = \boxed{-1 \times 10^{-10}\ \text{C}}$, and the number of electrons gained is
   $N_2 = (1 \times 10^{-10}\ \text{C})/(1.602 \times 10^{-19}\ \text{C/electron}) = \boxed{6.2 \times 10^{8}\ \text{electrons}}$.
   $q_3 = q_2 = \boxed{-1 \times 10^{-10}\ \text{C},\ 6.2 \times 10^{8}\ \text{electrons}}$.

8. From symmetry considerations, each time two identical cork balls touch, the charge is shared evenly. If we touch the first cork ball with an uncharged cork ball, the first cork ball (and the second cork ball) will have 1/2 of the original charge. If we now touch the second cork ball with an uncharged cork ball, the second cork ball and the third cork ball will have 1/4 of the original charge. If we now touch the third cork ball with the last uncharged cork ball, the third cork ball and the fourth cork ball will have 1/8 of the original charge, which is
   $q = (1/8)q_0 = (1/8)(-1.04 \times 10^{-13}\ \text{C}) = -0.13 \times 10^{-13}$ C, as desired.
   If we discharge one of the cork balls after touching, we need only one extra cork ball.

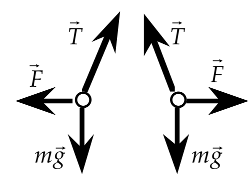
$\boxed{9}$. The total number of electrons in 0.1 g of aluminum is

$N = [(0.1 \text{ g})/(27.0 \text{ g/mol})](6.02 \times 10^{23} \text{ atoms/mol})(13 \text{ electron/atom})$
$= 2.9 \times 10^{22}$ electrons.

The fractional increase in the number of electrons is

fraction $= (1 \times 10^{-6} \text{ C})/(1.602 \times 10^{-19} \text{ C/electron})]/(2.9 \times 10^{22} \text{ electrons}) = \boxed{2.2 \times 10^{-10}}$.

10. (a)



(b) Before the charge is added, the cork balls are hanging vertically, so the tension is

$T_1 = mg = (0.2 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2) = \boxed{2.0 \times 10^{-3} \text{ N}}$.

After the charge is added, the charge will be shared equally by the two cork balls, and there is a horizontal Coulomb force.

From the force diagram, we apply $\sum \vec{F} = 0$:

horizontal: $T \sin \theta = F = kqq/r^2$;

vertical: $T \cos \theta = mg$.

If we divide the two equations, we get

$\tan \theta = F/mg = kq^2/r^2mg = kq^2/(2L \sin \theta)^2 mg$
$= (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1 \times 10^{-7} \text{ C})^2/[2(0.20 \text{ m}) \sin \theta]^2(0.2 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2) = 0.0065/(\sin^2 \theta)$.

This equation has only one unknown, θ, but the presence of trigonometric functions makes the algebra a little messy. When we calculate both sides for a range of angles, we get

$\sin \theta = 0.19, \quad \theta = 11°$.

The tension is

$T = mg/(\cos \theta) = (0.2 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)/(\cos 11°) = \boxed{2.0 \times 10^{-3} \text{ N}}$.

(c) From the analysis in part (b), we have $\theta = \boxed{11°}$.

11. (a) Because a silicon atom has 14 electrons, we find the number of electrons from

$N = [(5.98 \times 10^{27} \text{ g})/(28 \text{ g/mol})](6.02 \times 10^{23} \text{ atoms/mol})(14 \text{ electrons/atom})$
$= \boxed{1.8 \times 10^{51} \text{ electrons}}$.

(b) We find the fractional change from

$\Delta q/q = (1 \times 10^{-6} \text{ C})/(1.8 \times 10^{51} \text{ electrons})(1.6 \times 10^{-19} \text{ C/electron}) = \boxed{3.5 \times 10^{-39}}$.

12. Because charge is conserved, the two positive charges on the left must be balanced by two positive charges on the right. The charge of particle X is the proton charge:

$q_X = \boxed{+1.6 \times 10^{-19} \text{ C}}$.

13. (a) For the reaction $p + \bar{p} \rightarrow e^+ + e^- + e^+ + e^- + 2n$, the charges (as multiples of e) are

$+1 - 1 = +1 - 1 + 1 - 1 + 0$. Thus, we have $0 = 0$, so charge is $\boxed{\text{conserved}}$.

(b) For the reaction $e^+ + e^- \rightarrow 2p + n + 2\gamma$, the charges (as multiples of e) are

$+1 - 1 = +2$. Thus, we have $0 \neq 2$, so charge is $\boxed{\text{not conserved}}$.

(c) For the reaction $e^+ + e^- \rightarrow e^+ + e^- + p + \bar{p} + 2\gamma$, the charges (as multiples of e) are

$+1 - 1 = +1 - 1 + 1 - 1 + 0$. Thus, we have $0 = 0$, so charge is $\boxed{\text{conserved}}$.

(d) For the reaction $n + p \rightarrow e^- + p + \bar{p}$, the charges (as multiples of e) are

$0 + 1 = -1 + 1 - 1$. Thus, we have $1 \neq -1$, so charge is $\boxed{\text{not conserved}}$.

14. We let units help us find the charge:

$q = [(6.5 \times 10^{-4} \text{ g})/(9.11 \times 10^{-28} \text{ g/electron})](1.60 \times 10^{-19} \text{ C/electron})$
$= \boxed{1.1 \times 10^5 \text{ C}}$.

15. The proton at rest has charge $+ e_0$. The electron has charge $- e_0(1 + v^2/c^2)$, so the net charge is

$$\begin{aligned} Q_{\text{net}} &= + e_0 - e_0(1 + v^2/c^2) \\ &= (1.60 \times 10^{-19}\text{ C})\{1 - [1 + (1/137)^2]\} \\ &= \boxed{-8.5 \times 10^{-24}\text{ C}}. \end{aligned}$$

16. For the Coulomb force to be equal to the weight, we have

$k\,e^2/r^2 = mg;$

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2/r^2 = (1.67 \times 10^{-27}\text{ kg})(9.8\text{ m/s}^2)$, which gives

$r = 1.2 \times 10^{-1}\text{ m} = \boxed{12\text{ cm}}$.

17. The two up quarks will repel each other with a force

$$\begin{aligned} F_{\text{up-up}} &= kq_1q_2/r_{12}^2 \\ &= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})(1.6 \times 10^{-19}\text{ C})/(1.5 \times 10^{-15}\text{ m})^2 \\ &= \boxed{46\text{ N repulsion}}. \end{aligned}$$

The up and down quarks will attract each other with a force

$$\begin{aligned} F_{\text{up-down}} &= kq_1q_3/r_{13}^2 \\ &= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})(1.6 \times 10^{-19}\text{ C})/(1.5 \times 10^{-15}\text{ m})^2 \\ &= \boxed{23\text{ N attraction}}. \end{aligned}$$

18. We equate the two forces:

$F = kq_1q_2/r^2 = mg;$

$r = (kq_1q_2/mg)^{1/2} = [(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.5 \times 10^{-9}\text{ C})^2/(0.016\text{ kg} \times 9.8\text{ m/s}^2)]^{1/2} = \boxed{2.0 \times 10^{-3}\text{ m}}$.

19. The two ions will repel each other. The magnitude of the Coulomb force is

$F = kq_1q_2/r^2;$

$(1.1 \times 10^{-11}\text{ N}) = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(4.5 \times 10^{-9}\text{ m})^2$, which gives $q = \boxed{1.6 \times 10^{-19}\text{ C}}$.

We find the number of electron charges from

$N = q/e = (1.6 \times 10^{-19}\text{ C})/(1.6 \times 10^{-19}\text{ C/electron}) = \boxed{1\text{ electron}}$.

20. We assume that the cork balls are small, so they can be treated as point charges. For the Coulomb force, we have

$F = kq_1q_2/r^2;$

$0.18\text{ N} = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(2 \times 10^{-2}\text{ m})^2$, which gives

$q = \boxed{8.9 \times 10^{-8}\text{ C}}$.

If the pith balls were not small, the force between the charges would move some charge to the opposite sides of the pith balls. The center of the charge would not be at the center of the pith balls.

21. Both forces are inverse-square forces, so we have

$$\begin{aligned} F_E/F_g &= (kq^2/r^2)/(Gm^2/r^2) = kq^2/Gm^2 \\ &= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2/(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.10 \times 10^{-3}\text{ kg})^2 \\ &= \boxed{3.5 \times 10^{-10}}. \end{aligned}$$

This result is so different from Example 21-6 because the masses are so much larger than they are at the atomic scale.

22. For the Coulomb force to be 0.05% of the measured force, we have

$F = kq_1q_2/r^2;$

$(0.05 \times 10^{-2})(7 \times 10^{-7}\text{ N}) = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(0.10\text{ m})^2$, which gives

$q = \boxed{2.0 \times 10^{-11}\text{ C}}$.

$\boxed{23}$. (a) We consider two equal charges of magnitude 1 C separated by 1 cm. We call q_1 the magnitude in esu. If we equate the force in the two systems of units, we have

$F = q_1^2/r^2 = kq^2/r^2;$

$[q_1^2/(1\text{ cm})^2](10^{-5}\text{ N/dyne}) = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1\text{ C})^2/(1 \times 10^{-2}\text{ m})^2$, which gives

$q_1 = \boxed{3 \times 10^9\text{ esu in 1 C}}$.

(b) The electron charge is

$e = (1.6 \times 10^{-19}\text{ C})(3 \times 10^9\text{ esu/C}) = \boxed{4.8 \times 10^{-10}\text{ esu}}$.

24. (a) The attractive Coulomb force provides the centripetal acceleration:

$F = mv^2/r = mr\omega^2;$

$k\,e^2/r^2 = mr\omega^2$, which we write as $ke^2 = mr^3(2\pi/T)^2;$

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2 = (9.11 \times 10^{-31}\text{ kg})r^3[2\pi/(24\text{ h})(3600\text{ s/h})]^2,$

which gives $r = \boxed{3.6 \times 10^3\text{ m}}$.

(b) For the hydrogen orbit, we have

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2 = (9.11 \times 10^{-31}\text{ kg})r^3[2\pi/(4 \times 10^{-16}\text{ s})]^2,$

which gives $r = \boxed{1.0 \times 10^{-10}\text{ m}}$.

25. The Coulomb force is

$F = kq_1q_2/r^2 = kq_1(q - q_1)/r^2 = (qq_1 - q_1^2)k/r^2$, with q_1 as the variable.

To find q_1 that maximizes the force, we set $\mathrm{d}F/\mathrm{d}q_1 = 0$:

$\mathrm{d}F/\mathrm{d}q_1 = (q - 2q_1)k/r^2 = 0$, which gives $\boxed{q_1/q = \tfrac{1}{2}}$.

This means that $\boxed{q_2/q = \tfrac{1}{2}}$, which we would expect from the symmetry of the force law.

26. The two particles repel each other. At the position of closest approach, we have

$F = kq_1q_2/r^2 = k(2e)(74e)/r^2$

$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(74)(1.6 \times 10^{-19}\text{ C})^2/(6.0 \times 10^{-12}\text{ m})^2$

$= \boxed{9.5 \times 10^{-4}\text{ N}}$ repulsion.

27. (a) The opposite charges attract. We find the magnitude of the Coulomb force from

$F = ke^2/r^2 = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2/(3 \times 10^{-10}\text{ m})^2$

$= \boxed{2.6 \times 10^{-9}\text{ N toward the proton (centripetal)}}$.

(b) The attractive Coulomb force provides the centripetal acceleration:

$F = mv^2/r;$

$(2.6 \times 10^{-9}\text{ N}) = (9.11 \times 10^{-31}\text{ kg})v^2/(3 \times 10^{-10}\text{ m})$, which gives

$v = \boxed{9.2 \times 10^5\text{ m/s}}$.

(c) We find the frequency from

$f = v/2\pi r = (9.2 \times 10^5\text{ m/s})/2\pi(3 \times 10^{-10}\text{ m}) = \boxed{4.9 \times 10^{14}\text{ Hz}}$.

(d) We find the spring constant from

$k = (2\pi f)^2 m = [2\pi(4.9 \times 10^{14}\text{ Hz})]^2(9.11 \times 10^{-31}\text{ kg}) = \boxed{8.6\text{ N/m}}$.

28. (a) The acceleration of each particle is caused by the same force:

$F = m_1a_1 = m_2a_2$, which gives

$m_2 = (a_1/a_2)m_1 = [(1.93\text{ m/s}^2)/(5.36\text{ m/s}^2)](31.3\text{ g}) = \boxed{11.3\text{ g}}$.

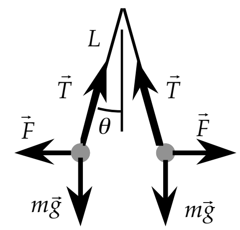
(b) Because the particles have equal charges, we have

$k\,q^2/r^2 = m_1a_1;$

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(8.75 \times 10^{-2}\text{ m})^2 = (31.3 \times 10^{-3}\text{ kg})(1.93\text{ m/s}^2)$, which gives

$q = \boxed{2.27 \times 10^{-7}\text{ C}}$.

29. From the force diagram, we apply $\sum \vec{F} = 0$:
horizontal: $T \sin \theta = F = kqq/r^2$;
vertical: $T \cos \theta = mg$.
If we divide the two equations, we get
$\tan \theta = F/mg = kq^2/r^2mg = kq^2/(2L \sin \theta)^2mg$
$\tan 10° = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q^2/[2(0.20\ \mathrm{m}) \sin 10°]^2(0.20 \times 10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})$,
which gives $q = \boxed{1.4 \times 10^{-8}\ \mathrm{C}}$.



30. (a) We find the magnitude of the electrical force from
$F_E = kq^2/r^2$
$= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(8.5 \times 10^{15}\ \mathrm{C})^2/(3.8 \times 10^8\ \mathrm{m})^2 = \boxed{4.5 \times 10^{24}\ \mathrm{N}}$.
(b) The ratio of forces is
$F_E/F_g = (kq^2/r^2)/(GmM/r^2) = kq^2/GmM$
$= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(8.5 \times 10^{15}\ \mathrm{C})^2/(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.36 \times 10^{22}\ \mathrm{kg})(5.98 \times 10^{24}\ \mathrm{kg})$
$= \boxed{2.2 \times 10^4}$.
(c) The density of charge of the earth is
$\rho = Q/V = (8.5 \times 10^{15}\ \mathrm{C})/(4/3)\pi(6.4 \times 10^6\ \mathrm{m})^3 = \boxed{7.7 \times 10^{-6}\ \mathrm{C/m^3}}$.
(d) The density of protons to produce the charge density of part (c) is
$\rho_p = \rho/e = (7.7 \times 10^{-6}\ \mathrm{C/m^3})/(1.6 \times 10^{-19}\ \mathrm{C/proton}) = \boxed{4.8 \times 10^{13}\ \mathrm{protons/m^3}}$.
(e) The density of all protons in Earth is
$\rho_p' = \frac{1}{2}\rho_M/m_p = \frac{1}{2}(5.52 \times 10^3\ \mathrm{kg/m^3})/(1.67 \times 10^{-27}\ \mathrm{kg/proton}) = \boxed{1.7 \times 10^{30}\ \mathrm{protons}}$

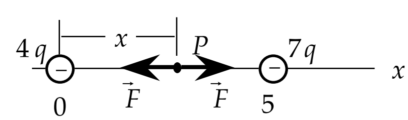
31. Because q_1 and q_2 attract each other, they must have opposite signs and their product will be negative. We can take this into account by giving the force a negative value:
$F_{12} = kq_1q_2/r_{12}^2$;
$-1.4 \times 10^{-2}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q_1q_2/(15.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
$q_1q_2 = -3.5 \times 10^{-14}\ \mathrm{C^2}$.
Because q_2 and q_3 attract each other, they must have opposite signs and their product will be negative. We can take this into account by giving the force a negative value:
$F_{23} = kq_2q_3/r_{23}^2$;
$-\ 3.8 \times 10^{-3}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q_2q_3/(20.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
$q_2q_3 = -1.7 \times 10^{-13}\ \mathrm{C^2}$.
Because q_1 and q_3 repel each other, they must have the same sign and their product will be positive. We can take this into account by giving the force a positive value:
$F_{13} = kq_1q_3/r_{13}^2$;
$+\ 5.2 \times 10^{-3}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q_1q_3/(10.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
$q_1q_3 = +5.8 \times 10^{-14}\ \mathrm{C^2}$.
We have three equations for three unknowns, q_1, q_2, and q_3. If we assume that q_1 is positive, when we combine the equations we get
$q_1 = \boxed{+1.1 \times 10^{-7}\ \mathrm{C}}$,
$q_2 = \boxed{-3.2 \times 10^{-7}\ \mathrm{C}}$,
$q_3 = \boxed{+5.3 \times 10^{-7}\ \mathrm{C}}$.
If we took q_1 to be negative, we would get the same magnitudes, with q_1 and q_3 negative and q_2 positive.

32. (a) In order to have a repulsion, Q must be negative. For forces with equal magnitudes, we have
$GMm/r^2 = kQe/r^2$, so we get
$Q = -\,GMm/ke$
$= -\,(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(6 \times 10^{24}\ \mathrm{kg})(0.9 \times 10^{-30}\ \mathrm{kg})/(9 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})$
$= \boxed{-2.5 \times 10^{-7}\ \mathrm{C}}$.

(b) The number of protons in the earth is
$N_p = \frac{1}{2}M/m_p$.
The discrepancy between the proton and the electron charges as a fraction of the electron charge is
$\Delta = (Q/N_p)/e = 2Qm_p/Me$
$= 2(2.5 \times 10^{-7}\ \mathrm{C})(1.6 \times 10^{-27}\ \mathrm{kg})/(6 \times 10^{24}\ \mathrm{kg})(1.6 \times 10^{-19}\ \mathrm{C})$
$= \boxed{8 \times 10^{-40}\ \text{of electron charge}}$.

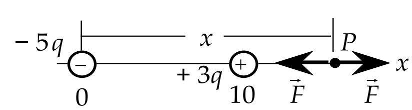
33. The force on a mass m attracted to a fixed mass M is $F_g = GMm/r^2$. The potential energy of the mass m is
$U = -\,GMm/r$, with $U = 0$ when $r = \infty$.
The negative sign for U is due to the force being attractive; the potential energy decreases as the masses come closer.
The force on a charge q repelled by a fixed charge Q of the same sign is $F_E = kQq/r^2$. The potential energy of the charge q is
$U_E = +\,kQq/r$, with $U = 0$ when $r = \infty$.
The positive sign for U is due to the force being repulsive; the potential energy increases as the charges come closer.
If the electrical force is the only one present, we have energy conservation. If the moving point charge is aimed straight at the fixed charge, at the distance of closest approach the charge will momentarily come to rest. Thus we have
$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$;
$K_\mathrm{i} + 0 = 0 + (kq_1q_2/r_\mathrm{f})$;
$1\ \mathrm{J} = (9 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})(10^{-6}\ \mathrm{C})(10^{-4}\ \mathrm{C})/r_\mathrm{f}$, which gives
$r_\mathrm{f} = \boxed{0.9\ \mathrm{m}}$.

34. Because the two charges have the same sign, the charge Q must be between the two on the x-axis, where the two forces on Q will be in opposite directions. The net force will be zero when the two magnitudes are equal:



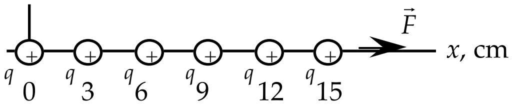
$k4qQ/r_1^{\,2} = k7qQ/r_2^{\,2}$, or, when we cancel common factors,
$4/x^2 = 7/(5 - x)^2$, which gives $x = 2.15$, and -15.4.
The point is between the two charges at $\boxed{x = 2.15}$.

35. Because the two charges have opposite signs, the charge Q must be on the x-axis outside the two, where the two forces on Q will be in opposite directions. The net force will be zero when the two magnitudes are equal



$k5qQ/r_1^{\,2} = k3qQ/r_2^{\,2}$, or, when we cancel common factors,
$5/x^2 = 3/(x - 10)^2$, which gives $x = 5.6$, and 44.4.
The point is outside the two charges at $\boxed{x = 44.4}$.
Compare with the answer to Problem 34.

36.



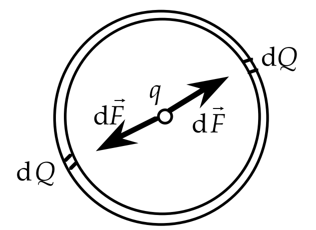
Because all of the charges have the same sign, the charge at $x = 25$ cm is repelled by all of the others. The net force will be toward $+x$, with a magnitude equal to the sum of the magnitudes of the individual forces:

$$F = \sum_{n=1}^{5} \frac{kqq}{r_i^2} = kqq\left[\frac{1}{(0.25\text{ m})^2} + \frac{1}{(0.20\text{ m})^2} + \frac{1}{(0.15\text{ m})^2} + \frac{1}{(0.10\text{ m})^2} + \frac{1}{(0.05\text{ m})^2}\right]$$

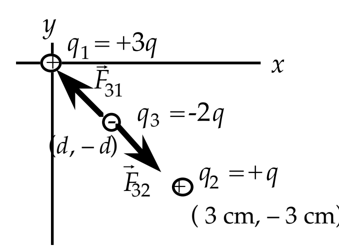
$$= \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.4\times10^{-7}\text{ C})^2}{(0.05\text{ m})^2}\left[\frac{1}{(5)^2} + \frac{1}{(4)^2} + \frac{1}{(3)^2} + \frac{1}{(2)^2} + \frac{1}{(1)^2}\right] = 0.30\text{ N}.$$

The net force is $\boxed{0.30\text{ N in the } +x\text{ -direction}}$.

$\boxed{37}$. The ring of charge Q can be thought of as an infinite number of differential charges spread uniformly on the ring. The positive charge q is attracted by one of the differential charges, which has a matching charge on the opposite side of the ring. The sum of the forces from the pair is zero, thus when all pairs are considered, the net force on q must be $\boxed{\text{zero}}$.



38. There will be two forces acting on the third charge. When the third charge is in equilibrium, the net force is zero, so the two forces must be in opposite directions. Because the sign of the third charge is opposite to the other two charges, it is attracted by the other two, so the third charge must be between the other two charges, which are separated by $3\sqrt{2}$ cm along the line $y = -x$. We place the third charge at ($+d$ cm, $-d$ cm), with $d < 3$ cm. The magnitudes of the two forces must be equal:



$F_{31} = F_{32}$;

$k\,q_3q_1/r_{31}^2 = kq_3q_2/r_{32}^2$, which reduces to

$q_1/r_{31}^2 = q_2/r_{32}^2$;

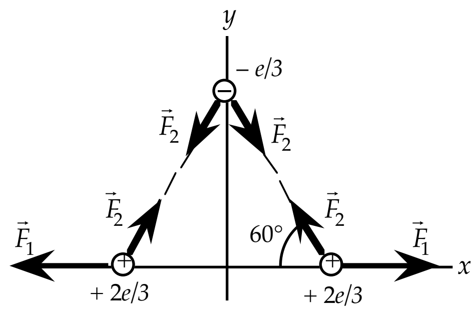
$q/(d\sqrt{2})^2 = 3q/[(3\sqrt{2}\text{ cm}) - (d\sqrt{2})]^2$, which gives $d = 1.10$ cm.

The third charge must be at $\boxed{(+1.10\text{ cm}, -1.10\text{ cm})}$.

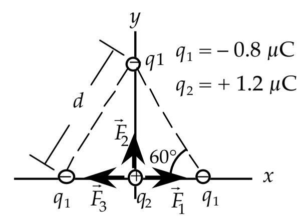
If the third charge is displaced slightly along the line joining the charges, the charge toward which it is moved will exert a larger attracting force, so the net force will be away from the equilibrium position. The equilibrium will be unstable.

If the third charge is displaced slightly away from the line joining the charges, both attracting forces will have a component back toward the line, so the net force will be toward the equilibrium position. The equilibrium will be stable.

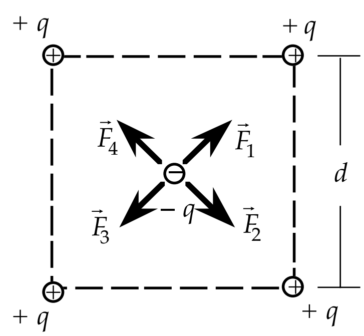
39. The forces on each quark are shown in the diagram.
For the positive "up" quark on the right, we have
$\sum F_x = F_1 - F_2 \cos 60°$
$= (46\text{ N}) - (23\text{ N}) \cos 60° = 34\text{ N};$
$\sum F_y = F_2 \sin 60°$
$= (23\text{ N}) \sin 60° = +20\text{ N}.$
When we combine these components, we get
$F_+ = \boxed{39\text{ N } 30° \text{above the line joining the two "up" quarks}}$
(the x-axis).
From symmetry, the force on the left "up" quark will be 39 N 30°above the $-x$-axis.
For the negative "down" quark at the top, we have
$\sum F_x = F_2 \cos 60° - F_2 \cos 60° = 0;$
$\sum F_y = -F_2 \sin 60° - F_2 \sin 60°$
$= -2(23\text{ N}) \sin 60° = -40\text{ N}.$
The force on the "down" quark is
$F_- = \boxed{40\text{ N toward the center of the line joining the two "up" quarks}}.$
Note that the sum of the three forces is zero, within the limitation of significant figures.



40. The three forces acting on the positive charge are shown in the diagram.
Their magnitudes are
$F_1 = F_3 = k|q_1|\,|q_2|/(d/2)^2$
$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(0.6 \times 10^{-6}\text{ C})(1.5 \times 10^{-6}\text{ C})/(9.0 \times 10^{-2}\text{ m})^2$
$= 1.76\text{ N};$
$F_2 = k|q_1|\,|q_2|/(d \sin 60°)^2$
$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(0.6 \times 10^{-6}\text{ C})(1.5 \times 10^{-6}\text{ C})/(18\times 10^{-2} \sin 60°\text{ m})^2$
$= 0.59\text{ N}.$
From the symmetry of the forces, we have
$\vec{F}_1 + \vec{F}_3 = 0;$
$\vec{F} = \vec{F}_2 = (0.59\text{ N})\,\hat{j}.$
The net force is $\boxed{0.59\text{ N toward the opposite corner}}$.



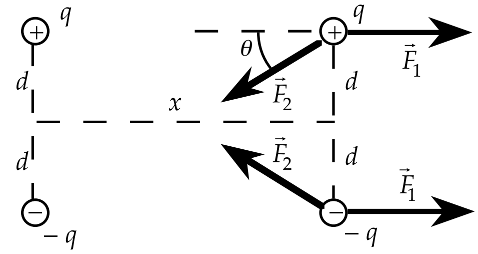
$\boxed{41}$. (a) From the symmetry of the charges and the distances, we have
$F_1 = F_2 = F_3 = F_4$, so
$\sum \vec{F} = 0$, the negative charge is in equilibrium.
(b) If the negative charge is moved slightly toward one of the positive charges, the attractive force toward that charge will increase, while the attractive force toward the opposite corner will decrease. The net force will be away from the equilibrium point, so it will be $\boxed{\text{unstable}}$.
(c) If the negative charge is moved perpendicular to the plane a small distance, each of the four attractive forces will have a component pointing back toward the plane. The net force, the sum of these four forces, will be toward the equilibrium point, so it will be $\boxed{\text{stable}}$.



42. From the symmetry of the charge distribution, we know that the forces on the dipoles will have the same magnitude and opposite directions, so we consider one dipole. Each charge of the dipole will have two forces acting on it, as shown in the diagram. From the symmetry of the charges and distances, we see that the net force will be horizontal, and the force on the positive charge will be the same as that on the negative charge. The net force on the dipole is

$$F_{\text{net}} = 2(F_1 - F_2 \cos\theta) = 2\left\{\frac{kq^2}{x^2} - \frac{kq^2}{x^2 + (2d)^2}\,\frac{x}{\left[x^2 + (2d)^2\right]^{1/2}}\right\}$$

$$= \frac{2kq^2}{x^2}\left\{1 - \frac{1}{\left[1 + (4d^2/x^2)\right]^{3/2}}\right\} = \frac{2kq^2}{x^2}\left\{1 - \left[1 + (4d^2/x^2)\right]^{-3/2}\right\}.$$



When $d \ll x$, $[1 + (4d^2/x^2)]^{-3/2} \simeq 1 - \frac{3}{2}(4d^2/x^2)$, so we have

$$F_{\text{net}} \simeq \frac{2kq^2}{x^2}\left\{1 - \left[1 - \frac{3}{2}\left(\frac{4d^2}{x^2}\right)\right]\right\} = \frac{12kd^2q^2}{x^4}.$$

43. (a) The three forces acting on q are shown in the figure. Their magnitudes are

$F_1 = F_2 = k2qq/(2L)^2 = \frac{1}{2}kq^2/L^2;$

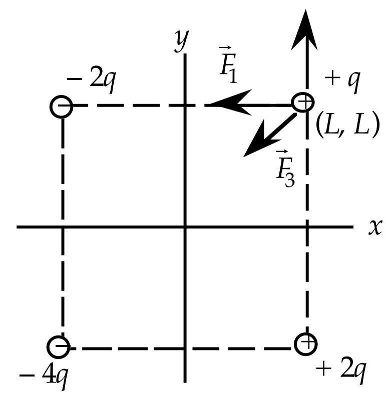
$F_3 = k4qq/(2L\sqrt{2})^2 = \frac{1}{2}kq^2/L^2.$

The net force acting on q is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-\tfrac{1}{2}kq^2/L^2)\,\hat{i} + (\tfrac{1}{2}kq^2/L^2)\,\hat{j} - \{[(\tfrac{1}{2}kq^2/L^2)\cos 45°]\,\hat{i} + [(\tfrac{1}{2}kq^2/L^2)\sin 45°]\,\hat{j}\}$$

$$= (\tfrac{1}{2}kq^2/L^2)\{[-(2+\sqrt{2})/2]\,\hat{i} + [(2-\sqrt{2})/2]\,\hat{j}\}$$

$$= \boxed{(\sqrt{3})kq^2/2L^2,\ 9.7° \text{ above the } -x\text{-axis}}.$$



(b) The four forces acting on Q are shown in the figure. Their magnitudes are

$F_1 = F_3 = k2qQ/(L\sqrt{2})^2 = kqQ/L^2;$

$F_2 = kqQ/(L\sqrt{2})^2 = kqQ/2L^2;$

$F_4 = k4qQ/(L\sqrt{2})^2 = 2kqQ/L^2.$

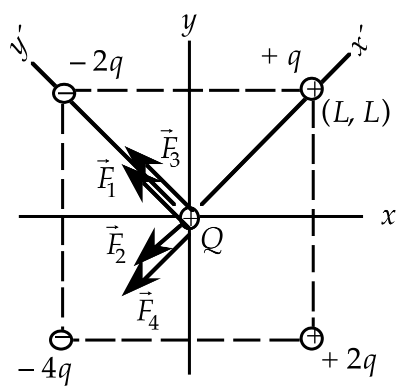
To find the net force, we use a rotated $x'y'$-coordinate system, as shown on the diagram. Thus

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

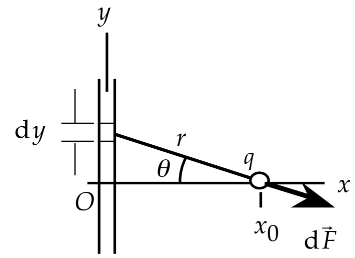
$$= (kqQ/L^2)\,\hat{j}\,' - (kqQ/2L^2)\,\hat{i}\,' + (kqQ/L^2)\,\hat{j}\,' - (2kqQ/L^2)\,\hat{i}\,'$$

$$= (kqQ/L^2)[-2.5\,\hat{i}\,' + 2\,\hat{j}\,']$$

$$= 3.2kqQ/L^2,\ 38.7° \text{ above the } -x'\text{-axis, or } \boxed{3.2kqQ/L^2,\ 6.3° \text{ below the } -x\text{-axis}}.$$



44. (a) Because the charge Q is symmetrically distributed about $y = 0$, the force on q will be along the x-axis toward positive x.



(b) Because Q is distributed uniformly, the linear charge density is $\lambda = Q/2L$, and the charge on a segment is

$$dQ = \boxed{(Q/2L)\,dy}.$$

(c) The force vector, shown in the figure, has magnitude

$$dF = (kq/r^2)\,dQ = \boxed{(kqQ/2Lr^2)\,dy}.$$

(d) From part (a), we know that we add (integrate) the x-components:

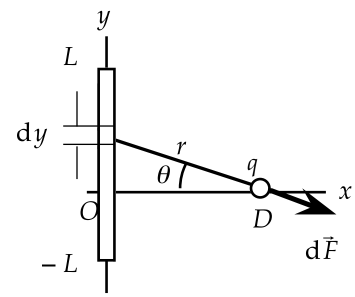
$$F_x = \int dF_x = \int_{-L}^{L} \frac{kqQ}{2Lr^2}\cos\theta\,dy = \int_{-L}^{L} \frac{kqQ}{2L}\frac{D}{\left(D^2 + y^2\right)^{3/2}}\,dy.$$

(e) The result of the integration is

$$F_x = \frac{kqQD}{2L}\frac{y}{D^2\left(D^2 + y^2\right)^{1/2}}\Bigg|_{-L}^{L} = \frac{kqQ}{2LD}\left[\frac{L}{D^2\left(D^2 + L^2\right)^{1/2}} - \frac{-L}{D^2\left(D^2 + L^2\right)^{1/2}}\right]$$

$$= \frac{kqQ}{D\left(D^2 + L^2\right)^{1/2}}.$$

45. Because the line charge is symmetrically distributed about $y = 0$, the force on q will be along the x-axis toward positive x.



The charge on the segment dy is

$$dQ = \lambda\,dy.$$

The force vector, shown in the figure, has magnitude

$$dF = (kq/r^2)\,dQ = (kq\lambda/r^2)\,dy.$$

To find the force, we add (integrate) the x-components:

$$F = \int dF_x = \int_{-\infty}^{\infty} \frac{kq\lambda}{r^2}\cos\theta\,dy.$$

From the figure, we see that $r = x_0/\cos\theta$, and $y = x_0\tan\theta$.

We change variable to θ, with

$$dy = x_0\sec^2\theta\,d\theta = (x_0/\cos^2\theta)\,d\theta:$$

$$F = \int_{-\pi/2}^{\pi/2} \frac{kq\lambda}{\left(x_0/\cos\theta\right)^2}\left(\cos\theta\right)\left(x_0/\cos^2\theta\right)d\theta = \int_{-\pi/2}^{\pi/2} \frac{kq\lambda}{x_0}\cos\theta\,d\theta$$

$$= \frac{kq\lambda}{x_0}\left(\sin\theta\right)\Big|_{-\pi/2}^{\pi/2} = \frac{2kq\lambda}{x_0}.$$

$$\vec{F} = (2kq\lambda/x_0)\,\hat{i}.$$

46. We use the figure from Problem 45, with the line charge from $y = 0$ to $y = +\infty$. The force on q will have both an x-component and a y-component. We find each component by integrating, using the same change of variable that we used in Problem 43:

$$F_x = \int dF_x = \int_0^\infty \frac{kq\lambda}{r^2}\cos\theta\,dy$$

$$= \int_0^{\pi/2} \frac{kq\lambda}{(x_0/\cos\theta)^2}(\cos\theta)(x_0/\cos^2\theta)\,d\theta = \int_0^{\pi/2}\frac{kq\lambda}{x_0}\cos\theta\,d\theta$$

$$= \frac{kq\lambda}{x_0}(\sin\theta)\Big|_0^{\pi/2} = \frac{kq\lambda}{x_0};$$

$$F_y = \int dF_y = -\int_0^\infty \frac{kq\lambda}{r^2}\sin\theta\,dy.$$

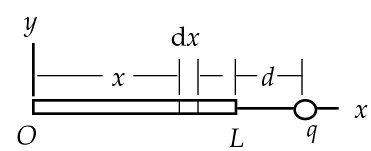
$$= -\int_0^{\pi/2} \frac{kq\lambda}{(x_0/\cos\theta)^2}(\sin\theta)(x_0/\cos^2\theta)\,d\theta = -\int_0^{\pi/2}\frac{kq\lambda}{x_0}\sin\theta\,d\theta$$

$$= -\frac{kq\lambda}{x_0}(-\cos\theta)\Big|_0^{\pi/2} = -\frac{kq\lambda}{x_0}.$$

The force on q is

$\vec{F} = (kq\lambda/x_0)\,\hat{i} - (kq\lambda/x_0)\,\hat{j}$, or $\boxed{(kq\lambda/x_0)\sqrt{2}\ \ 45° \text{ below the } x\text{-axis}}$.

$\boxed{47}$. We align the rod along the x-axis with one end at the origin, as shown in the figure. The linear charge density is $\lambda = Q/L$, so the charge on the element dx is $dQ = (Q/L)\,dx$. All elements of the rod produce a force in the $+x$-direction. The total force is



$$\vec{F} = \int \hat{i}\,dF_x = \int_0^L \frac{kq\lambda}{r^2}\hat{i}\,dx = \frac{kqQ}{L}\hat{i}\int_0^L \frac{dx}{(L - x + d)^2}$$

$$= \frac{kqQ}{L}\hat{i}\left(\frac{1}{L - x + d}\right)\Big|_0^L = \frac{kqQ}{L}\hat{i}\left(\frac{1}{d} - \frac{1}{L + d}\right) = \frac{kqQ}{d(L + d)}\hat{i}.$$

The force on q is $\boxed{kqQ/d(L + d) \text{ away from the rod}}$.

48. In Example 21-10 we found the electric force exerted on a point charge Q located on the axis of a uniformly charged ring of charge q and radius R to be

$F = kqQL/(R^2 + L^2)^{3/2}$,

where L is the distance between Q and the center of the ring. In our case there are two rings, each exerting a force on Q. Since these two forces are opposite in direction and we want the net force to be zero, we need to place Q where the two forces have the same magnitude. Let the distance between the first ring and Q be L_1, etc, then we have

$F_1 = F_2$;

$kqQL_1/(R_1^2 + L_1^2)^{3/2} = kqQL_2/(R_2^2 + L_2^2)^{3/2}$.

Also, $L_1 + L_2 = 100$ cm. Plug in $R_1 = 25$ cm and $R_2 = 40$ cm and solve for L_1: $L_1 = \boxed{1.3 \text{ cm}}$, i.e., the charge should be placed 1.3 cm from the center of the ring with a radius of 25 cm (and 98.3 cm from the other).

49. Let the charge on each ring of radius R be q and the center-to-center separation between the two rings be $2L$. Draw an x-axis from the center of one ring (at $x = -L$) to that of the other (where $x = +L$). By symmetry the net force of both rings on a point charge Q is zero at $x = 0$, midway between the two rings. To examine the stability of the equilibrium position at $x = 0$, imagine moving the point charge to a new location x, away form (yet very close to) the equilibrium: $|x| << L$. The point charge is now a distance $(L + x)$ from the one ring and $(L - x)$ from the other. The net force exerted on the point charge is now

$$F = kqQ(L + x)/[R^2 + (L + x)^2]^{3/2} - kqQ(L - x)/[R^2 + (L - x)^2]^{3/2}.$$

For $|x| << L$ we may use the approximation

$$\begin{aligned}(L + x)[R^2 + (L + x)^2]^{-3/2} &\approx (L + x)(R^2 + L^2 + 2Lx)^{-3/2} \\ &= (L + x)(R^2 + L^2)^{-3/2}\,[1 + 2Lx/(R^2 + L^2)]^{-3/2} \\ &\approx (L + x)(R^2 + L^2)^{-3/2}\,[1 + (-3/2)2Lx/(R^2 + L^2)] \\ &\approx L/(R^2 + L^2)^{3/2} + [(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x \quad \text{(up to the first power in } x\text{)}\end{aligned}$$

and

$$[R^2 + (L - x)^2]^{-3/2} \approx L/(R^2 + L^2)^{3/2} - [(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x.$$

so

$$F \approx [2kqQ(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x = Cx.$$

If $C > 0$, then $F(x)$ has the same sign as x. Thus if Q moves toward one of the two rings the net force on it tends to push it further toward that ring — the equilibrium is unstable. Similarly, if $C < 0$ then $F(x)$ and x have opposite signs, and the net force always tends to push the charge back to $x = 0$. The equilibrium is stable. (Just think of the restoring force of a spring, $F = -kx$, where $k > 0$.)
Thus the stability of the equilibrium depends on the sign of the expression

$$qQ(R^2 - 2L^2).$$

$\boxed{\text{If } q \text{ and } Q \text{ have the same sign, then the equilibrium is stable if } R > \sqrt{2}L\text{, and unstable if } R < \sqrt{2}L.}$

$\boxed{\text{If } q \text{ and } Q \text{ have opposite signs, then the equilibrium is stable if } R < \sqrt{2}L\text{, and unstable if } R > \sqrt{2}L.}$

50. We pair an element of the ring dQ with the element diametrically opposite. The two forces exerted on q at the center will have the same magnitude but opposite directions. Their sum will be zero, and thus, for all of the elements of the ring, we have

$$\vec{F} = 0.$$

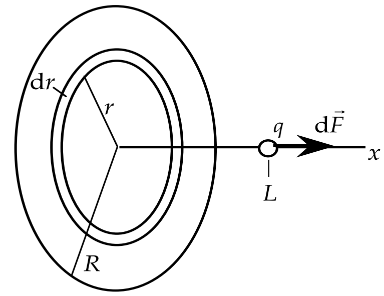
We assume that q and Q have the same sign. If q is moved in the xy-plane, the dQ toward which it moves will exert a greater repulsion, while the dQ on the opposite side will exert a smaller repulsion. The net force will be toward the center of the ring; the equilibrium is stable. If q and Q have opposite signs, the forces become attractive. The net force will be away from the center of the ring; the equilibrium is unstable.
The Coulomb force is an inverse-square force, like the gravitational force. A mass anywhere inside a uniform spherical shell of mass will experience no gravitational force. A charge inside a uniformly charged spherical shell will experience no electrical force.

51. If we consider the plate to be a series of concentric rings, each ring will produce a force away from the plate, as shown in the figure. We choose a representative ring of radius r and thickness dr. The area charge density of the plate is

$\sigma = Q/\pi R^2$, so the charge on the ring is

$dq = \sigma 2\pi r\, dr = (2Q/R^2) r\, dr$.

We use the result for a ring from Example 21–10 to find the total force by summing (integrating) the forces from all of the rings:



$$\vec{F} = \int d\vec{F} = \int_0^R \frac{2kqQL}{R^2} \frac{r\, dr}{\left(r^2 + L^2\right)^{3/2}} \hat{i} = \frac{2kqQL}{R^2} \hat{i} \left[\frac{-1}{\left(r^2 + L^2\right)^{1/2}} \right] \Bigg|_0^R$$

$$= \frac{2kqQL}{R^2} \left[\frac{-1}{\left(R^2 + L^2\right)^{1/2}} - \frac{-1}{L} \right] \hat{i} = \frac{2kqQ}{R^2} \left[1 - \frac{L}{\left(R^2 + L^2\right)^{1/2}} \right] \hat{i}.$$

For the given data, we get

$$\vec{F} = \frac{2\left(9 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right)\left(0.65 \times 10^{-6}\ \mathrm{C}\right)\left(1.6 \times 10^{-6}\ \mathrm{C}\right)}{\left(8 \times 10^{-2}\ \mathrm{m}\right)^2} \times$$

$$\left\{ 1 - \frac{5\ \mathrm{cm}}{\left[(8\ \mathrm{cm})^2 + (5\ \mathrm{cm})^2\right]^{1/2}} \right\} \vec{i}$$

$= 1.4$ N away from the center.

52. We can consider the plane sheet as a plate with an infinite radius. The analysis of Problem 51 can be used:

$$F = \int dF = \int_0^\infty kq\sigma L \frac{2\pi r\, dr}{\left(r^2 + L^2\right)^{3/2}}$$

$$= 2\pi kq\sigma L \left[\frac{-1}{\left(r^2 + L^2\right)^{1/2}} \right] \Bigg|_0^\infty = 2\pi kq\sigma L \left(\frac{1}{L}\right) = 2\pi kq\sigma$$

$= \dfrac{q\sigma}{2\varepsilon_0}$ away from the plane sheet.

53. Because the plates attract each other, equal and opposite forces are required to separate them. The two plates are so close that we can say that, to any small element of charge on one plate, the other plate will appear infinite. This neglects small effects at the edge. The field of the plate is uniform due to the charge density $\sigma = Q/A$. From the result of Problem 52, the force on a small segment of charge ΔQ due to the charge on the other plate is

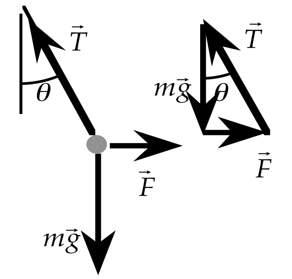
$\Delta F = \Delta Q\, (Q/A)/2\varepsilon_0$.

When we add the forces on all of the charge elements, we have

$F = \sum \Delta Q\, (Q/2\varepsilon_0 A) = Q^2/2\varepsilon_0 A$;

$0.1\ \mathrm{N} = /2(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(0.05\ \mathrm{m^2})$, which gives $Q = \boxed{3.0 \times 10^{-7}\ \mathrm{C}}$.

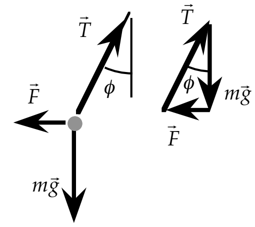
54. From the result of Problem 52, we know that the force exerted by the sheet of charge will be perpendicular to the sheet.



(a) A positive charge is repelled by the sheet. The cork ball will hang at an angle where $\sum \vec{F} = 0$. From the diagram of the vector sum of the forces, we have

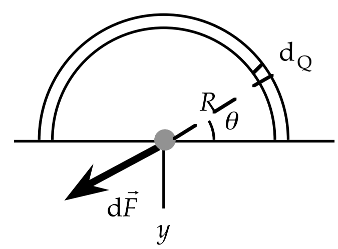
$$\tan\theta = F/mg = (q\sigma/2\varepsilon_0)/mg = q\sigma/2\varepsilon_0 mg$$
$$= (0.8\times10^{-8}\ \text{C})(1.2\times10^{-6}\ \text{C/m}^2)/[2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)]$$
$$= 0.0069, \text{ so } \theta = \boxed{0.4^\circ \text{ from the vertical away from the sheet}}.$$

(b) A negative charge is attracted by the sheet. We find the angle from



$$\tan\phi = q\sigma/2\varepsilon_0 mg$$
$$= (3\times10^{-8}\ \text{C})(2\times10^{-6}\ \text{C/m}^2)/[2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)]$$
$$= 0.026, \text{ so } \phi = \boxed{1.5^\circ \text{ from the vertical toward the sheet}}.$$

55. We place the wire in a vertical plane, as shown. From the symmetry of the charge distribution, we know that the force on q will be down. The linear charge density of the wire is $\lambda = Q/\pi R$. We use an element $dQ = \lambda R\, d\theta$ at an angle θ from the horizontal. We find the net force by summing (integrating) the vertical components:



$$F = \int dF_y = \int_0^\pi \frac{kq}{R^2}\sin\theta\, dQ = \int_0^\pi \frac{kqQ}{\pi R^3}(\sin\theta)\, R\, d\theta$$
$$= \frac{kqQ}{\pi R^2}(-\cos\theta)\Big|_0^\pi = \frac{2kqQ}{\pi R^2}.$$

From the given data, we get

$$F = 2(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.30\times10^{-6}\ \text{C})(0.75\times10^{-6}\ \text{C})/\pi(0.050\ \text{m})^2 = \boxed{0.52\ \text{N}}.$$

56. (a) All of the forces from the charges will lie along the line of the charges, with those from the positive charges to the right and those from the negative charges to the left. We allow for the possibility that the charges are not the same. We label a representative charge with m, where m goes from 0 to n. The distance from the mth charge to Q is $D - md$, and we can take the direction of the force into account by using a factor of $(-1)^m$. Thus we have

$$F = kQ\sum_{m=0}^{n}\frac{q_m(-1)^m}{(D-md)^2}. \qquad \text{When } q_m = q, \text{ we have } F = kQq\sum_{m=0}^{n}\frac{(-1)^m}{(D-md)^2}.$$

(b) Because $D >> md$, we can use the approximation $(D - md)^{-2} \simeq D^{-2}[1 + (2md/D)]$:

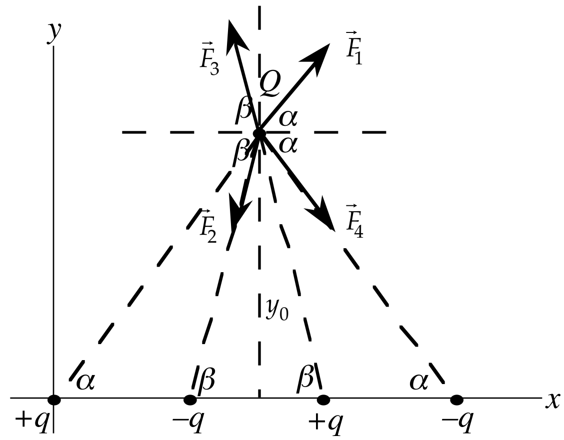
$$F = kQ\sum_{m=0}^{n}\frac{q_m(-1)^m}{D^2}\left(1+\frac{2md}{D}\right) = \frac{kQ}{D^2}\sum_{m=0}^{n}q_m(-1)^m + \frac{2kQd}{D^3}\sum_{m=0}^{n}mq_m(-1)^m.$$

When $q_m = q$, we have

$$F = \frac{kQq}{D^2}\sum_{m=0}^{n}(-1)^m + \frac{2kQqd}{D^3}\sum_{m=0}^{n}m(-1)^m, \text{ where we have different results for } n \text{ being odd or even:}$$

$$F_{\text{odd}} = 0 - \frac{2kQqd}{D^3}\left(\frac{n+1}{2}\right) = -\frac{(n+1)kQqd}{D^3}, \quad \text{and} \quad F_{\text{even}} = +\frac{kQq}{D^2} + \frac{2kQqd}{D^3}\left(\frac{n}{2}\right) = +\frac{kQq}{D^2} + \frac{nkQqd}{D^3}.$$

57. By symmetry the net force, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$, is parallel to the x-axis, as $\vec{F}_1 + \vec{F}_4$ is along the positive x-direction and $\vec{F}_2 + \vec{F}_3$ is along the negative x-direction. So we only need to consider the x-component of each force. We have



$F_{1x} = F_{4x} = F_1 \cos\alpha$
$= \{kq(3q)/[(1.5\text{ cm})^2 + y_0^2]\}\{1.5\text{ cm}/[(1.5\text{ cm})^2 + y_0^2]^{1/2}\}$
$= (4.5\text{ cm})kq^2/[(1.5\text{ cm})^2 + y_0^2]^{3/2}$, and
$F_{2x} = F_{3x} = -F_2 \cos\beta$
$= -\{kq(3q)/[(0.5\text{ cm})^2 + y_0^2]\}\{0.5\text{ cm}/[(0.5\text{ cm})^2 + y_0^2]^{1/2}\}$
$= -(1.5\text{ cm})kq^2/[(0.5\text{ cm})^2 + y_0^2]^{3/2}$.

Thus
$\vec{F} = (F_{1x} + F_{2x} + F_{3x} + F_{4x})\hat{i} = (3.0\text{ cm})kq^2\{3/[(1.5\text{ cm})^2 + y_0^2]^{3/2} - 1/[(0.5\text{ cm})^2 + y_0^2]^{3/2}\}\hat{i}$.

58. The charge is uniformly distributed over the entire sphere, with $\rho = e/V = e/(4\pi R^3/3)$. The portion of the charge that is contained in the spherical region of radius r is then $q = \rho(4\pi r^3/3) = er^3/R^3$. According to the textbook the net force exerted by the charged sphere on the negative point charge is then
$F = -keq/r^2 = -e^2r/R^3$.
Set this to equal to ma, with $a = \mathrm{d}^2r/\mathrm{d}t^2$ the acceleration of the point charge of mass m:
$-e^2r/R^3 = m\,\mathrm{d}^2r/\mathrm{d}t^2$, or
$m\,\mathrm{d}^2r/\mathrm{d}t^2 = -(e^2/R^3)r$.
This equation is analogous to the standard equation for a spring-mass system, namely, $m\,\mathrm{d}^2x/\mathrm{d}t^2 = -kx$, with x replaced by r and k by ke^2/R^3. Thus the solution to our equation also yields a simple-harmonic motion, equivalent to that with an effective spring constant of $k_{\text{eff}} = ke^2/R^3$. The frequency of the oscillation is
$f = \omega/2\pi = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(ke^2/R^3m)^{1/2}}$.

59. We choose a differential charge element of one of the plates as $\mathrm{d}q = \sigma\,\mathrm{d}A$, which is a point charge. We find the force on $\mathrm{d}q$ exerted by the other plate from Problem 52:
$\mathrm{d}F = \mathrm{d}q\,(\sigma/2\varepsilon_0) = \sigma\,\mathrm{d}A(\sigma/2\varepsilon_0)$.
The force per unit area is
$\mathrm{d}F/\mathrm{d}A = \sigma^2/2\varepsilon_0 = (10^{-5}\text{ C/m}^2)^2/2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{5.7\text{ N/m}^2}$.
The force per unit area is independent of the separation of the plates. If the distance is doubled, the force per unit area is $\boxed{5.7\text{ N/m}^2}$.

60. The surface area of the cone is
$A = \pi R(h^2 + R^2)^{1/2}$.
If the charge Q is uniformly distributed over its surface then the surface charge density is
$\sigma = Q/A = \boxed{Q/[\pi R(h^2 + R^2)^{1/2}]}$.

61. The surface charge density of Earth is
$\sigma = Q/A = Q/4\pi R^2$.
The electric field just outside its surface due to this charge density is $E = \sigma/2\varepsilon_0$, which exerts an electrostatic force of
$F_E = qE = q(\sigma/2\varepsilon_0) = q(Q/4\pi R^2)/2\varepsilon_0$
on a charge q placed near Earth's surface. For mechanical equilibrium for the charge of mass m, set
$F_E = F_g$; $q(Q/4\pi R^2)/2\varepsilon_0 = mg$; which gives
$q = 8\pi\varepsilon_0 R^2mg/Q$
$= 8\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(6.37\times10^6\text{ m})^2)(10\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)/(6\times10^5\text{ C}) = \boxed{1.5\times10^{-3}\text{ C}}$.

62. Because 1 mm is very small compared to the dimensions of the plate, we can treat the plate as an infinite plate with density $\sigma = Q/L^2$ and use the result of Problem 52. The upward electrical force is balanced by the downward force of gravity:

$F_E = mg$, or $q\sigma/2\varepsilon_0 = mg$;

$(0.8 \times 10^{-6}\ \text{C})Q/[2(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.60\ \text{m})^2] = (1.5 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)$; so

$Q = \boxed{1.2 \times 10^{-7}\ \text{C}}$.

If $d = 2$ mm, the plate would still appear to be an infinite plate. The electrical force would not depend on distance, so Q remains 1.2×10^{-7} C.

If $d = 1$ m, the plate would no longer appear to be infinite. The inverse-square dependence of the electrical force means that Q would have to be larger to exert the same magnitude force. For large distances, the plate would appear to be a point charge.

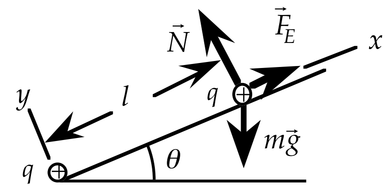
63. In the equilibrium position, the net force is zero. From the diagram,

$\sum F_x = F_E - mg \sin\theta = 0$;

$kqq/\ell^2 = mg \sin\theta$;

$(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-8}\ \text{C})^2/(0.08\ \text{m})^2 = (0.5 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2) \sin\theta$, which gives

$\sin\theta = 0.115, \quad \theta = \boxed{6.6^\circ}$.



64. There is a Coulomb force of repulsion between two like point charges:

$F = (1/4\pi\varepsilon_0)qq/d^2$

$= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})(1.6 \times 10^{-19}\ \text{C})/(8 \times 10^{-15}\ \text{m})^2$

$= \boxed{3.6\ \text{N repulsion}}$.

$\boxed{65}$. (a) The attractive Coulomb force provides the centripetal acceleration:

$F = (1/4\pi\varepsilon_0)(e^2/R^2) = mv^2/R$, which gives $v = \boxed{(e^2/4\pi\varepsilon_0 mR)^{1/2}}$.

(b) The magnitude of the angular momentum is

$L = mvR = \boxed{(e^2 mR/4\pi\varepsilon_0)^{1/2}}$.

(c) We use the result of part (a):

$v = (e^2 v/4\pi\varepsilon_0 L)^{1/2}$, which gives $v = \boxed{e^2/4\pi\varepsilon_0 L}$.

(d) We use the result of part (b):

$R = L/mv = \boxed{4\pi\varepsilon_0 L^2/me^2}$.

(e) The time to go around the circle is the period:

$\tau = 2\pi R/v = 2\pi(4\pi\varepsilon_0 L^2/me^2)/(e^2/4\pi\varepsilon_0 L) = \boxed{32\pi^3\varepsilon_0{}^2 L^3/me^4}$.

(f) We are given $L = 1.05 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$. For the others, we have

$v = (9 \times 10^9\ \text{m/s}^2)(1.6 \times 10^{-19}\ \text{C})^2/(1.05 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}) = \boxed{2.2 \times 10^6\ \text{m/s}}$.

$R = [1/(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)](1.05 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s})^2/(9.11 \times 10^{-31}\ \text{kg})(1.6 \times 10^{-19}\ \text{C})^2 = \boxed{5.3 \times 10^{-11}\ \text{m}}$.

$\tau = [2\pi/(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)^2](1.05 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s})^3/(9.11 \times 10^{-31}\ \text{kg})(1.6 \times 10^{-19}\ \text{C})^4 = \boxed{1.5 \times 10^{-16}\ \text{s}}$.

66. The electrical force will be the repulsive force between the excess positive charge on the Sun and Earth. In each case, this will be the number of protons times δe. Set

$F_E = F_g$

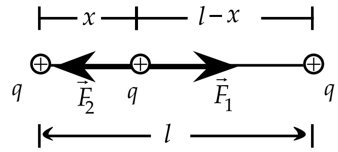
$(1/4\pi\varepsilon_0)(\Delta q_{\text{sun}}\Delta q_{\text{earth}}/R^2) = (1/4\pi\varepsilon_0)(N_{\text{sun}}\ \delta e\ N_{\text{earth}}\ \delta e/R^2) = GM_{\text{sun}}M_{\text{earth}}/R^2$;

$(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.25 \times 10^{57})\ \delta(1.15 \times 10^{44})\ \delta(1.6 \times 10^{-19}\ \text{C})^2 =$

$(6.7 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2 \times 10^{30}\ \text{kg})(6 \times 10^{24}\ \text{kg})$, which gives

$\delta = \boxed{5 \times 10^{-15}}$.

67. (a) The middle charge is repelled by each of the other charges.
The net force is



$F_{\text{net}} = F_1 - F_2$;

$F_{\text{net}} = kq^2[1/x^2 - 1/(\ell - x)^2]$

$= \boxed{kq^2[\ell(\ell - 2x)/x^2(\ell - x)^2]\text{, away from the closer charge}}$.

For the net force to be zero, we have

$\ell - 2x = 0$, or $x = \ell/2$, as expected from symmetry.

(b) We call the displacement from equilibrium $\Delta x = x - (\ell/2)$. When we substitute this into the expression for the net force, we get

$F_{\text{net}} = kq^2[\ell(-2\,\Delta x)/(\ell/2 + \Delta x)^2(\ell/2 - \Delta x)^2]$, or

$\vec{F}_{\text{net}} = \boxed{-2kq^2\ell\,\Delta\vec{x}\,/[(\ell/2)^2 - (\Delta x)^2]^2}$.

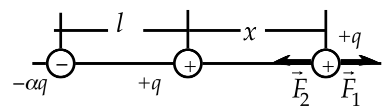
(c) When $\Delta x << \ell$, we can drop the Δx^2 term in the denominator:

$\vec{F}_{\text{net}} = -kq^2(32\,\Delta x/\ell^3)\,\hat{i}$, which has the form of a restoring spring force.

The oscillation frequency for small displacements is

$f = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(32kq^2/\ell^3 m)^{1/2}}$.

68. (a) The net force will be



$$F_{\text{net}} = F_1 - F_2 = \frac{kq^2}{x^2} - \frac{k\alpha q^2}{(x+\ell)^2} = kq^2\frac{(x+\ell)^2 - \alpha x^2}{x^2(x+\ell)^2}.$$

(b) Because the charges have opposite signs, the moving charge must be outside of the two charges where the two forces will be in opposite directions and farther from the larger negative charge. The net force will be zero when

$(x + \ell)^2 - \alpha x^2 = 0$, or $(\alpha - 1)x^2 - 2\ell x - \ell^2 = 0$.

The positive solution to this quadratic equation is

$x = x_0 = \boxed{\ell[(1 + \alpha^{1/2})/(\alpha - 1)]}$.

The negative solution corresponds to a position between the two charges, where the equal magnitude forces will be in the same direction.

(c) If $\alpha = 40$, the equilibrium position is

$x_0 = \ell[(1 + \alpha^{1/2})/(\alpha - 1)] = \ell[(1 + 40^{1/2})/(40 - 1)] \approx 0.1878\ell$. To find the restoring force, we consider a small displacement Δx from the equilibrium position: $x = x_0 + \Delta x$, with $|\Delta x| << \ell$. The net force is

$$\begin{aligned}
F(x) &\approx F(x_0) + (\mathrm{d}F/\mathrm{d}x)\Delta x = 0 + \{\mathrm{d}[kq^2/x^2 - \alpha kq^2/(x+\ell)^2]/\mathrm{d}x\}\,\Delta x \\
&= 2kq^2[-1/x^3 + \alpha/(x+\ell)^3]\,\Delta x \\
&\approx 2kq^2[-1/(0.1878\ell)^3 + 40/(0.1878\ell + \ell)^3]\,\Delta x \\
&\approx -(254kq^2/\ell^3)\Delta x \\
&= -k_{\text{effective}}\,\Delta x.
\end{aligned}$$

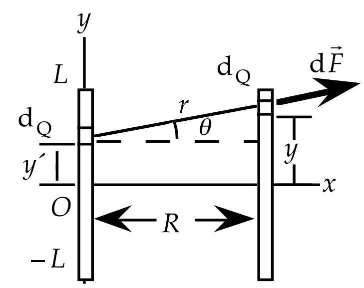
Thus the effective force constant is $254kq^2/\ell^3$, and the frequency of oscillations is

$f = (1/2\pi)(k_{\text{effective}}/m)^{1/2} = \boxed{(1/2\pi)(254kq^2/\ell^3 m)^{1/2}}$.

69. From the analogy with the gravitational force, we know that if we replace distribution 2 with a point charge q_2, the force exerted on q_2 by distribution 1 is the same as if distribution 1 were a point charge. From Newton's third law, the force on distribution 1 by q_2 is the reaction to the force on q_2, thus distribution 1 can be treated as a point charge when there is an external point charge.
Similarly, if we replace distribution 1 with a point charge q_1, the force exerted on q_1 by distribution 2 is the same as if distribution 2 were a point charge. From Newton's third law, the force on distribution 2 by q_1 is the reaction to the force on q_1, thus distribution 2 can be treated as a point charge when there is an external point charge. Thus we can simultaneously treat both spherically symmetric distributions as point charges to find the force between them.

70. We find the force between the two rods by choosing a differential element for each rod, as shown in the diagram. The charge density of each rod is $\lambda = Q/2L$, so we have $dQ = (Q/2L)\,dy$ and $dQ' = (Q/2L)\,dy'$. These two elements are equivalent to two point charges, so the force between them is

$$dF = \frac{k\,dQ\,dQ'}{r^2} \text{ repulsion.}$$



From symmetry, the force between the two rods must be perpendicular to the rods. We need to add (integrate) only the x-component

$$F = \int dF_x = \iint \frac{k\,dQ\,dQ'}{r^2}\cos\theta$$

$$= \int_{-L}^{L}\int_{-L}^{L} k\left(\frac{Q}{2L}\right)^2 \frac{dy\,dy'}{r^2}\left(\frac{R}{r}\right) = \frac{kQ^2R}{4L^2}\int_{-L}^{L}\int_{-L}^{L}\frac{dy\,dy'}{\left[(y-y')^2+R^2\right]^{3/2}} \text{ repulsion.}$$

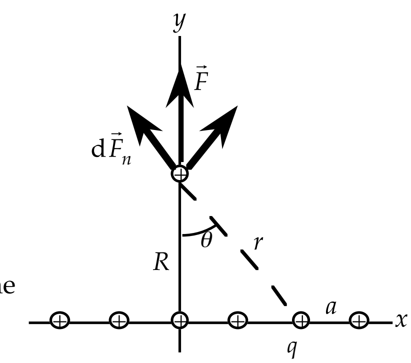
If $R >> L$, to each rod the other rod will be equivalent to a point charge, so $\boxed{F = kQ^2/R^2 \text{ repulsion}}$.

71. (a) If we consider a pair of charges equidistant from $x = 0$, we see that the x-component of the net force from the pair is zero. Thus the total force from the line of charges will be in the y-direction. We need to add only the y-components of the forces. This component from the nth charge is

$$F_{ny} = (kqQ/r^2)\cos\theta = (kqQ/r^2)(R/r)$$
$$= kqQR/r^3 = kqQR/[(na)^2 + R^2]^{3/2}.$$

The total force from all charges is

$$\vec{F} = \sum_{n=-\infty}^{\infty} F_{ny}\hat{j} = kqQR\sum_{n=-\infty}\left\{\frac{1}{\left[(na)^2+R^2\right]^{3/2}}\right\}\hat{j}.$$



(b) When $a \to 0$ and $q \to 0$ such that $q/a \to \lambda$, the distribution becomes a line charge. Each charge becomes dq; the location of the nth charge, na, becomes x; and the separation of charges, a, becomes dx. The summation becomes an integral:

$$\vec{F} = k\left(\frac{q}{a}\right)QR\sum_{n=-\infty}^{\infty}\left\{\frac{a}{\left[(na)^2+R^2\right]^{3/2}}\right\}\hat{j};$$

$$\vec{F} = k\lambda QR\int_{-\infty}^{\infty}\frac{dx}{\left(x^2+R^2\right)^{3/2}}\hat{j}.$$

We scale the integral by making the substitutions $x = uR$ and $dx = R\,du$:

$$\vec{F} = k\lambda QR\int_{-\infty}^{\infty}\frac{R\,du}{\left[(uR)^2+R^2\right]^{3/2}}\hat{j} = \frac{k\lambda Q}{R}\int_{-\infty}^{\infty}\frac{du}{\left(u^2+1\right)^{3/2}}\hat{j}.$$

Since u is dimensionless, so is the last integral. From the factor in front we see that $\vec{F}$ varies as $1/R$.

CHAPTER 22 Electric Field

Answers to Understanding the Concepts Questions

1. The moving truck picks up electric charge as it moves. Rubber tires are good insulators; the charge will not automatically flow to the ground. There is danger that when enough charge builds up, a breakdown can occur with the formation of a spark, and such a spark is extremely dangerous when gasoline is present. For just this reason, tires are made today with materials that conduct well, and a dragging chain is no longer necessary.

2. The direction of the electric field is tangent to the electric field lines. If two field lines intersect then there are two possible tangents at the intersection, and yet the actual electric field can point only at one direction.

3. The introduction of a gravitational field $\vec{g} = \vec{F}_g/m$ is indeed useful for the same reasons that the introduction of an electric field is useful. The field resembles that of the electric field in that in the absence of matter (charge) the field lines are continuous, and their density represents the strength of the field. It differs in that mass comes in only one sign: gravity is a uniquely *attractive* force, so that field lines have only one end on matter. The other end must be at infinity, since there is no mass of opposite sign for the line to attach to. In other words, there is no analogue of an overall neutral charge distribution, in which lines start in part of the distribution and end elsewhere.

4. When placed against a metal wall, the excess charge on the balloon would induce a buildup of opposite charge on the wall, causing the balloon to be attracted to it. When placed against an insulating wall the excess charges on the balloon would polarize the molecules on the wall, and the resulting attractive force would also make the balloon stick to the wall.

5. The electric field lines emerge from positive charges and end up at negative charges. The density of the field lines indicates the strength (magnitude) of the field, which is proportional to the charge that produces the field. Since five times the field lines leave one charge (q_1) as end up at the other (q_2), q_1 is positive and q_2 is negative, and $q_1/q_2 = -5$.

6. Not really, since we may think of a negative charge $-Q$ distributed uniformly over a spherical surface at infinity to accompany our single positive point charge Q. The charge density everywhere is zero, so that this depiction has no practical consequence other than the satisfying notion that the universe involving single charges is still electrically neutral.

7. To the left of q_1 the net force is to the left (nonzero), and in between q_2 and q_3 the net force is to the right. There are two points where E = 0, one is in between q_1 and q_2 (-2 cm $< x <$ +4 cm), and the other is somewhere to the right of q_3 ($x > 10$ cm).

8. The magnitude of the field of a dipole decreases with the distance r between the center of the dipole and the point of interest. In fact it can be shown that $E(r)$ goes like $1/r^3$. While $E(r)$ is nonzero for any finite r, as r approaches infinity $E(r)$ approaches zero.

9. We know that because the electric field above Earth's surface points downward, toward Earth.

10. In order to visualize a sphere with an induced dipole moment, think of the induction of a positive charge $+Q$ at the sphere's north pole and a negative charge $-Q$ at the south pole in response to an external field oriented along the north-south axis. Suppose that we now suddenly change the external field so that it is now perpendicular to the north-south axis. The charges will move in response so that now the dipole moment is oriented along the direction of the new external field. But since these charges are not attached to the conductor — they move freely on the conducting surface — their motion does not induce a rotation of the sphere. With a long rod, the situation is different. Even if the charges are free to move within the conductor, the shape of the conductor itself restricts the movement of the charges. Thus there will be equal and opposite forces on the two ends, tending to rotate the rod. At the same time, the original inducing field is now gone, and the charges rush back to each other under the influence of the coulomb forces between them. Whether there is an actual motion of the rod depends on how rapidly the charges move back together compared to how rapidly the new field acts.

11. The net electric charge present on the comb causes the molecules in the paper to polarize; so the region that's closer to the comb has a net charge that's opposite in sign to that of the comb, resulting in a net attraction.

12. In principle, yes it can. The field lines of an electrostatic field can only begin and end where there is a charge present. For example, suppose there is an isolated positive charge from which field lines emerge. These field lines will not end unless there is a negative charge. If no other charges are present the field lines will extend to infinity, even though the density of the field lines becomes zero now that they are spread out infinitely far from each other, meaning that the magnitude of the electric field there is zero.

13. The total charge of the water molecule is zero. The charge distribution shown in the figure suggests that the electric field is that of two dipoles touching at one end. The superposition of two electric dipole fields is again an electric dipole field (they both fall as $1/r^3$), except under very special circumstances in which there is a cancellation, so that only the $1/r^4$ terms are left. This is not the case here.

14. A small hole can be drilled on the negatively charged receptor plate to allow protons to pass through.

15. The density of the field lines (number of lines per unit cross-sectional area) represents the magnitude of the electric field. Suppose there are a total of N field lines which emerge from a positive charge. A distance r from the charge, these field lines are evenly distributed over a spherical surface of radius r, so the density of the field lines there is $N/A = N/4\pi r^2$, which is proportional to $1/r^2$, an accurate representation of the r-dependency of the magnitude of the electric field. If the electric field changes with r in any other power, then the field line density, which always goes like $1/r^2$, would no longer represent the magnitude of the field.

16. The velocity field has features common to the electric field. Sources (like faucets) correspond to positive charges, and sinks (like drains) correspond to negative charges. The velocity field is represented by a vector at every point in space, just like the electric field. The major difference is that in a liquid there is something that actually moves along the lines (look back at Chapter 16), whereas the electric field lines do not represent motion except in the sense that a test charge would accelerate along the tangent of a field line. The electric field is thus more like an "acceleration field," something which is of little interest in the study of fluids.

17. An electric field line can only end up at a negative charge. It is therefore impossible to construct such an arrangement for the electric field lines to be directed into a point where no charge is present.

18. From the figure we can see that the forces will align the small dipole in such a way that the attraction is maximized, or such that the potential energy is minimized. In other words, the small dipole will align its electric dipole moment to be antiparallel to the large fixed dipole's electric dipole moment.

19. This configuration can indeed be thought of as two electric dipoles of equal strength pointing at opposite directions. It is an example of an electric *quadrupole*. The field produced by this setup is not exact zero. This is because the two dipoles, while equal in magnitude and opposite in orientation, are slightly displaced from one another so their fields do not completely cancel out, even though the net field does drop rapidly as the distance r from the origin (as $1/r^4$, as can be shown).

20. The positive charges from these dipoles form an infinite, uniform sheet of charge, while the negative charges form its own sheet, parallel to the first one, with exactly the opposite density of charge. The net field is the superposition of those from the two sheets, so it must be zero.

21. Both the force of gravity and the electrical force are independent of the height. If the gravitational force of attraction is stronger than the repulsive force, the pellet will fall down, albeit with an acceleration smaller than that due to gravity. If the repulsive force is stronger, then the particle will accelerate away from the plate, and go upwards.

22. Two forces are exerted on the pellet: the electrostatic repulsive force from the sphere, up; and the gravitational force, down. The motion of the pellet depends on the relative strengths of these two forces. If the charge on the sphere is relatively weak then the electrostatic repulsion cannot prevent the pellet from colliding with the top of the sphere. If the electric repulsion is relatively strong then the pellet will not be able to reach the surface of the sphere. Rather, it is accelerated (by gravity) toward the sphere until it reaches the equilibrium position (where the electrostatic repulsion is equal to its weight), then decelerates while continuing to move downward toward the sphere. Eventually it comes to a momentary stop due to the strong repulsion of the sphere, and then starts to move back up, accelerating towards (and past) the equilibrium point before decelerating to a stop. Afterwards the motion repeats itself, with the pellet oscillating up and down above the sphere, reaching the greatest speed upon passing the equilibrium position.

23. The electric field at the origin is now dominated by the charge q_1 located at $x_1 = -1$ mm, as q_1 is so much closer to the origin than any other charge. As a good approximation we may just calculate the field due to q_1 and neglect those due to q_1 and q_2.

24. When the distance between a point and the surface of a sphere is much less than the radius of the sphere, the sphere can be approximated as a plane from the perspective of that point. Such is the case of Example 22-10.

Solutions to Problems

1. The displacement $\vec{r}$ from the charge is shown in the diagram. We find its magnitude from

$$r^2 = x^2 + y^2$$
$$= (3\text{ cm})^2 + (4\text{ cm})^2, \text{ which gives } r = 5\text{ cm}.$$

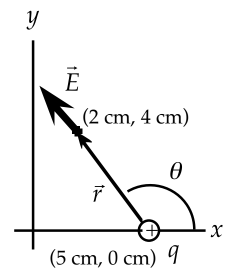
We find its direction from

$$\tan\theta = y/x$$
$$= (4\text{ cm})/(-3\text{ cm}) = -1.33,$$

which gives $\theta = 127°$.

The electric field at (2 cm, 4 cm) is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$
$$= \left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5\times10^{-6}\ \text{C}\right)}{\left(5\times10^{-2}\ \text{m}\right)^2}\left(\cos\theta\ \hat{i} + \sin\theta\ \hat{j}\right)$$
$$= 1.8\times10^7\left(-0.60\hat{i} + 0.80\hat{j}\right)\text{N/C}.$$

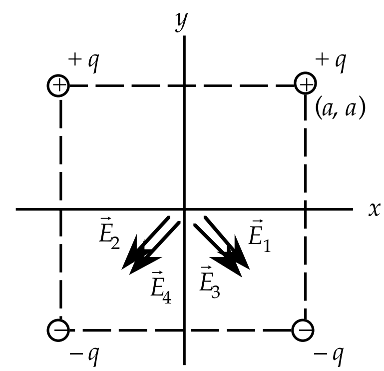


2. Because the origin is equidistant from the equal charges, the electric fields will have the same magnitude:

$$E_i = (1/4\pi\varepsilon_0)[q/(a\sqrt{2})^2] = (1/4\pi\varepsilon_0)(q/2a^2).$$

The electric fields are shown in the diagram. From the symmetry, we have

$$\vec{E} = \sum E_{iy}\ \hat{j} = -4[(1/4\pi\varepsilon_0)(q/2a^2)]\cos 45°\ \hat{j}$$
$$= \boxed{-(1/4\pi\varepsilon_0)(q\sqrt{2}/a^2)\ \hat{j}}.$$



3. Because we can treat the nucleus as a point charge, the field will be radial:

$$\vec{E} = (1/4\pi\varepsilon_0)(q/r^2)$$
$$= [(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(79)(1.60\times10^{-19}\ \text{C})/(1\times10^{-9}\ \text{m})^2] = \boxed{(1.14\times10^{11}\ \text{N/C})}.$$

The force on an electron is

$$\vec{F} = q\vec{E} = -e\vec{E}$$
$$= -(1.60\times10^{-19}\ \text{C})(1.14\times10^{11}\ \text{N/C}) = \boxed{-(1.82\times10^{-8}\ \text{N})\ \ (\text{toward the nucleus})}.$$

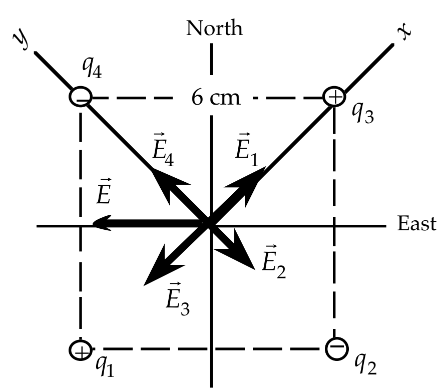
4. Because the electric field from each of the charges is along a diagonal of the square, we choose the xy-coordinate system in the following way: the direction from -2μC to -5μC is the positive x-axis (east),
and the direction from $+7\ \mu$C to $+3\mu$C is positive y-axis (north).
We have

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$
$$= (k/r^2)(5\mu\text{C} - 2\mu\text{C})\hat{i} + (k/r^2)(7\mu\text{C} - 3\mu\text{C})\hat{j},$$

where $r = a/\sqrt{2}$, $a = 0.040$ m, and $k = 9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

The result is

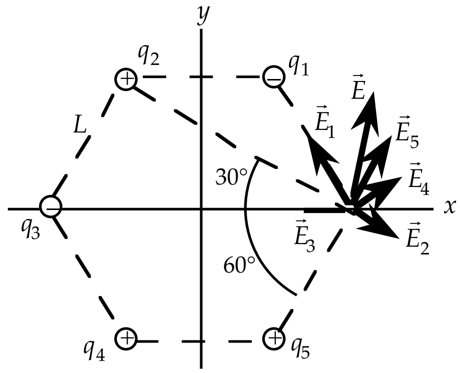
$$\vec{E} = [(3.4\times10^7\hat{i}) + 4.5\times10^7\hat{j})]\ \text{N/C}$$
$$= 5.6\times10^6\ \text{N/C},\ 53° \text{ above the } x\text{-axis, i.e.,}$$
$$\vec{E} = \boxed{5.6\times10^6\ \text{N/C},\ 53° \text{ north of east}}.$$



5. For a regular hexagon, we have the angles shown. The edge is $L = 10$ cm. For the distances from the charges we have

   $r_1 = r_5 = L = 10$ cm; $r_2 = r_4 = 2L \cos 30° = 17.3$ cm; $r_3 = 2L = 20$ cm.

   We take advantage of the symmetry of the charges to simplify the vector addition of the individual fields. Because $q_1 = -q_5$, and $r_1 = r_5$, the magnitudes of $\vec{E}_1$ and $\vec{E}_5$ will be equal. Their resultant will be in the y-direction:



$$\vec{E}_1 + \vec{E}_5 = 2[(1/4\pi\varepsilon_0)q_1/r_1^2] \sin 60° \hat{j}$$
$$= 2[(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-6} \text{ C})/(0.10 \text{ m})^2] \sin 60° \hat{j}$$
$$= (3.12 \times 10^6 \text{ N/C})\hat{j}.$$

   Because $q_2 = q_4$, and $r_2 = r_4$, the magnitudes of $\vec{E}_2$ and $\vec{E}_4$ will be equal. Their resultant will be in the x-direction:

$$\vec{E}_2 + \vec{E}_4 = 2[(1/4\pi\varepsilon_0)q_2/r_2^2] \cos 30° \hat{i}$$
$$= 2[(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3 \times 10^{-6} \text{ C})/(0.173 \text{ m})^2] \cos 30° \hat{i}$$
$$= (1.56 \times 10^6 \text{ N/C})\hat{i}.$$

   For $\vec{E}_3$ we have

$$\vec{E}_3 = -[(1/4\pi\varepsilon_0)q_3/r_3^2]\hat{i}$$
$$= -[(9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4 \times 10^{-6} \text{ C})/(0.20 \text{ m})^2]\hat{i} = -(0.90 \times 10^6 \text{ N/C})\hat{i}.$$

   The resultant electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 = [(0.66 \times 10^6 \hat{i}) + 3.12 \times 10^6 \hat{j})] \text{ N/C}$$
$$= \boxed{3.19 \times 10^6 \text{ N/C, } 78° \text{ above the } +x\text{-axis}}.$$

6. (a) The electric field of q_1 will be away from q_1 with a magnitude

$$E_1 = (1/4\pi\varepsilon_0)(q_1/r_1^2)$$
$$= (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.5 \times 10^{-6} \text{ C})/(0.22 \text{ m})^2$$
$$= \boxed{2.8 \times 10^5 \text{ N/C}} \text{ away from } q_1.$$

   (b) This field produces an attractive force on q_2:

$$F_2 = q_2E_1 = (3.5 \times 10^{-6} \text{ C})(2.8 \times 10^5 \text{ N/C}) = 0.98 \text{ N toward } q_1.$$

   (c) At the midpoint, both fields will be toward q_2. The resultant field is

$$E_1 + E_2 = [(1/4\pi\varepsilon_0)q_1/r_1^2] + [(1/4\pi\varepsilon_0)q_2/r_2^2]$$
$$= (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\{[(1.5\times 10^{-6} \text{ C})/(0.11 \text{ m})^2] + [(3.5 \times 10^{-6} \text{ C})/(0.11 \text{ m})^2]\}$$
$$= \boxed{3.7 \times 10^6 \text{ N/C}} \text{ toward } q_2.$$

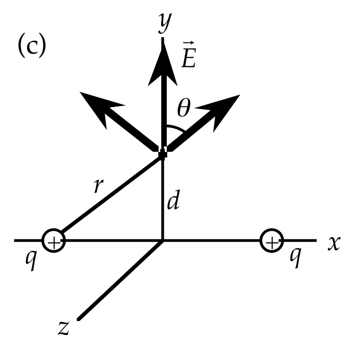
7. (a) With the charges on the x-axis, the electric fields produced by the charges will have the same magnitude and point in the $-x$-direction. The resultant field will be

$$\vec{E} = 2(1/4\pi\varepsilon_0)[q/(\ell/2)]^2 (-\hat{i}) = -(1/4\pi\varepsilon_0)(8q/\ell^2)\hat{i}.$$

   (b) The fields produced by the charges will have the same magnitude and point in opposite directions. The resultant field will be $\vec{E} = \boxed{0}$.

   (c) We take a representative point on the y-axis. From the diagram, we see that the electric fields produced by the charges will have the same magnitude, and the resultant field will point away from the origin. If we call the distance from the origin d, we have

(c)



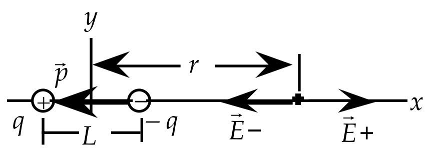
$$E = 2(1/4\pi\varepsilon_0)(q/r^2) \cos \theta = (1/4\pi\varepsilon_0)(2q/r^2)(d/r)$$
$$= (1/4\pi\varepsilon_0)\{2qd/[d^2 + (\ell/2)^2]^{3/2}\}.$$

   From the symmetry in the yz-plane, at a point d from the origin we have

$$E = \boxed{(1/4\pi\varepsilon_0)\{2qd/[d^2 + (\ell/2)^2]^{3/2}\} \text{ away from the origin}}.$$

8. (a) The electric fields produced by the charges will have the same magnitude and point in opposite directions. The resultant field will be $\vec{E} = \boxed{0}$.
(b) Because there is no field at $x = 0$, there will be no force on the test charge. If we displace the test charge a small distance δ away from the x-axis, the two charges will produce a resultant field that will point away from the origin. This is similar to the situation shown in the diagram for Problem 7. Thus there will be a force on the test charge, $\vec{F} = q_0\vec{E}$, that will point away from the origin. The equilibrium will be $\boxed{\text{unstable}}$.

9.



From the diagram, we see that the resultant electric field is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r + (L/2)\right]^2}\hat{i} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\left[r - (L/2)\right]^2}\hat{i}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{\left[r + (L/2)\right]^2} - \frac{1}{\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= \frac{q}{4\pi}\left\{\frac{\left[r - (L/2)\right]^2 - \left[r + (L/2)\right]^2}{\left[r + (L/2)\right]^2\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{-2rL}{\left[r + (L/2)\right]^2\left[r - (L/2)\right]^2}\right\}\hat{i}$$

$$= -\frac{2qL}{4\pi\varepsilon_0 r^3}\left\{\frac{1}{\left[1 + (L/2r)\right]^2\left[1 - (L/2r)\right]^2}\right\}\hat{i}.$$

We express this in terms of the dipole moment:

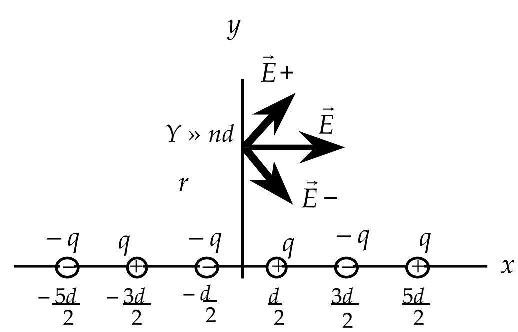
$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}\left\{\frac{1}{\left[1 + (L/2r)\right]^2\left[1 - (L/2r)\right]^2}\right\}.$$

When $r >> L$, the electric field along the axis of the dipole far from the dipole becomes

$$\boxed{\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}}.$$

10. We treat the line of charges as n pairs symmetrically placed about the y-axis. From the diagram, we see that a pair of charges produces an electric field parallel to the x-axis. For a pair with $r^2 = Y^2 + x^2$, we add the x-components to get the magnitude of the field:

$$E = 2(1/4\pi\varepsilon_0)(q/r^2)(x/r) = 2qx/4\pi\varepsilon_0(Y^2 + x^2)^{3/2}.$$



For all pairs, we have $Y >> x$, so we get

$$E \simeq 2qx/4\pi\varepsilon_0 Y^3.$$

Because the pairs alternate in sign, the direction of E will alternate. The electric field of the ith pair is

$$\vec{E}_i = [(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i}, \text{ with } i = 1, 2, 3, \ldots, n.$$

The values of x_i are $d/2$, $3d/2$, $5d/2$, …, so when we sum the n pairs, we get

$$\vec{E} = \sum\vec{E}_i = \sum[(-1)^i 2qx_i/4\pi\varepsilon_0 Y^3]\hat{i} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1 + 3 - 5 + 7 - \ldots)\hat{i}.$$

For the first few terms, the result of the summation is – 1, + 2, – 3, + 4, … . Thus the general result of the summation is $(-1)^n n$. The resultant electric field is

$$\vec{E} = (2q/4\pi\varepsilon_0 Y^3)(d/2)(-1)^n n\hat{i} = (-1)^n(qnd/4\pi\varepsilon_0 Y^3)\hat{i}.$$

11. We assume that the charge is displaced a small distance δ toward positive x. The net electric field at that point is

$$\vec{E} = (1/4\pi\varepsilon_0)[Q/(a-\delta)^2](-\hat{i}) + (1/4\pi\varepsilon_0)[Q/(a+\delta)^2]\hat{i}$$
$$= (Q/4\pi\varepsilon_0)\{-[1/(a-\delta)^2] + [1/(a+\delta)^2]\}\hat{i}.$$

With $\delta << a$, we use the approximation $1/(a+\delta)^2 \simeq (1/a^2) - (2\delta/a^3)$:

$$\vec{E} \simeq (Q/4\pi\varepsilon_0)(-1/a^2 - 2\delta/a^3 + 1/a^2 - 2\delta/a^3)\hat{i} = -(4Q\delta/4\pi\varepsilon_0 a^3)\hat{i}.$$

The force on the test charge is

$$\vec{F} = q_0\vec{E} = -(4Q\delta/4\pi\varepsilon_0 a^3)\hat{i} = -(Q/\pi\varepsilon_0 a^3)\delta\hat{i}.$$

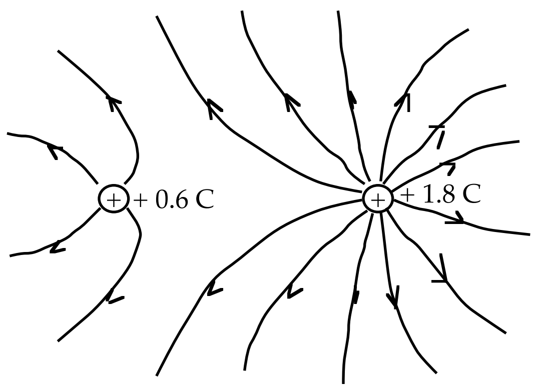
Thus the force is a restoring force, so the equilibrium is $\boxed{\text{stable}}$.

The effective force constant of the system is

$k_{\text{eff}} = Q/\pi\varepsilon_0 a^3$, so the frequency is

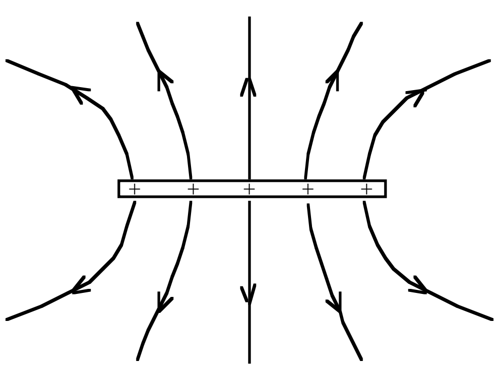
$$f = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(Qq/\pi\varepsilon_0 a^3 m)^{1/2}}.$$

12.



13. The density of field lines represents the magnitude of the electric field. Because the electric field between parallel plates depends linearly on the charge density on the plates, the density of the field lines should be $\boxed{\text{tripled}}$.

14.



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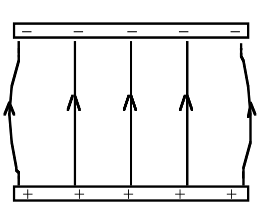

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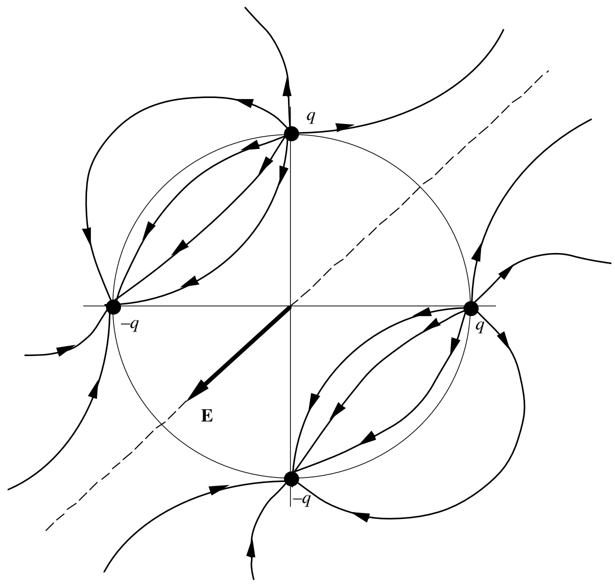
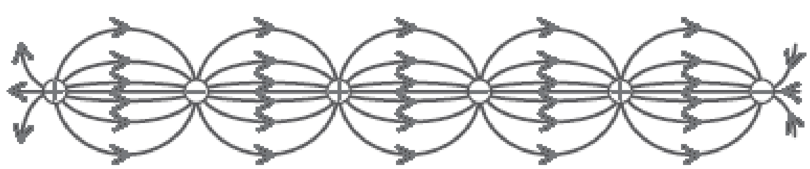


The combined electric field at the center of the turntable due to the two charges at 3- and 9-o'clock is $2kq/R^2$, pointing from the 3 o'clock position to the 9 o'clock position; while that due to the two charges at 12- and 6-o'clock is also $2kq/R^2$ in magnitude, pointing from the 12-o'clock position to the 6-o'clock position. By symmetry the net electric field is

$$E = \sqrt{2}\,(2kq/R^2)$$
$$= 2\sqrt{2}\,(9\times10^9\ \mathrm{N\cdot m^2/C^2})(8\times10^{-5}\ \mathrm{C})/(0.15\ \mathrm{m})^2 = \boxed{9.1\times10^7\ \mathrm{N/C}},$$

pointing midway between the 6- and 9-o'clock positions (i.e., toward the 7:30 position).

17.



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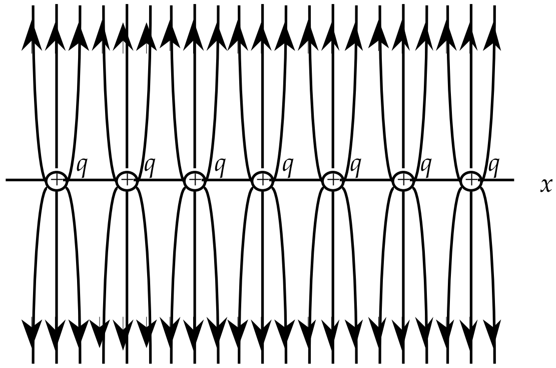

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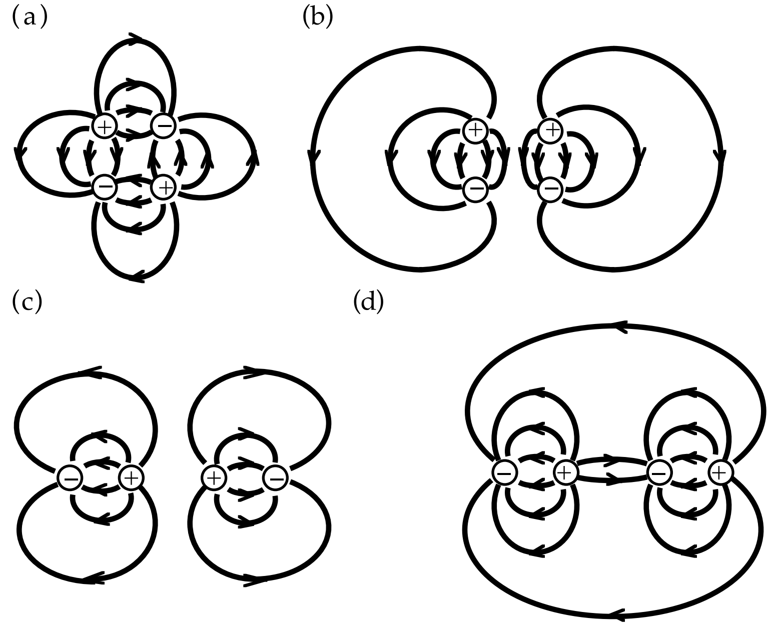


20. From the dependence of the field on $1/r^2$, close to any single charge that charge will be the major contributor to the field. At a distance of $d = 0.080$ cm from $-q$, the electric field is dominated by that charge. So we have

$$\vec{E} \approx kq/d = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q/(8.0 \times 10^{-4}\ \mathrm{m})^2$$
$$= \boxed{(1.4 \times 10^{16}\ \mathrm{N/C^2})q\ \text{ toward } -q}.$$

Now consider the field a distance $d_3 = 35$ m from $-q$. Taking the $-q$ position as the origin, east as $+x$ and north as $+y$, the positions of the two $+q$'s are:

$q_1\,(-0.060\ \mathrm{m}, -0.10\ \mathrm{m})$, and $q_2\,(0.06\ \mathrm{m}, -0.104\ \mathrm{m})$.

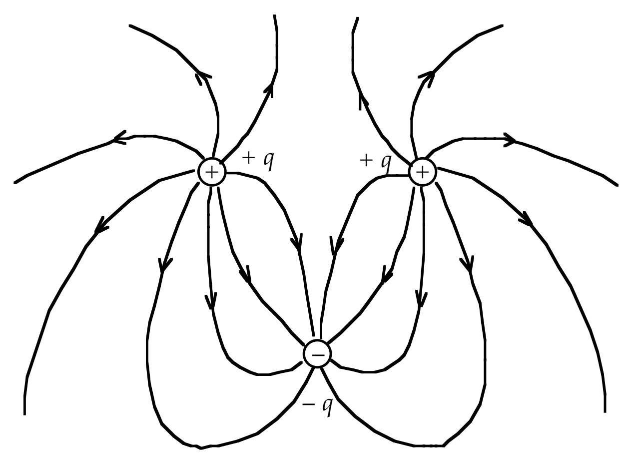
Therefore, the distance between q_1 and (0, 35 m) is

$d_1 = [(0.060\ \mathrm{m})^2 + (35.10\ \mathrm{m})^2]^{1/2} = 35.1\ \mathrm{m}$, the same as that between q_2 and (0, 35 m). We have

$E_x = 0$ (by symmetry) and

$$E_y = E_{1y} + E_{2y} + E_3 \approx 2kq/d_1^{\,2} - kq/d_3^{\,2}$$
$$= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})q[2/(35.1\ \mathrm{m})^2 - 1/(35\ \mathrm{m})^2] = (7.3 \times 10^6\ \mathrm{N/C^2})q.$$ Thus

$$\vec{E} = \boxed{[(7.3 \times 10^6\ \mathrm{N/C^2})q]\hat{j}}\ \text{(away from } -q\text{)}.$$



Note that this result can also be obtained by treating the system as one single net charge of $+q$, located 35 m from the point of interest. This is a good approximation since the size of the triangle is much less than 35 m.

21. We find the magnitude of the electric field from an infinitely long line of charge from

$$E = \lambda/2\pi\varepsilon_0 R$$
$$= (0.3 \times 10^{-6}\ \mathrm{C/m})(2)(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})/(20 \times 10^{-2}\ \mathrm{m})$$
$$= 2.7 \times 10^4\ \mathrm{N/C}.$$

The electric field is

$$\vec{E} = \boxed{2.7 \times 10^4\ \mathrm{N/C}\ \text{perpendicular to and away from the line}}.$$

22. The linear charge density is

$$\lambda = Q/L = (6 \times 10^{-6}\text{ C})/(0.25\text{ m}) = 2.4 \times 10^{-5}\text{ C/m}.$$

On the x-axis (6 cm, 0 cm, 0 cm) the electric field is

$$dE_x = (k\lambda dz/z^2 + x^2)[x/(z^2 + x^2)^{1/2}] = k\lambda\, x\, dz/(z^2 + x^2)^{3/2}$$

$$E_x = 2k\lambda x \int dz/(z^2 + x^2)^{3/2} \quad \text{(where } z \text{ starts at 0, ends at } 0.25/2\text{ m)}$$

$$= 2k\lambda\,(\sin\theta_f - \sin\theta_i)/x \quad \text{(where } \theta_i = \text{at } 0^\circ \text{ and } \theta_f = 64.4^\circ)$$

$$= 2(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\,(2.4 \times 10^{-5}\text{ C/m})(\sin 64.4^\circ - 0)/0.06\text{ m}$$

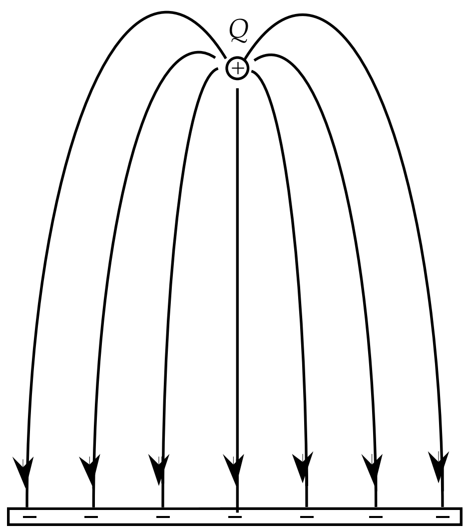
$$= (6.5 \times 10^6\text{ N/C}),\text{ so}$$

$$\vec{E}_1 = E_x\,\hat{i} = \boxed{(6.5 \times 10^6\text{ N/C})\hat{i}}.$$

At the same distance from the rod along the y-axis, at (0 cm, 6 cm, 0 cm), the electric field will have the same magnitude but will be in the y-direction:

$$\vec{E}_2 = \boxed{(6.5 \times 10^6\text{ N/C})\hat{j}}.$$

23.



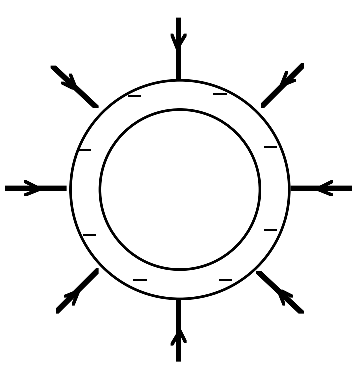
24. Each infinite plane sheet produces a uniform electric field. If we assume both charge densities are positive, between the sheets the fields will be in opposite directions:

$$E = E_1 - E_2 = (\sigma_1 - \sigma_2)/2\varepsilon_0 \text{ away from the first sheet, independent of } L.$$

For the force on charge Q we have

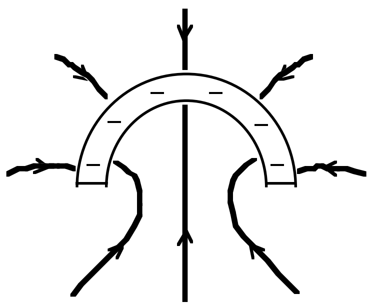
$$F = QE = \boxed{Q(\sigma_1 - \sigma_2)/2\varepsilon_0 \text{ away from the first sheet}}.$$

25.



26. (a) At a point in the plane of the circle outside the circle, the charge distribution will appear as a line. There can be no preference toward either side of the line, so the direction of the electric field must be in the plane of the circle. If the point is moved around the circle, the charge distribution does not change, so the electric field is directed radially out from the center.
    (b) From symmetry, the field along the axis of the circle is directed along the axis. At a distance L, with $L >> R$, the charge appears to be a point charge, so we have
    $E_{\text{axis}} = (1/4\pi\varepsilon_0)(Q/L^2) = (1/4\pi\varepsilon_0)(2\pi R\lambda/L^2) = \boxed{(R\lambda/2\varepsilon_0 L^2),\ L >> R}$.

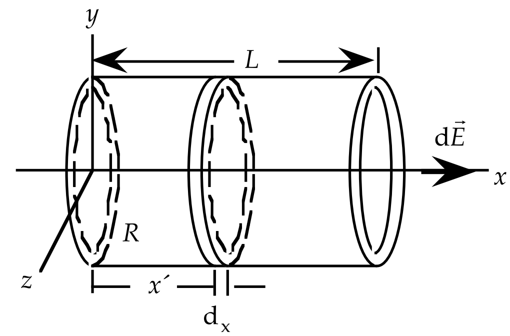
27.



28. Each plate produces an electric field parallel to the x-axis and away from the plate with a magnitude
    $E = \sigma/2\varepsilon_0$.
    (a) Between the plates, the fields from the two plates are in opposite directions, so we have
    $E_{0,0,0} = (\sigma/2\varepsilon_0) - (\sigma/2\varepsilon_0) = \boxed{0}$.
    (b) Outside the two plates, the fields from the two plates are in the same direction, so we have
    $\vec{E}_{8,0,0} = (\sigma/2\varepsilon_0)\,\hat{i} + (\sigma/2\varepsilon_0)\,\hat{i} = (\sigma/\varepsilon_0)\,\hat{i}$
    $= [(1.2 \times 10^{-6}\,\text{C/m}^2)/(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)]\,\hat{i}$
    $= \boxed{(1.4 \times 10^5\,\text{N/C})\hat{i}}$.
    (c) The field outside the plates is independent of y and z, so we have
    $\vec{E}_{8,1,2} = \boxed{(1.4 \times 10^5\,\text{N/C})\hat{i}}$.

29. We assume that the plates are large enough that they may be considered infinite plates. Each plate produces an electric field perpendicular to and away from the plate with a magnitude
    $E = \sigma/2\varepsilon_0$.
    Outside the two plates, the fields from the two plates are in the same direction, so we have
    $E = (\sigma/2\varepsilon_0) + (\sigma/2\varepsilon_0)$
    $= \boxed{\sigma/\varepsilon_0 \text{ perpendicular to the plates and away from them}}$.
    Between the plates, the fields from the two plates are in opposite directions, so we have
    $E = (\sigma/2\varepsilon_0) - (\sigma/2\varepsilon_0) = \boxed{0}$.

30. Because we know the electric field for a hoop, we choose a circular segment of length dx' as the differential element. The charge on this segment is $dq = (q/L)\,dx'$. The field of this element at a point x on the x-axis will be in the $+x$-direction, with a magnitude



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(x-x')\,dq}{\left[R^2+(x-x')^2\right]^{3/2}}$$

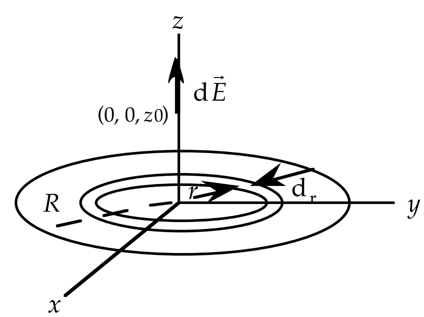
$$= \frac{q}{4\pi\varepsilon_0 L}\frac{(x-x')\,dx'}{\left[R^2+(x-x')^2\right]^{3/2}}.$$

We find the total field by integrating over the length of the tube:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 L}\int \frac{(x-x')\,dx'}{\left[R^2+(x-x')^2\right]^{3/2}}\hat{i} = \frac{q}{4\pi\varepsilon_0 L}\left\{\frac{1}{\left[R^2+(x-x')^2\right]^{1/2}}\right\}\hat{i}\,\Bigg|_0^L$$

$$= \frac{q}{4\pi\varepsilon_0 L}\left\{\frac{1}{\left[R^2+(x-L)^2\right]^{1/2}} - \frac{1}{\left[R^2+x^2\right]^{1/2}}\right\}\hat{i}\,.$$

$\boxed{31}$. (a) From the symmetry of the charge distribution, we know that the electric field on the z-axis is along the z-axis. For a differential element we choose a ring of radius r and thickness dr. The charge on the ring is $dq = (Q/\pi R^2)2\pi r\,dr = (2Qr\,dr)/R^2$. Using the result for the field of a hoop of charge, we integrate over the disk:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{z_0\,dq}{\left(r^2+z_0^2\right)^{3/2}}\hat{k} = \frac{z_0 2Q}{4\pi\varepsilon_0 R^2}\int_0^R \frac{r\,dr}{\left(r^2+z_0^2\right)^{3/2}}\hat{k}$$

$$= \frac{z_0 Q}{2\pi\varepsilon_0 R^2}\left[\frac{-1}{\left(r^2+z_0^2\right)^{1/2}}\right]\Bigg|_0^R \hat{k}$$

$$= \frac{z_0 Q}{2\pi\varepsilon_0 R^2}\left[\frac{1}{z_0} - \frac{1}{\left(R^2+z_0^2\right)^{1/2}}\right]\hat{k}.$$

(b) To find the field in the limit $z_0 \to \infty$, we rearrange and use the approximation $(1+x)^{-1/2} \simeq 1-(x/2)$:

$$\vec{E} = \frac{Q}{2\pi\varepsilon_0 R^2}\left\{1 - \left[1+\left(\frac{R}{z_0}\right)^2\right]^{-1/2}\right\}\hat{k} = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - 1 + \frac{1}{2}\left(\frac{R}{z_0}\right)^2\right]\hat{k}$$

$$= \frac{Q}{4\pi\varepsilon_0 z_0^2}\hat{k}.$$

As we expect, the field is that of a point charge.

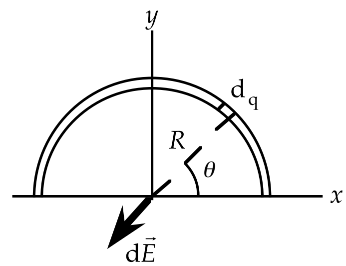
(c) To find the field in the limit $R \to \infty$, we consider the result from part (a). The second term will go to zero, so we have $\vec{E} = Q/2\pi\varepsilon_0 R^2\hat{k}$.

The charge density of the disk is $\sigma = Q/\pi R^2$, so we can write $\vec{E} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}$.

As we expect, the field is that of an infinite plane.

The limits of parts (b) and (c) are not the same. Part (b) is equivalent to the disk being a point charge, while part (c) is equivalent to being very close to the disk.

32. Let the length of the rod be L, then the radius of the semicircle is $R = L/\pi$. Because the charge distribution is symmetric about the y-axis, we know that the electric field at the center will be directed along the $-y$-axis. We choose a differential element of the rod at an angle θ with charge



$$dq = Q\,(d\theta/\pi).$$

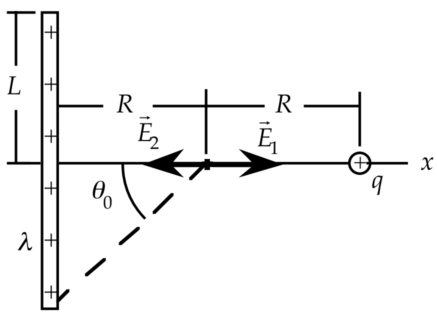
We find the total field at the center by integrating the y-components over the rod:

$$dE_y = -(k\,dq\sin\theta/R^2) = -k[(Q/\pi)\,d\theta]\sin\theta/(L/\pi)^2$$
$$= -(\pi kQ/L^2)\sin\theta\,d\theta;$$
$$E_y = -(\pi kQ/L^2)\int\sin\theta\,d\theta,$$ where θ starts at 0° and ends at 180°.

Thus

$$\vec{E} = E_y\hat{j} = (-2\pi kQ/L^2)\hat{j} = [-2\pi\,(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.36\times10^{-6}\ \text{C})/(0.18\ \text{m})^2]\hat{j}$$
$$= \boxed{-(6.3\times10^5\ \text{N/C})\hat{j}}.$$

33.



We find the electric field from the vector sum of the field of a rod and the field of a point charge:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$= (\lambda/2\pi\varepsilon_0 R)\sin\theta_0\,\hat{i} - (q/4\pi\varepsilon_0 R^2)\,\hat{i}$$
$$= (1/4\pi\varepsilon_0)[(2\lambda\sin\theta_0)/R - q/R^2]\,\hat{i}.$$

The angle for the endpoint of the rod is

$$\theta_0 = \tan^{-1}(L/R) = \tan^{-1}(1) = 45^\circ.$$

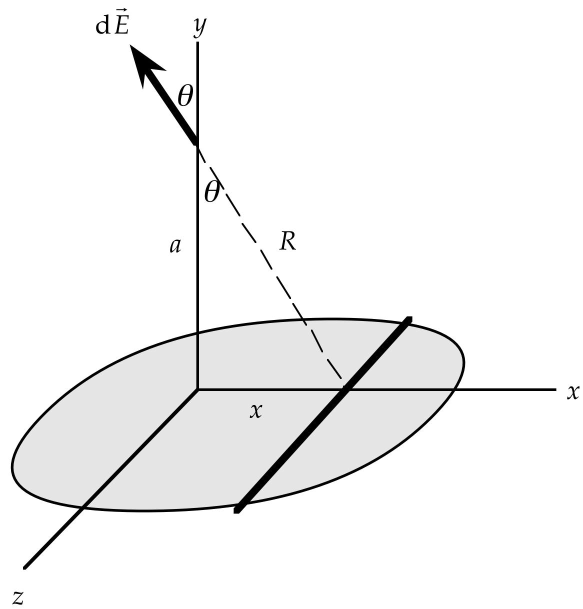
The magnitude of the field is

$$E = (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)[2(15\times10^{-6}\ \text{C/m})(\sin 45^\circ)/(0.15\ \text{m}) - (3\times10^{-6}\ \text{C})/(0.15\ \text{m})^2]$$
$$= 7.3\times10^4\ \text{N/C}.$$

The resultant field is

$$\vec{E} = \boxed{7.3\times10^4\ \text{N/C}\ \ \text{toward the point charge}}.$$

34.



Imagine that the infinitely large sheet is made of infinitely long rods running in the z-direction in the xz plane. Consider one such rod, of width dx, a distance x from the z-axis. The charge per unit length of the rod is $\lambda = dq/(\text{length of the rod}) = \sigma\, dx$. According to the result of Example 22-7, with the length of the rod approaching infinity, the magnitude of the electric field dE due to the rod at a point on the y-axis a distance a from the charged sheet is

$$dE = \lambda/2\pi\varepsilon_0 R = \sigma\, dx/2\pi\varepsilon_0 R.$$

By symmetry, the net field at this point is along the y-axis, so we only need to consider the y-component of $d\vec{E}$:

$$dE_y = dE\cos\theta = (\lambda/2\pi\varepsilon_0 R)(a/R)$$
$$= \sigma a\, dx/2\pi\varepsilon_0 R^2 = \sigma a\, dx/2\pi\varepsilon_0(x^2 + a^2).$$

Integrate over the entire range of x:

$E = \int dE_y = (\sigma a/2\pi\varepsilon_0)\int dx/(x^2 + a^2)$, where x ranges from $-\infty$ to $+\infty$.

The integral in the last step is equal to $(1/a)\tan^{-1}(x/a)$, and with the upper- and lower-limits given above it yields π/a. Thus

$E = (\sigma a/2\pi\varepsilon_0)(\pi/a) = \sigma/2\varepsilon_0$, as expected.

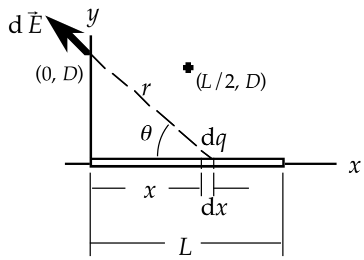
35. Since the electric field at the radius $(R + h)$ vanishes, and the charge distribution within the radius is spherically symmetrical, the total charge enclosed within the radius must be zero. This means that the charge on the shell of thickness h must be $+Q$. Since $R >> h$ the shell is very thin, so its radius is nearly uniform, and is approximately equal to R. The volume of the shell is then

$$V \approx 4\pi R^2 h.$$

The charge density on the shell follows as

$$\rho = Q/V \approx \boxed{Q/4\pi R^2 h}.$$

36.



To find the electric field at the point (0, D), we choose a differential element of the rod, as shown in the diagram. The charge of this element is $dq = (Q/L)\,dx$. We find the field produced by the element, which has both x- and y-components, by integrating along the rod:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=L} \frac{dq}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_{x=0}^{x=L} \frac{dx}{r^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right).$$

To perform the integration, we must eliminate variables until we have one, for which we choose θ.

From the diagram we see that $r = D/\sin\theta$, and $x = D\cot\theta$. This gives

$$dx = -D\csc^2\theta\,d\theta = -(D\,d\theta)/\sin^2\theta.$$

The limits for θ are $\pi/2$ rad to $\theta_0 = \cos^{-1}[L/(D^2 + L^2)]$. When we make these substitutions, we have

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 L}\int_{\pi/2}^{\theta_0} \frac{(-d\theta)/\sin^2\theta}{(D/\sin\theta)^2}\left(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\int_{\pi/2}^{\theta_0} d\theta\left(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 LD}\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)\Big|_{\pi/2}^{\theta_0}$$

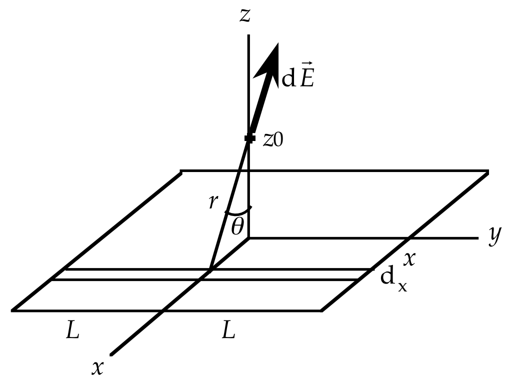
$$= \frac{Q}{4\pi\varepsilon_0 LD}\left[(\sin\theta_0 - 1)\,\hat{i} + (\cos\theta_0 - 0)\,\hat{j}\right];$$

$$\vec{E}(0, D) = \frac{Q}{4\pi\varepsilon_0 LD}\left[\left(\frac{D}{\sqrt{D^2 + L^2}} - 1\right)\hat{i} + \left(\frac{L}{\sqrt{D^2 + L^2}}\right)\hat{j}\right].$$

Because the point $(L/2, D)$ is opposite the midpoint of the rod, we know that the field there will have only a y-component. Instead of doing another integration, we use the result from the text:

$$\vec{E}(L/2, D) = \frac{2\lambda}{4\pi\varepsilon_0 D}\left[\frac{L/2}{\sqrt{D^2 + (L/2)^2}}\right]\hat{j} = \frac{Q}{4\pi\varepsilon_0 D}\left(\frac{2}{\sqrt{4D^2 + L^2}}\right)\hat{j}.$$

37. (a) We choose a strip of the square parallel to the y-axis, so that we can use the result for the electric field of a rod. The strip has length $2L$ and thickness dx. The charge of the strip is d$q = \sigma 2L$ dx, which gives a linear charge density $\lambda = \sigma$ dx.



The magnitude of the field produced by the strip is

$$dE = \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}}.$$

From the diagram, we see that the charge distribution is symmetric about the y-axis. The resultant field will be in the z-direction, so we integrate the z-components for the region from $x = -L$ to $x = L$.

From the diagram, we have $\cos\theta = z_0/r$.

$$\vec{E} = \int_{x=-L}^{x=L} \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}}\cos\theta\,\hat{k} = \frac{\sigma L z_0}{2\pi\varepsilon_0}\int_{x=-L}^{x=L}\frac{dx}{r^2 L^2 + r^2}\,\hat{k}$$

$$= \frac{\sigma L z_0}{2\pi\varepsilon_0}\,\hat{k}\int_{x=-L}^{x=L}\frac{dx}{(x^2 + z_0^2)\sqrt{L^2 + x^2 + z_0^2}}.$$

(b) We can simplify the integral by doubling the z-components for the region from $x = 0$ to $x = L$:

$$\vec{E} = 2\int_{x=0}^{x=L}\frac{\sigma\, dx}{2\pi\varepsilon_0 r}\frac{L}{\sqrt{L^2 + r^2}}\cos\theta\;\hat{k}.$$

We choose θ for the variable by using $x = z_0 \tan\theta$, $dx = (z_0/\cos^2\theta)\, d\theta$, $r = z_0/\cos\theta$.

The limits for θ are 0 and θ_0, which is determined from $\tan\theta_0 = L/z_0$.

The integral becomes

$$\vec{E} = \frac{2\sigma}{2\pi\varepsilon_0}\int_0^{\theta_0}\frac{(z_0/\cos^2\theta)\, d\theta}{(z_0/\cos\theta)}\frac{L}{\sqrt{L^2 + (z_0/\cos\theta)^2}}\cos\theta\;\hat{k}$$

$$= \frac{\sigma}{\pi\varepsilon_0}\int_0^{\theta_0}\frac{\cos\theta\, d\theta}{\sqrt{\cos^2\theta + (z_0/L)^2}}\;\hat{k}.$$

As $L \to \infty$, the denominator in the integrand becomes $\cos\theta$, so the integral is $\int d\theta$. We find the upper limit of the integral from $\tan\theta_0 \to \infty$, which gives $\theta_0 = \pi/2$. The value of the integral is $\pi/2$, and the field becomes

$$\vec{E} \to (\sigma/\pi\varepsilon_0)(\pi/2)\hat{k} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}.$$

(c) As $z_0 \to 0$, the denominator in the integrand becomes $\cos\theta$, so the integral is $\int d\theta$. We find the upper limit of the integral from $\tan\theta_0 \to \infty$, which gives $\theta_0 = \pi/2$. The value of the integral is $\pi/2$, and the field becomes

$$\vec{E} \to (\sigma/\pi\varepsilon_0)(\pi/2)\hat{k} = \boxed{(\sigma/2\varepsilon_0)\hat{k}}.$$

This is the result from part (b). In both cases, the square looks like an infinite plane.

38. Because the electric field produced by the infinite plate is constant, there will be a constant downward force on the charge and thus constant acceleration of the pellet:

$$a = |q|E/m = |q|\sigma/2\varepsilon_0 m.$$

We find the speed from

$$v^2 = v_0^2 + 2a(y - y_0) = 0 + 2(|q|\sigma/2\varepsilon_0 m)(d - 0)$$

$$= [(1.08 \times 10^{-6}\ \mathrm{C})(2.17 \times 10^{-6}\ \mathrm{C/m^2})/(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.555 \times 10^{-3}\ \mathrm{kg})](0.175\ \mathrm{m}),$$

which gives $\boxed{v = 9.1\ \mathrm{m/s}}$.

39. The force from the electric field produces the acceleration:
$qE = ma;$
$q(850\ \mathrm{N/C}) = (120 \times 10^{-6}\ \mathrm{kg})(4.6\ \mathrm{m/s^2})$, which gives $q = 6.5 \times 10^{-7}\ \mathrm{C} = \boxed{0.65\ \mu\mathrm{C}}$.

40. The uniform electric field from the sheet produces a force on the electron toward the sheet which gives the electron an acceleration:
$qE = q\sigma/2\varepsilon_0 = ma$, or $a = q\sigma/2\varepsilon_0 m$.
We find the velocity from
$v = v_0 + at = 0 + (q\sigma/2\varepsilon_0 m)t$
$= (1.6 \times 10^{-19}\ \mathrm{C})(6.1 \times 10^{-9}\ \mathrm{C/m^2})/[2(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(9.1 \times 10^{-31}\ \mathrm{kg})](17.5 \times 10^{-9}\ \mathrm{s})$
$= \boxed{1.1 \times 10^6\ \mathrm{m/s}}$ away from the sheet.
We check the distance traveled by the electron:
$d = \frac{1}{2}(v + v_0)t = \frac{1}{2}(1.01 \times 10^6\ \mathrm{m/s})(17.5 \times 10^{-9}\ \mathrm{s}) = \boxed{9.6 \times 10^{-3}\ \mathrm{m}}$.

41. Because the electric field produced by the infinite plate is constant, there must be a constant upward force on the charge that balances the downward force of gravity. To produce an upward force on a positive charge, the plate must have a positive charge. We find the density from
$qE = q(\sigma/2\varepsilon_0) = mg;$
$(8.5 \times 10^{-7}\ \mathrm{C})\sigma/2(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}) = (0.83 \times 10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})$, which gives
$\sigma = \boxed{1.7 \times 10^{-7}\ \mathrm{C/m^2}}$.

42. Because we can treat the nucleus as a point charge, the field will be radial:
$E = [(1/4\pi\varepsilon_0)(q/r^2)]$
$= [(9 \times 10^9\ \mathrm{N\cdot m^2/C^2})(79)(1.60 \times 10^{-19}\ \mathrm{C})/(10^{-11}\ \mathrm{m})^2] = \boxed{(1.14 \times 10^{15}\ \mathrm{N/C})\ \text{radial}}$.
The acceleration of the alpha particle is
$a = QE/m = 2(1.60 \times 10^{-19}\ \mathrm{C})(1.14 \times 10^{15}\ \mathrm{N/C})/[4(1.67 \times 10^{-27}\ \mathrm{kg})]$
$= \boxed{5.4 \times 10^{22}\ \mathrm{m/s^2}\ \text{(away from the nucleus)}}$.

43. The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force:
$F = mv^2/r;$
$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v = \boxed{(q\lambda/2\pi\varepsilon_0 m)^{1/2}}$, which does not depend on r.

44. The force produced by the electric field of the wire on the negative charge is toward the wire.
We choose the x-axis along the wire and the y-axis perpendicular to the wire to apply $\sum \vec{F} = m\vec{a}$:
x-component: $0 = m\,\mathrm{d}^2x/\mathrm{d}t^2$, which we normally write as $\boxed{m\,\mathrm{d}^2x/\mathrm{d}t^2 = 0}$;
y-component: $-q\lambda/2\pi\varepsilon_0 y = m\,\mathrm{d}^2y/\mathrm{d}t^2$, which we normally write as $\boxed{m\,\mathrm{d}^2y/\mathrm{d}t^2 = -q\lambda/2\pi\varepsilon_0 y}$.

45. The force produced by the electric field of the wire on the negative charge is toward the wire and provides the centripetal force:
$F = mv^2/r;$
$q(\lambda/2\pi\varepsilon_0 r) = mv^2/r$, which gives a speed $v = (q\lambda/2\pi\varepsilon_0 m)^{1/2}$.
The period of the orbit is
$T = 2\pi r/v = [2\pi/(q\lambda/2\pi\varepsilon_0 m)^{1/2}]r$.
If the centripetal force is provided by a point charge, we have
$(1/4\pi\varepsilon_0)(qQ/r^2) = mv^2/r$, which gives
$v = (qQ/4\pi\varepsilon_0 mr)^{1/2}$.
The period of the orbit is
$T_{\text{point charge}} = 2\pi r/v = 2\pi(4\pi\varepsilon_0 m/qQ)^{1/2} r^{3/2}$, which has a different r dependence.

46. The electric field of each plate is up, and the electric force must be up to balance the force of gravity; therefore the charge must be positive. Because the acceleration is zero, we have

$qE_+ + qE_- = mg;$

$q = mg/[(\sigma_+/2\varepsilon_0) + (\sigma_2/2\varepsilon_0)] = 2mg\varepsilon_0/(\sigma_+ + \sigma_-)$

$= 2(5.6\times 10^{-3}\text{ kg})(9.8\text{ m/s}^2)(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)/[(1.6 + 0.22)\times 10^{-6}\text{ C/m}^2] = \boxed{5.3\times 10^{-7}\text{ C}}.$

47. We use the coordinate system from Example 22–12. The initial horizontal component of the velocity is

$v_{0x} = (v_0^2 - v_{0y}^2)^{1/2} = [(5.0\times 10^6\text{ m/s})^2 - (2.0\times 10^5\text{ m/s})^2]^{1/2} = 5.0\times 10^6\text{ m/s}.$

The time for the electron to travel between the plates is

$t_1 = L_1/v_{0x} = (3\times 10^{-2}\text{ m})/(5.0\times 10^6\text{ m/s}) = 6.0\times 10^{-9}\text{ s}.$

The deflection at this time is

$y_1 = v_{0y}t_1 + \frac{1}{2}at_1^2 = v_{0y}t_1 + \frac{1}{2}(qE/m)t_1^2$

$= (2.0\times 10^5\text{ m/s})(6.0\times 10^{-9}\text{ s}) + \frac{1}{2}[(1.6\times 10^{-19}\text{ C})(10^3\text{ N/C})/(9.1\times 10^{-31}\text{ kg})](6.0\times 10^{-9}\text{ s})^2$

$= 4.4\times 10^{-3}\text{ m}.$

The vertical component of the velocity as the electron leaves the plates is

$v_{1y} = v_{0y} + at_1 = v_{0y} + (qE/m)t_1$

$= (2.0\times 10^5\text{ m/s}) + [(1.6\times 10^{-19}\text{ C})(10^3\text{ N/C})/(9.1\times 10^{-31}\text{ kg})](6.0\times 10^{-9}\text{ s})$

$= 1.25\times 10^6\text{ m/s}.$

After it leaves the plates, the electron travels in a straight line with a direction given by

$\tan\theta = v_{1y}/v_{0x} = (1.25\times 10^6\text{ m/s})/(5.0\times 10^6\text{ m/s}) = 0.25.$

The deflection while the electron travels this straight line is

$y_2 = L_2\tan\theta = (12\times 10^{-2}\text{ m})(0.25) = 3.0\times 10^{-2}\text{ m}.$

The total deflection is

$y = y_1 + y_2 = (0.44\times 10^{-2}\text{ m}) + (3.0\times 10^{-2}\text{ m}) = 3.4\times 10^{-2}\text{ m} = \boxed{3.4\text{ cm}}.$

48. The electric field between the oppositely-charged parallel plates is uniform and will produce a constant acceleration:

$qE = e\sigma/\varepsilon_0 = ma,$ or $a = e\sigma/\varepsilon_0 m.$

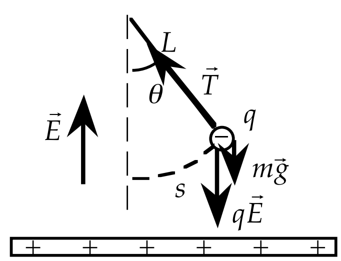
For a constant acceleration over the separation of the plates d, we have

$v^2 = v_0^2 + 2ad;$

$(3.0\times 10^7\text{ m/s})^2 = (1.6\times 10^6\text{ m/s})^2 + 2[(1.6\times 10^{-19}\text{ C})\sigma/(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(9.1\times 10^{-31}\text{ kg})](2\times 10^{-2}\text{ m}),$

which gives $\sigma = \boxed{1.1\times 10^{-6}\text{ C/m}^2}.$

49. The electric field produced by the plate is



$E = \sigma/2\varepsilon_0$

$= (10^{-6}\text{ C/m}^2)/2(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 5.6\times 10^4\text{ N/C}.$

Using the force diagram, we find the equation of motion for the tangential direction:

$-(mg + qE)\sin\theta = m\,d^2s/dt^2.$

If the angle is small, we have

$\sin\theta \simeq \theta = s/L$, and the equation of motion becomes

$-[(mg + qE)/L]s = m\,d^2s/dt^2.$

This is the equation for simple harmonic motion. The effective force constant is

$k_{\text{eff}} = (mg + qE)/L.$

The angular frequency of the motion is

$\omega = (k_{\text{eff}}/m)^{1/2} = [(mg + qE)/Lm]^{1/2}$

$= \{[(5\times 10^{-3}\text{ kg})(9.8\text{ m/s}^2) + (2\times 10^{-6}\text{ C})(5.6\times 10^4\text{ N/C})]/(1\text{ m})(5\times 10^{-3}\text{ kg})\}^{1/2} = \boxed{5.7\text{ s}^{-1}}.$

50. To find the force on the proton, we need the electric field. We write the linear dependence on x and find the constants:

$E = E_0 - bx;$

$500\ \text{N/C} = E_0 - 0$, which gives $E_0 = 500\ \text{N/C};$

$0 = 500\ \text{N/C} - b(3\ \text{m})$, which gives $b = (500/3)\ \text{N/C}\cdot\text{m}$.

Because the electric force is the only force on the proton, the equation of motion is

$m\,\mathrm{d}^2x/\mathrm{d}t^2 = q(E_0 - bx).$

If we change variable to $x' = x - 3$, $\mathrm{d}^2x'/\mathrm{d}t^2 = \mathrm{d}^2x/\mathrm{d}t^2$, and we have

$m\,\mathrm{d}^2x'/\mathrm{d}t^2 = q(E_0 - bx' - 3b) = -qbx'.$

We rewrite this as

$\mathrm{d}^2x'/\mathrm{d}t^2 = -(qb/m)x' = -\omega^2x',$

which is the equation for simple harmonic motion, with angular frequency

$\omega = (qb/m)^{1/2} = [(1.60 \times 10^{-19}\ \text{C})(167\ \text{N/C}\cdot\text{m})/(1.67 \times 10^{-27}\ \text{kg})]^{1/2} = 1.26 \times 10^5\ \text{s}^{-1}.$

We write the solution:

$x'(t) = A\sin(\omega t + \delta)$, and $v(t) = A\omega\cos(\omega t + \delta)$,

and determine A and δ from the initial conditions:

$x'(0) = -3\ \text{m} = A\sin(0 + \delta);\quad v(0) = 5 \times 10^5\ \text{m/s} = -A\omega\cos(0 + \delta).$

The solution of these two equations is

$A = 5.0\ \text{m}$ and $\delta = -37°$.

The time to traverse the region is the time when $x = 3$ m, or $x' = 0$:

$0 = (5.0\ \text{m})\sin(\omega t - 37°)$, which gives $\omega t = (37°)(\pi/180°)$, or

$t = (37°)(\pi/180°)/(1.26 \times 10^5\ \text{s}^{-1}) = \boxed{5.1 \times 10^{-6}\ \text{s}}$.

51. The torque on the dipole is

$$\begin{aligned}\vec{\tau} = \vec{p} \times \vec{E} &= qL(\cos 45°\,\hat{i} + \sin 45°\,\hat{j}) \times E\hat{i}\\ &= -qLE\sin 45°\,\hat{k}\\ &= (2 \times 10^{-6}\ \text{C})(0.10\ \text{m})(10\ \text{N/C})\sin 45°\,\hat{k}\\ &= \boxed{-(1.41 \times 10^{-6}\ \text{N}\cdot\text{m})\hat{k}}.\end{aligned}$$

52. The torque is directly proportional to the magnitude of the dipole moment. The new dipole moment is

$p_2 = q_2L_2 = (5q_1)(3L_1) = 15q_1L_1 = \boxed{15p_1}$.

The torque will be increased by a factor of 15.

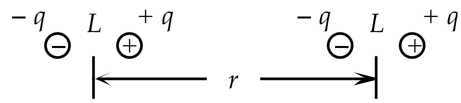
$\boxed{53}$. We estimate the field along the bisector:

$$\begin{aligned}E &\simeq (1/4\pi\varepsilon_0)(p/r)^3\\ &= [(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6 \times 10^{-30}\ \text{C}\cdot\text{m})/(3 \times 10^{-9}\ \text{m})^3]\\ &= \boxed{2 \times 10^6\ \text{N/C}}.\end{aligned}$$

54. From Problem 51, the torque acts to align the dipole with the electric field. As the dipole passes the x-axis, the torque direction will reverse. The dipole will oscillate around the direction of the electric field. The work done by the electric field is

$$\begin{aligned}W &= -\Delta U = -[(-\vec{p}\cdot\vec{E})_f - (-\vec{p}\cdot\vec{E})_i]\\ &= -[(-pE) - (-pE\cos 45°)]\\ &= qLE(1 - \cos 45°)\\ &= (2 \times 10^{-6}\ \text{C})(0.10\ \text{m})(10\ \text{N/C})(1 - 0.707)\\ &= \boxed{5.9 \times 10^{-7}\ \text{J}}.\end{aligned}$$

55. Each dipole is a pair of charges q separated by a distance L, such that $p = qL$. To find the force on the dipole on the right, we find the electric field at each of the charges produced by the charges of the other dipole. The field at the positive charge is



$$E_+ = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r^2} - \frac{q}{(r+L)^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{\left[1 + (L/r)\right]^2}\right\} \text{ to the right.}$$

The field at the negative charge is

$$E_- = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-L)^2} - \frac{q}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r^2}\left\{\frac{1}{\left[1 - (L/r)\right]^2} - 1\right\} \text{ to the right.}$$

The force on the dipole is

$$F = qE_+ + (-q)E_- = \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \frac{1}{\left[1 + (L/r)\right]^2} 1 - \frac{1}{\left[1 - (L/r)\right]^2} + 1\right\} \text{ to the right.}$$

Because $L << r$, we make use of the approximation $(1 \pm x)^{-2} \simeq 1 \mp 2x + 3x^2 \mp \ldots$ and expand the terms:

$$F \simeq \frac{q^2}{4\pi\varepsilon_0 r^2}\left\{1 - \left[1 - 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] - \left[1 + 2\frac{L}{r} + 3\left(\frac{L}{r}\right)^2\right] + 1\right\} = \frac{q^2}{4\pi\varepsilon_0 r^2}\left[-6\left(\frac{L}{r}\right)^2\right] = -\frac{6q^2L^2}{4\pi\varepsilon_0 r^4} = \quad -\frac{6p^2}{4\pi\varepsilon_0 r^4} \text{(attraction).}$$

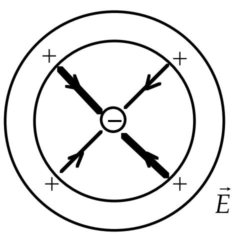
56. The potential energy of a dipole in an electric field is $U = -\vec{p} \cdot \vec{E}$. The maximum energy occurs when $\vec{p}$ and $\vec{E}$ are in opposite directions, and the minimum energy occurs when $\vec{p}$ and $\vec{E}$ are parallel:

$U_{max} = pE$, $U_{min} = -pE$, and

$\Delta U = 2pE$;

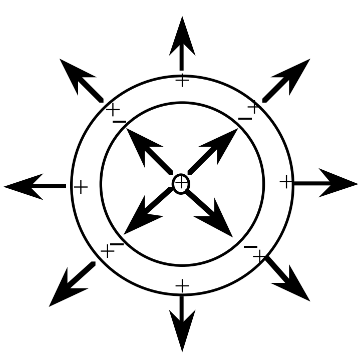
4.4×10^{-25} J $= 2p(10^4$ N/C), which gives $p = \boxed{2.2 \times 10^{-29}\ \text{C}\cdot\text{m}}$.

$\boxed{57}$.

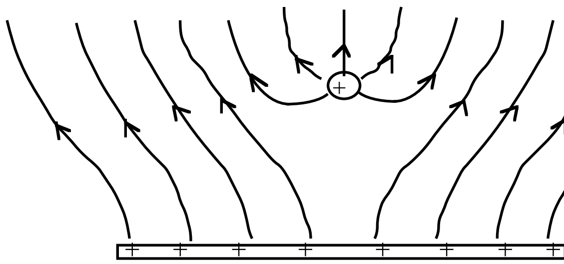


$\vec{E} = 0$ outside

58.



59.



60. In the region outside a uniformly charged sphere, the electric field is the same as that of a point charge:

$$E = (1/4\pi\varepsilon_0)(q/r^2).$$

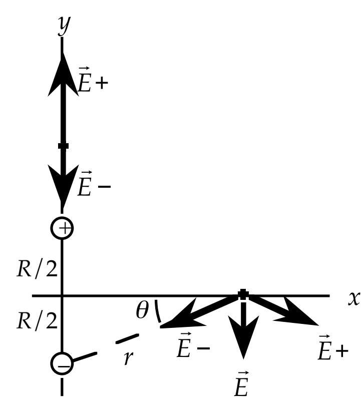
For the cork ball, we have

$$E = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.5 \times 10^{-9}\ \text{C})/(1.2 \times 10^{-2}\ \text{m})^2 = \boxed{2.2 \times 10^5\ \text{N/C}}.$$

For the uranium nucleus, we have

$$E = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(28)(1.6 \times 10^{-19}\ \text{C})/(5 \times 10^{-15}\ \text{m})^2 = \boxed{1.6 \times 10^{21}\ \text{N/C}}.$$

61. We choose the coordinate system shown in the diagram, with the rods aligned parallel to the z-axis.



(a) $\vec{E} = \vec{E}_+ + \vec{E}_- = (\lambda/2\pi\varepsilon_0)[1/(y - R/2) - 1/(y + R/2)]\,\hat{j}$

$$= \boxed{\{(\lambda R/2\pi\varepsilon_0)/[y^2 - (R/2)^2]\}\hat{j}}.$$

(b) We see from the diagram that the symmetry along the x-axis means that the resultant field will have only a y-component. We find the field by doubling the y-component from one rod:

$$\vec{E} = -2E_-\sin\theta\,\hat{j} = -(2\lambda/2\pi\varepsilon_0 r)[(R/2)/r]\,\hat{j}$$

$$= -(\lambda R/2\pi\varepsilon_0 r^2)\,\hat{j} = \boxed{-\{(\lambda R/2\pi\varepsilon_0)/[x^2 + (R/2)^2]\}\hat{j}}.$$

62. The electric field at the positive rod produced by the negative rod is

$$E = \lambda/2\pi\varepsilon_0 R \text{ toward the negative rod.}$$

The force on a charge Q of the rod is $F = QE$, so the force per unit length is

$$F/L = (Q/L)E$$

$$= \lambda(\lambda/2\pi\varepsilon_0 R) = \boxed{\lambda^2/2\pi\varepsilon_0 R\ \ (\text{attraction})}.$$

63. Each infinite plate produces a constant field perpendicular to the plate. The total electric field is

$$\vec{E} = (\sigma_1/2\varepsilon_0)\,\hat{j} + (\sigma_2/2\varepsilon_0)\,\hat{i}.$$

The force produced by this field on the particle causes a constant acceleration:

$$\vec{a} = q\vec{E}/m = (q/2\varepsilon_0 m)(\sigma_1\hat{j} + \sigma_2\hat{i}).$$

If the particle starts from rest, its position is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 = (1\ \text{m})\hat{i} + (1\ \text{m})\hat{j} + 0 + (q/4\varepsilon_0 m)(\sigma_2\hat{i} + \sigma_1\hat{j})t^2$$

$$= \{1 + [(1 \times 10^{-7}\ \text{C})/4(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1 \times 10^{-3}\ \text{kg})](+3 \times 10^{-6}\ \text{C/m}^2)t^2\}\,\hat{i}\ +$$

$$\{1 + [(1 \times 10^{-7}\ \text{C})/4(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1 \times 10^{-3}\ \text{kg})](-5 \times 10^{-6}\ \text{C/m}^2)t^2\}\,\hat{j}$$

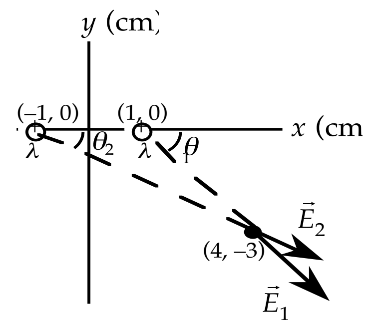
$$= \boxed{[(1 + 8.5t^2)\hat{i} + (1 - 14t^2)\hat{j}]\ \text{m, with } t \text{ in s}}.$$

64. (a) At the origin the fields from the line charges are in opposite directions. From the symmetry of the charges, we see that the total field at the origin is

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 = \boxed{0}.$$

(b) The force on a charge at the origin is

$$\vec{F}_0 = q\vec{E}_0 = \boxed{0}.$$



(c) We find the angles and distances for each line charge:

$\tan\theta_1 = 3/(4\ - 1)$, which gives $\theta_1 = 45.0°$;

$\tan\theta_2 = 3/(4\ + 1)$, which gives $\theta_2 = 31.0°$;

$r_1^2 = (3\text{ cm})^2 + (3\text{ cm})^2$, which gives $r_1 = 4.24$ cm;

$r_2^2 = (5\text{ cm})^2 + (3\text{ cm})^2$, which gives $r_2 = 5.83$ cm.

The total field is

$$\begin{aligned}\vec{E} &= \vec{E}_1 + \vec{E}_2\\ &= (\lambda/2\pi\varepsilon_0 r_1)(\cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j}) + (\lambda/2\pi\varepsilon_0 r_2)(\cos\theta_2\,\hat{i} - \sin\theta_2\,\hat{j})\\ &= (\lambda/2\pi\varepsilon_0)\{[(\cos\theta_1\,\hat{i} - \sin\theta_1\,\hat{j})/r_1] + [(\cos\theta_2\,\hat{i} - \sin\theta_2\,\hat{j})/r_2]\}\\ &= [(5\times10^{-6}\text{ C/m})/2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)]\\ &\qquad [(\cos 45.0°\,\hat{i} - \sin 45.0°\,\hat{j})/(4.24\times10^{-2}\text{ m}) + (\cos 31.0°\,\hat{i} - \sin 31.0°\,\hat{j})/(5.83\ \times10^{-2}\text{ m})]\\ &= \boxed{(2.8\times10^{6}\text{ N/C})\hat{i} - (1.5\times10^{6}\text{ N/C})\hat{j}}.\end{aligned}$$

The force on the charge is

$$\begin{aligned}\vec{F} = q\vec{E} &= (6\times10^{-6}\text{ C})[(2.8\ \times10^{-6}\text{ N/C})\hat{i} - (1.5\ \times10^{-6}\text{ N/C})\ \hat{j}]\\ &= \boxed{(17\hat{i} - 9\hat{j})\text{ N}}.\end{aligned}$$

65. (a) The electric field of the plate is perpendicular to and away from the plate. The force on the positive charge is away from the plate:

$$\begin{aligned}F = qE &= q\sigma/2\varepsilon_0\\ &= (1.6\times10^{-19}\text{ C})(8.0\times10^{-6}\text{ C/m}^2)/2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)\\ &= \boxed{7.2\times10^{-14}\text{ N away from the plate}}.\end{aligned}$$

(b) We find the work from the work-energy theorem:

$$\begin{aligned}W &= \Delta K\\ &= 0 - (2\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV}) = \boxed{-3.2\times10^{-13}\text{ J}}.\end{aligned}$$

(c) Because the work is done by the electric field, we have

$$W = -Fd$$

$-3.2\times10^{-13}\text{ J} = -(7.2\times10^{-14}\text{ N})d$, which gives $d = \boxed{4.4\text{ m}}$.

66. (a) When there is no charge on the drop, the forces acting on the drop are the downward force of gravity and the upward drag force;

$mg - F_{\text{drag}} = 0$, or

$\rho(\frac{4}{3}\pi r^3)g = 6\pi\eta r v_0$, which gives $v_0 = 2r^2\rho g/9\eta$.

(b) With a positive charge, the electric force is up, so we have

$mg - F_{\text{drag}} - qE = 0$, or

$qE = mg - F_{\text{drag}} = \rho(\frac{4}{3}\pi r^3)g - 6\pi\eta r v_1$.

If we use the result of part (a), we get

$qE = 6\pi\eta r v_0 - 6\pi\eta r v_1 = 6\pi\eta(v_0 - v_1)r = 6\pi\eta(v_0 - v_1)(9v_0\eta/2\rho g)^{1/2}$, which gives

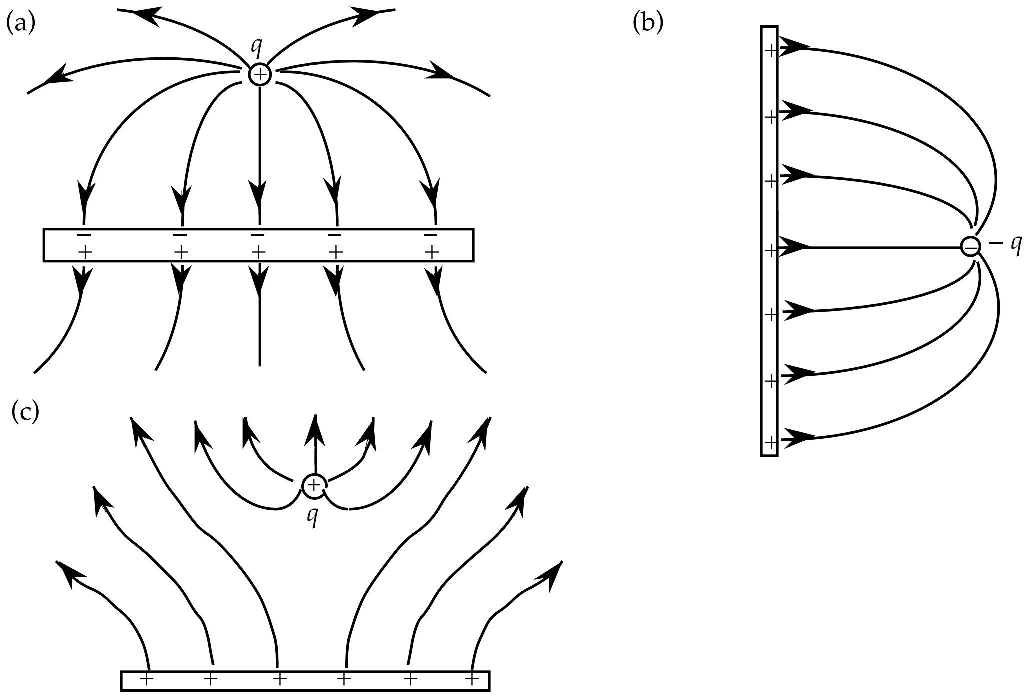
$q = [18\pi(v_0 - v_1)/E](v_0\eta^3/2\rho g)^{1/2}$.

(c) Because the droplet is stationary, there is no drag force, so we have

$qE = mg = \rho(\frac{4}{3}\pi r^3)g$;

$E = \frac{4}{3}\pi r^3\rho g/q = \frac{4}{3}\pi(2.0\times10^{-6}\text{ m})(0.85\times10^{3}\text{ kg/m}^3)(9.8\text{ m/s}^2)/(1.6\ \text{x}10^{-19}\text{ C}) = \boxed{1.74\times10^{6}\text{ N/C}}$.

67.



68. We write the relation as

$t = \lambda^\alpha q^\beta m^\gamma R^\delta \varepsilon_0{}^\mu$. When we substitute the dimensions, we get

$[t] = [\lambda]^\alpha [q]^\beta [m]^\gamma [R]^\delta [\varepsilon_0]^\mu$;

$[T] = [QL^{-1}]^\alpha [Q]^\beta [M]^\gamma [L]^\delta [Q^2T^2M^{-1}L^{-3}]^\mu$.

We equate the exponents for each dimension:

Q: $0 = \alpha + \beta + 2\mu$;

L: $0 = -\alpha + \delta - 3\mu$;

M: $0 = \gamma - \mu$;

T: $1 = 2\mu$.

We have four equations with five unknowns. Two we can find directly:

$\mu = \frac{1}{2}$, $\gamma = \frac{1}{2}$.

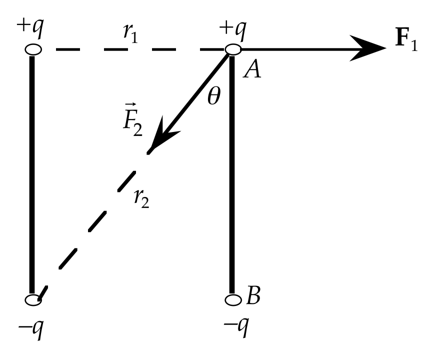
To find the others we must use the fact that the field of a line charge depends on λ/ε_0. This is the only contributor to the force, so we expect the exponent of λ to be the negative of the exponent of ε_0:

$\alpha = -\mu = -\frac{1}{2}$; which gives $\beta = -\frac{1}{2}$, and $\delta = 1$.

When we use these results, we have

$\boxed{t \propto R(m\varepsilon_0/q\lambda)^{1/2}}$.

69. By symmetry the net force exerted on one rod by the other is perpendicular to each rod. Also, the forces from the other rod on the two charges at points A and B are the same. So the force between the two rods is



$F = 2(F_1 - F_2 \sin\theta) = 2[kq^2/r_1{}^2 - (kq^2/r_2{}^2)(r_1/r_2)]$

$= 2(9 \times 10^9\ \mathrm{N\cdot m^2/C^2})(2.2 \times 10^{-4}\ \mathrm{C})^2 \{1/(0.18\ \mathrm{m})^2 - (0.18\ \mathrm{m})/[(0.18\ \mathrm{m})^2 + (0.20\ \mathrm{m})^2]^{3/2}\}$

$= \boxed{1.9 \times 10^4\ \mathrm{N}\text{, directly away from each other}}$.

The net torque on each rod is $\boxed{\text{zero}}$, since each of the two charges on each rod receives the same amount of force and is equidistant form the axis.

70. Choose an xy coordinate system originated at the point where the electron enters the region, with the x-axis pointing east and y-axis pointing north. The motion in the x-direction is uniform, so

$x = v_x t = (v_0 \cos\theta)t$, where $v_0 = 3\times 10^6$ m/s and $\theta = 40^\circ$.

In the y-direction there is an acceleration:

$a_y = F_y/m = -eE/m$, so

$y = v_{0y} t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta)t + \frac{1}{2}(-eE/m)t^2$.

When the electron strikes the bottom plate $y = 0$, whereupon

$t = 2mv_0 \sin\theta / eE$. Plug this into the expression for x to obtain

$x = (v_0 \cos\theta)(2mv_0 \sin\theta/eE) = \boxed{mv_0^2 \sin(2\theta)/eE}$.

Note that this is analogous to the range formula for a projectile, with g replaced by eE/m.

71. We are given the force

$$\vec{F} = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left\{\ln\left[\frac{R-(L/2)}{R+(L/2)}\right] + R\left[\frac{1}{R-(L/2)} - \frac{1}{R+(L/2)}\right]\right\}\hat{i}.$$

If we change variable to $x = L/2R$, the magnitude of the force becomes

$$F = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\ln\left(\frac{1-x}{1+x}\right) + \left(\frac{1}{1-x} - \frac{1}{1+x}\right)\right] = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\ln(1-x) - \ln(1+x) + \left(\frac{1}{1-x} - \frac{1}{1+x}\right)\right].$$

Using the approximate expansions for small x, we get

$$F = \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\right) + \left(1 + x + x^2 + x^3 + \ldots\right) - \left(1 - x + x^2 - x^3 + \ldots\right)\right]$$

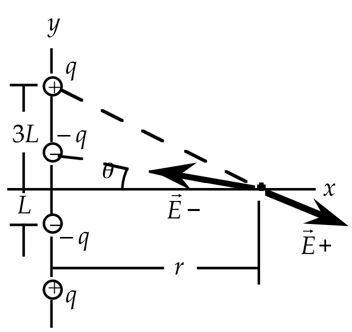
$$= \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left[\left(-2x - \frac{2x^3}{3} - \ldots\right) + \left(2x + 2x^3 + \ldots\right)\right] \approx \frac{q\lambda_0}{2\pi\varepsilon_0 L}\left(\frac{4x^3}{3}\right).$$

In terms of the distance R, the force is

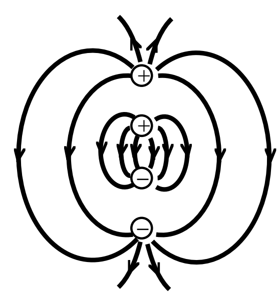
$F = (q\lambda_0/2\pi\varepsilon_0 L)(4/3)(L/2R)^3 = q\lambda_0 L^2/12\pi\varepsilon_0 R^3$.

The field of a dipole on the axis is $E = p/2\pi\varepsilon_0 R^3$, so the dipole moment is $p = \boxed{\lambda_0 L^2/6}$.

72. (a)



(b)



We orient the four charges along the y-axis, as shown in the diagram. From the symmetry of the charge distribution, we see that the resultant field will be along the x-axis, and we can double the difference between the component from the positive charge and that from the negative one. With r the distance along the x-axis, we have

$\vec{E} = 2(E_{+x} - E_{-x})\hat{i} = 2(q/4\pi\varepsilon_0)[\cos\theta_+/[r^2 + (3L)^2] - \cos\theta_-/(r^2 + L^2)]\hat{i}$

$= (q/2\pi\varepsilon_0)\{r/[r^2 + (3L)^2]^{3/2} - r/(r^2 + L^2)^{3/2}\}\hat{i}$.

If we use the approximation given in the hint, we get

$\vec{E} = (qr/2\pi\varepsilon_0)[1/r^3 - 3(3L)^2/2r^5 - 1/r^3 + 3L^2/2r^5]\hat{i} = -(6qL^2/\pi\varepsilon_0 r^4)\hat{i}$.

CHAPTER 23 Gauss' Law

Answers to Understanding the Concepts Questions

1. The value of temperature at each point forms a scalar field. Since there is no directionality associated with temperature, flux, which measures an abstract sort of flow across a surface, cannot be associated with a temperature field. However, given a temperature field, one can calculate at each point a vector, termed the *temperature gradient,* representing the change in temperature. This vector field has the components $(\partial T/\partial x,\ \partial T/\partial y,\ \partial T/\partial z)$, where you will recall that the symbol ∂ refers to partial differentiation. Because this field has directionality, it is possible to define a flux for it.

2. The opening is presumably very small in comparison with the size of the sphere. So the "open" sphere can be thought of as a closed one, only with a small patch removed.

3. According to Gauss' law, the net charge enclosed by the surface is zero. This does not, however, mean that the electric field over the surface is always zero. The simplest counter example would be a uniform field produced by some charge distribution outside the surface. The flux of a uniform field over any enclosed surface is always zero, yet the field itself is not. Also, the surface can enclose an equal amount of positive and negative charges, producing a non-zero field but a zero net flux over the surface.

4. Suppose we consider a charge-free region, and there is a break (discontinuity) in a field line. It is then possible to construct a Gaussian surface that envelops the tip of the break in the field line. There will be a net flux across that surface, but on the other hand, there is no charge in the region. Thus a break in an electric field line in a charge-free region violates Gauss' law. The only way to satisfy Gauss' law is to insist that when a field line ends, it ends on a charge.

5. Consider, for example, a spherical Gaussian surface of radius r centered at the location of a point charge q. The electric flux through this surface is $\Phi = EA = E(4\pi r^2)$. If $E = c/r$ then $\Phi = (c/r)(4\pi r^2) = 4\pi cr$, which depends on the radius r of the Gaussian surface, rather than just the charge q enclosed — this is contradictory to Gauss' law.

6. If the point charge is located at the center of a certain face (face A) of the cube, then by symmetry the electric flux through each of the four faces that are perpendicular to face A is identical. However, the flux through the remaining face that is directly opposite to face A is different. Therefore, in this case symmetry alone does not provide a simple answer to the flux through each face.

7. The reconciliation follows by considering a pill-box Gaussian surface on the first plate, with its flat ends, of area A, extending just outside of the plate itself. The charge density is not changed, so that the Q/ε_0 part of Gauss' law is unchanged. The flux through the end surfaces, however, is changed. On the side of the plate away from the second, negatively charged, plate, the flux through the end of the pill-box is $(\sigma/2\varepsilon_0)A$ from the positive plate, and $-(\sigma/2\varepsilon_0)A$ from the negative plate. These cancel. The flux through the end surface of the pill-box between the plates is $(\sigma/2\varepsilon_0)A$ from the positive plate and a like amount from the negative plate. These add to a total of $(\sigma/\varepsilon_0)A$. There is no conflict.

8. No. It only means that the net charge enclosed by the surface is zero, since the flux is proportional to the net charge enclosed. As an example, consider a spherical shell of radius r that is uniformly charged to a total charge q. At the center of the sphere is a point charge $-q$. If we draw a spherical Gaussian surface of radius $R > r$, concentric with the charged shell, then $E = 0$ everywhere on the gaussian surface — and yet there are charged enclosed by it (although no net charge).

9. The electric field produced by a uniformly charged spherical shell is zero anywhere inside the shell. The force it exerts on a charge there is therefore zero.

10. Gauss' law for fluid flow involves the fluid flux, given by $\Phi = \int \vec{v} \cdot d\vec{A}$. This flux describes the rate at which the fluid crosses the surface. For a closed surface there will be a net outflow of fluid only if there is a source of fluid somewhere within the enclosed volume. Thus Gauss' law will read $\Phi = S$, where S is the rate at which fluid is "created" inside the surface by a source, in m^3/s. If there are sources (faucets) in the region, then S is positive; if there are sinks (drains) in the region, then S is negative. Evaporation acts as a sink; that is, a negative contribution to the flux. Looking at the net flux, it is impossible to separate evaporation from any other type of sink. In the case of electricity, the analog of evaporation would be the disappearance of electric charge. There are deep principles that argue against that, and therefore one would not expect S to change with time unless charges actually cross the boundary of the enclosed surface.

11. Yes. The charges would be deposited over the exterior surface of the aluminum shell, in such a way that the electric field inside the shell remains zero.

12. Like the Coulomb force, the gravitational force is also a central force with inverse-square dependency on distance, so Gauss' law applies to it as well. If we compare the Coulomb force, $F_E = (1/4\pi\varepsilon_0)\, q_1 q_2/r^2$, with the gravitational force, $F_g = G\, m_1 m_2/r^2$, we find that in writing down Gauss' law for gravitational field we need to make the following substitution: q to m, $\vec{E}$ $(=\vec{F}_E/q)$ to $\vec{g}$ $(=\vec{F}_g/m)$, and $1/4\pi\varepsilon_0$ to G (or $1/\varepsilon_0$ to $4\pi G$). Also, the gravitational flux is negative due to the attractive nature of the force. Thus $\Phi_E = \oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$ becomes $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -4\pi G m$.

13. Symmetry does allow us to state that the electric field is parallel to the vector normal to the surface of the torus. Thus Gauss' law gives us a value for the integral $\int E\, dA$. Because of the curvature of the surface, E is not the same on the inner part of the torus as on the outer part, and therefore the integral cannot be converted to the form $E\int dA = EA$.

14. Assuming that the two point charges are fixed so that they cannot annihilate each other, the resulting electric field is zero inside the conductor . Outside the conductor, the field lines extend from the charges, and always intersect the surface of the conductor perpendicularly.

15. With Gauss' law we can show that there is no charge in the region of uniform electric field. Take a Gaussian surface in the shape of a can with the two ends perpendicular to the constant field direction. The net flux through the surface is zero, and so the net charge inside the region is zero. The surface can be anywhere within the large region, so that there is no net charge anywhere. If you are worried that this does not rule out equal positive and negative charges inside the region, just make the can smaller. No matter how small the can's volume, there is no net charge.

16. The flux also triples. This is because $\vec{E}$ is now $3\vec{E}$ (as it is proportional to the charge that produces it).

17. For a charged line of finite length, the electric field is not uniform over the Gaussian cylinder in question. For example, the value of the E-field close to one end of the line is not quite the same as that near the center of the line. Therefore we can no longer evaluate the electric flux as $\oint \vec{E} \cdot d\vec{A} = EA$.

18. The flux is proportional to the charge enclosed, so as q doubles the flux also doubles, to 8.0×10^6 N · m^2/C. Doubling the side length of the cube is irrelevant to the answer, since the flux depends only on the charge enclosed, not on the size and shape of the Gaussian surface.

19. Let's use Gauss' law together with a Gaussian surface in the form of a tiny pill-box whose flat ends are perpendicular to the z-axis. Since the z-component of $\vec{E}$ vanishes and the other components are independent of z, the net flux through this Gaussian surface is independent of z, and the net charge can only depend on x and y. Thus the charge density must also be independent of z.

20. Imagine a Gaussian surface that enclosed a certain volume V of the region. The charge enclosed by the surface is $q = \rho V$. By measuring the electric field on the surface we can find the flux Φ through the surface. But according to Gauss' law $\Phi = q/\varepsilon_0 = \rho V/\varepsilon_0$, so $\rho = \varepsilon_0 \Phi / V$.

21. No. If the charge distribution is not uniform then the E-field would not exhibit the symmetry that allowed us to write $\oiint \vec{E} \cdot d\vec{A} = EA$. The resulting E-field is not the same. Consider, for example, a highly asymmetrical case where all the charges on the shell is concentrated at one point on the shell. The resulting electric field is that of a point charge located on the shell, and that is certainly very different from the result of a uniformly charged shell.

Solutions to Problems

1. The electric field of the plate is perpendicular to the plate with magnitude $E = \sigma/2\varepsilon_0$.
   (a) Because the circle is parallel to the plate, the area vector is perpendicular to the plate. The flux through the circle is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = E\pi R^2 = \boxed{\sigma\pi R^2/2\varepsilon_0}.$$
   (b) The angle between the field vector and the area vector is 30°. The flux through the circle is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = (\sigma\pi R^2/2\varepsilon_0)\cos 30° = \boxed{0.866\ \sigma\pi R^2/2\varepsilon_0}.$$

2. The angle between the field vector and the area vector is 48°. The flux through the square is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = EA\cos 48° = (1325\ \text{N/C})(0.27\ \text{m})^2\cos 48° = \boxed{65\ \text{N}\cdot\text{m}^2/\text{C}}.$$

3. On the ends of the cylinder the electric field is not constant, but it is always perpendicular to the area vector of the surface. On the sides of the cylinder the electric field is constant and parallel to the area vector. The flux through the cylinder is
   $$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{end}} \vec{E}\cdot d\vec{A} + \iint_{\text{end}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$
   $$= 0 + 0 + EA_{\text{side}} = \frac{\lambda}{2\pi\varepsilon_0 R}(2\pi R h) = \frac{\lambda h}{\varepsilon_0}.$$
   We see that the result is independent of R, so we get the same flux through a cylinder of radius $2R$.

4. The electric field and the area vector are parallel. Because the electric field varies over the surface, we find the flux by integrating:
   $$\Phi = \iint_{\text{square}} \vec{E}\cdot d\vec{A} = \iint_{\text{end}} \left(5xz\,\hat{k}\right)\cdot\left(dx\,dy\,\hat{k}\right) = 5z\int_{-1}^{2} dy\int_{-1}^{2} x\,dx$$
   $$= 5z\,(y)\Big|_{1}^{2}\left(\frac{x^2}{2}\right)\Big|_{-1}^{2} = 5(3)[2-(-1)]\left[\frac{(2)^2}{2} - \frac{(-1)^2}{2}\right]$$
   $$= 68\ \text{N}\cdot\text{m}^2/\text{C}.$$

5. Because each side of the cube has an area vector parallel to one of the coordinate axes, the scalar product for the side involves only one component of the electric field. The total flux through the cube is
   $$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{x=0} \vec{E}\cdot d\vec{A} + \iint_{x=1} \vec{E}\cdot d\vec{A} + \iint_{y=0} \vec{E}\cdot d\vec{A} + \iint_{y=1} \vec{E}\cdot d\vec{A} + \iint_{z=0} \vec{E}\cdot d\vec{A} + \iint_{z=1} \vec{E}\cdot d\vec{A}$$
   $$= \iint_{x=0} (5x)\,dy\,dz + \iint_{x=1} (5x)\,dy\,dz + \iint_{y=0} (-3y)\,dx\,dz + \iint_{y=1} (-3y)\,dx\,dz\ +$$
   $$\iint_{z=0} (4z)\,dx\,dy + \iint_{z=1} (4z)\,dx\,dy$$
   $$= 0 + (5)(1)(1)(1) + 0 + (-3)(1)(1)(1) + 0 + (4)(1)(1)(1) = +6\ \text{N}\cdot\text{m}^2/\text{C}.$$

6. (a) The area vector is perpendicular to the plate and thus parallel to the elecric field. The flux through the loop is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = EA = (150\ \text{N/C})(4\times10^{-4}\text{m}^2) = \boxed{6\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}}.$$
   (b) The angle between the field vector and the area vector is 30°. The flux through the circle is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = EA\cos 25° = (150\text{N/C})(4\times10^{-4}\text{m}^2)\cos 25° = \boxed{5.4\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}.}$$
   (c) The angle between the field vector and the area vector is 330°. The flux through the circle is
   $$\Phi = \iint \vec{E}\cdot d\vec{A} = \vec{E}\cdot\vec{A} = EA\cos 335° = (150\ \text{N/C})(4\times10^{-4}\text{m}^2)\cos 335° = \boxed{5.4\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}}.$$
   There is no change.

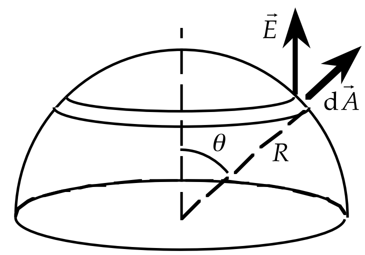
7. The electric field and the area vector are parallel. Because the electric field varies over the surface, we find the flux by integrating. We choose a circular ring of radius r and thickness dr as the differential element:

$$\Phi = \iint \vec{E} \cdot \mathrm{d}\vec{A} = \int_0^R E_0\left(1 - \frac{r}{R}\right)(2\pi r\, \mathrm{d}r)$$

$$= 2\pi E_0 \int_0^R \left(r - \frac{r^2}{R}\right) \mathrm{d}r = 2\pi E_0\left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^R = \frac{\pi E_0 R^2}{3}.$$

8. Because the angle between the electric field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a strip at an angle θ with a thickness $R\,\mathrm{d}\theta$, as shown in the diagram. The area of this strip is

$\mathrm{d}A = (2\pi R \sin\theta)R\,\mathrm{d}\theta = 2\pi R^2 \sin\theta\,\mathrm{d}\theta$.



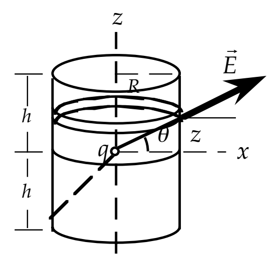
From the diagram, we see that θ is the angle between $\vec{E}$ and d$\vec{A}$, so we have

$$\Phi = \iint \vec{E} \cdot \mathrm{d}\vec{A} = \int_0^{\pi/2} E\,(\cos\theta)\, 2\pi R^2 \sin\theta\,\mathrm{d}\theta$$

$$= E 2\pi R^2\left(\frac{\sin^2\theta}{2}\right)\Bigg|_0^{\pi/2} = E\pi R^2.$$

This is the flux of a constant field through the area of a circle of radius R.

9. Because the angle between the electric field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a band at an elevation z, which corresponds to an angle θ such that $z = R\tan\theta$. The band has thickness $\mathrm{d}z = R\sec^2\theta\,\mathrm{d}\theta$ so the area of this band is



$\mathrm{d}A = 2\pi R\,\mathrm{d}z = 2\pi R^2 \sec^2\theta\,\mathrm{d}\theta$.

From the symmetry we see that the flux will be the same for the upper and lower halves of the surface, so we double the result of the integration over the top half. The angle θ ranges from 0 to θ_0, with $\sin\theta_0 = h/R$. From the diagram, we see that θ is the angle between $\vec{E}$ and d$\vec{A}$ for all elements of the band, so we have

$$\Phi = \iint \vec{E} \cdot \mathrm{d}\vec{A} = 2\int_0^{\theta_0} \frac{1}{4\pi\varepsilon_0}\frac{q}{(R/\cos\theta)^2} 2\pi R^2 \sec^2\theta\,(\cos\theta)\,\mathrm{d}\theta$$

$$= \frac{q}{\varepsilon_0}\int_0^{\theta_0} \cos\theta\,\mathrm{d}\theta = \frac{q}{\varepsilon_0}(\sin\theta)\Bigg|_0^{\theta_0} = \frac{q}{\varepsilon_0}\frac{1}{\sqrt{1 + (R^2/h^2)}}.$$

10. If the charge is placed a very small distance above the center, the radial electric field through the hemisphere is constant in magnitude and always perpendicular to the surface ($\vec{E}$ and d$\vec{A}$ parallel). The flux through the hemisphere is

$\Phi_{\text{hemisphere}} = \iint \vec{E}\cdot\mathrm{d}\vec{A} = EA = (q/4\pi\varepsilon_0 R^2)(\frac{1}{2}\,4\pi R^2) = q/2\varepsilon_0$.

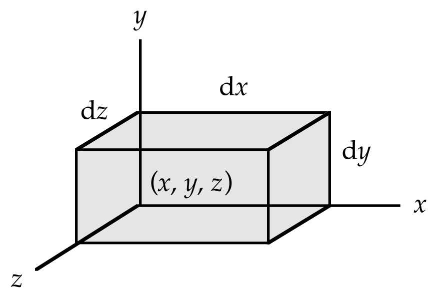
The direct calculation of the flux through the planar circle is more difficult; however we can use the symmetry of the electric field of the point charge. The flux above the horizontal plane must be equal to the flux below the horizontal plane:

$\Phi_{\text{circle}} = \Phi_{\text{hemisphere}}$.

Thus, the total flux is

$\Phi = \Phi_{\text{hemisphere}} + \Phi_{\text{circle}} = (q/2\varepsilon_0) + (q/2\varepsilon_0) = \boxed{q/\varepsilon_0}$.

11. Because each side of the parallelepiped has an area vector parallel to one of the coordinate axes, the scalar product for each side involves only the component of the electric field in the direction of the axis. Because the electric field is a function of x, y, and z, the magnitude of its components will be different on opposite sides of the parallelepiped.
For example, along the x-axis, we have
$$\vec{E}(x + dx, y, z) = \vec{E}(x, y, z) + [\partial \vec{E}(x, y, z)/\partial x]\, dx,$$
or the equivalent three component equations.
The area vector always points out of the surface. We find the differential flux through the two sides perpendicular to the x-axis, with area $dy\, dz$, from



$$\begin{aligned}\Phi_1 &= \vec{E}\cdot d\vec{A}\\ &= (E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot dy\, dz\,(-\hat{i}) + \{(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) + \\ &\qquad [(\partial E_x/\partial x)\hat{i} + (\partial E_y/\partial x)\,\hat{j} + (\partial E_z/\partial x)\hat{k}]\}\cdot dy\, dz\,\hat{i}\\ &= -E_x\, dy\, dz + E_x\, dy\, dz + (\partial E_x/\partial x)\, dx\, dy\, dz = (\partial E_x/\partial x)\, dx\, dy\, dz.\end{aligned}$$

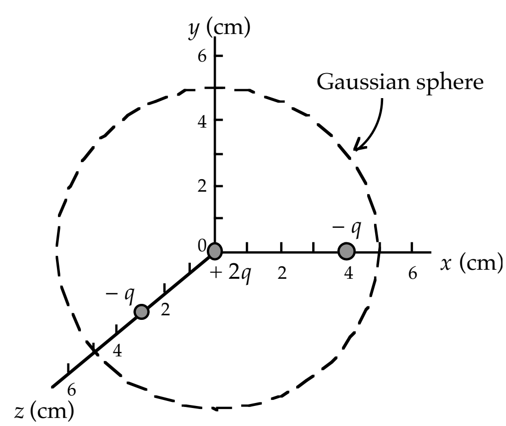
If we apply a similar analysis to the other pairs of sides, we have
$$\Phi_2 = (\partial E_y/\partial y)\, dy\, dx\, dz \quad\text{and}\quad \Phi_3 = (\partial E_z/\partial z)\, dz\, dx\, dy.$$
The total flux through the surface is
$$\begin{aligned}\Phi &= \Phi_1 + \Phi_2 + \Phi_3\\ &= (\partial E_x/\partial x)\, dx\, dy\, dz + (\partial E_y/\partial y)\, dy\, dx\, dz + (\partial E_z/\partial z)\, dz\, dx\, dy\\ &= [(\partial E_x/\partial x) + (\partial E_y/\partial y) + (\partial E_z/\partial z)]\, dx\, dy\, dz.\end{aligned}$$

12. The flux is directly dependent on the enclosed charge:
$$\Phi = Q/\varepsilon_0,\ \text{ or }\ Q = \varepsilon_0\Phi = (8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(-\,5.7\times10^{-5}\ \mathrm{N\cdot m^2/C}) = \boxed{-5.0\times10^{-16}\ \mathrm{C}}.$$

13. (a) We use the spherical surface within the charged surface as a Gaussian surface. Because there is no enclosed charge the total electric flux through the surface is $\boxed{\text{zero}}$.
(b) We use the spherical surface outside the charged surface as a Gaussian surface. Because all of the charge is enclosed, the total electric flux through the surface is
$$\begin{aligned}\Phi &= \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q/\varepsilon_0\\ &= (10^{-3}\ \mathrm{C})/(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}) = \boxed{1.13\times10^{8}\ \mathrm{N\cdot m^2/C}}.\end{aligned}$$

14. Because all of the charge is enclosed, the total electric flux through the surface is
$$\begin{aligned}\Phi &= \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q/\varepsilon_0\\ &= (120\times10^{-9}\ \mathrm{C})/(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}) = \boxed{1.36\times10^{4}\ \mathrm{N\cdot m^2/C}}.\end{aligned}$$

15. (a) For the Gaussian sphere we have
$$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0 = (-\,q + 2q - q)/\varepsilon_0 = 0.$$
The net electric flux through the surface is $\boxed{\text{zero}}$.
(b) Some electric field lines from the positive charge to the negative charges will pierce the sphere; however, every line that comes out through the sphere at some point will enter the sphere at some other point.



16. Because the charge is placed at the midpoint, we know from symmetry that the flux is the same through the two ends. From Gauss' law, we have

$$\Phi = Q/\varepsilon_0 = \Phi_{\text{ends}} + \Phi_{\text{curved}};$$

$$(1.2\times10^4\ \text{C})/\varepsilon_0 = 2(4.5\times10^6\ \text{N}\cdot\text{m}^2/\text{C}) + \Phi_{\text{curved}}, \text{ which gives } \Phi_{\text{curved}} = \boxed{4.6\times10^6\ \text{N}\cdot\text{m}^2/\text{C}}.$$

17. (a) The total electric flux through the surface depends only on the enclosed charge:

$$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q/\varepsilon_0;$$

$$-4\times10^2\ \text{N}\cdot\text{m}^2 = Q/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2), \text{ which gives } Q = \boxed{-3.54\times10^{-9}\ \text{C}}.$$

(b) The total electric flux through the closed surface does not depend on the shape of the surface. The enclosed charge is $Q = \boxed{-3.54\times10^{-9}\ \text{C}}$.

(c) Similar to (b), the enclosed charge is $Q = \boxed{-3.54\times10^{-9}\ \text{C}}$.

18. The total electric flux through the surface depends only on the enclosed charge:

$$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q/\varepsilon_0 = (420\times10^{-6}\ \text{C})/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 4.7\times10^7\ \text{N}\cdot\text{m}^2/\text{C}.$$

Because the charge is at the center of the cube, we know from symmetry that each of the six sides has the same flux through it:

$$\Phi_{\text{side}} = (1/6)\Phi_{\text{total}} = (1/6)(4.7\times10^7\ \text{N}\cdot\text{m}^2/\text{C}) = \boxed{7.9\times10^6\ \text{N}\cdot\text{m}^2/\text{C}}.$$

19. Because the charge at the origin is at the center of the cube, we know from symmetry that it will produce a flux out of each side that is 1/6 of the total flux it produces:

$$\Phi_1 = (1/6)\Phi_{\text{charge}} = (1/6)(Q/\varepsilon_0)$$

$$= (1/6)(5\times10^{-8}\ \text{C})/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 9.4\times10^2\ \text{N}\cdot\text{m}^2/\text{C}.$$

Because the uniform field is parallel to the x-axis, it produces no flux through the sides parallel to the x-axis. Through the sides parallel to the yz-plane, the uniform field produces a flux

$$\Phi_2 = EA = (3000\ \text{N/C})(0.20\ \text{m})^2 = 1.2\times10^2\ \text{N}\cdot\text{m}^2/\text{C}.$$

Because this flux enters the cube from the + x-axis and leaves the cube toward the – x-axis, we have

$$\boxed{\begin{array}{l} 9.4\times10^2\ \text{N}\cdot\text{m}^2/\text{C out of the sides parallel to the } xy\text{- or } yz\text{-planes,} \\ 10.6\times10^2\ \text{N}\cdot\text{m}^2/\text{C out of the side perpendicular to the } -x\text{-axis,} \\ 8.2\times10^2\ \text{N}\cdot\text{m}^2/\text{C out of the side perpendicular to the } +x\text{-axis.} \end{array}}$$

20. The gravitational field is

$$\vec{g} = \frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r}.$$

If we compare this to the electric field,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r},$$

we see that $\vec{E} \to \vec{g}$, $Q \to -M$, and $\varepsilon_0 \to 1/4\pi G$. If we make these substitutions in Gauss' law for the electric field, we have

$$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = q/\varepsilon_0 \to \Phi = \oint\!\!\oint \vec{g}\cdot d\vec{A} = -M/(1/4\pi G) = -4\pi GM,$$

which is Gauss' law for the gravitational field, where M is the enclosed mass.

To find the gravitational field within a sphere of uniform mass density, we must select a Gaussian surface. From symmetry, we know that the field is radial, since all directions must be equivalent. If we choose a spherical surface, the field will be constant and parallel to the area vector, so we have

$$\oint\!\!\oint \vec{g}\cdot d\vec{A} = g\oint\!\!\oint dA = -4\pi GM_{\text{enclosed}};$$

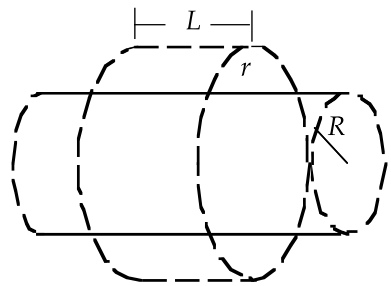
$$g4\pi r^2 = -4\pi G\rho(\tfrac{4}{3}\pi r^3), \text{ which gives } g = -\tfrac{4}{3}\pi G\rho r, \text{ or } g = \boxed{-GMr/R^3 \text{ toward the center, } r < R}.$$

At a point outside the sphere, we know from symmetry that the field is radial, since all directions must be equivalent. Over a spherical surface, the field will be constant and parallel to the area vector, so

$$\oint\!\!\oint \vec{g}\cdot d\vec{A} = g\oint\!\!\oint dA = -4\pi GM_{\text{enclosed}};$$

$$g4\pi r^2 = -4\pi GM, \text{ which gives } g = \boxed{-GM/r^2 \text{ toward the center, } r > R}.$$

21. From symmetry, we know that, outside the charge, the electric field will be radially away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius r, centered on the axis. On the ends of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E}\cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. For Gauss' law, we have



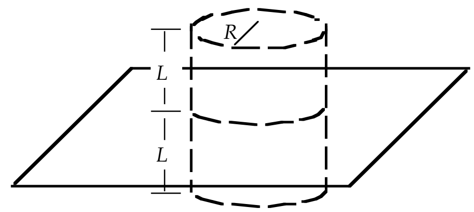
$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = r\pi R^2 L/\varepsilon_0$, which gives

$E = \boxed{\rho R^2/2\varepsilon_0 r}$.

Note that the result is independent of L, as it must be.

22. For a Gaussian surface, we choose a cylinder with radius R and length $2L$, with its axis perpendicular to the plate. Because the charge density of the plate is uniform, we know that the electric field must be perpendicular to the plate and may depend only on the distance from the plate. By arranging the surface so that the ends are equidistant from the plate, we know that the flux through the ends must be the same, while the flux through the side must be zero. When we apply Gauss' law, we have



$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 2EA_{\text{end}} + 0 = Q/\varepsilon_0;$$

$2E\pi R^2 = \sigma\pi R^2/\varepsilon_0$, which gives

$E = \boxed{\sigma/2\varepsilon_0}$.

$\boxed{23}$. For the long rod we have

$E_{\text{rod}} = (1/2\pi\varepsilon_0)\lambda/r = 2(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.5\times10^{-8}\ \text{C/m})/(10^{-2}\ \text{m}) = 1.17\times10^4\ \text{N/C}.$

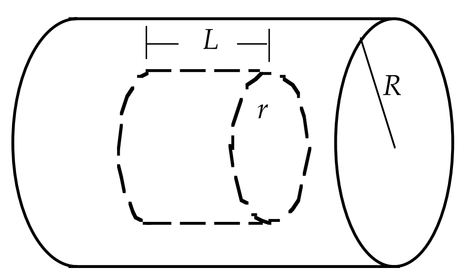
For the point charge we have

$E_{\text{point charge}} = (1/4\pi\varepsilon_0)q/r^2 = (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.5\times10^{-8}\ \text{C})/(10^{-2}\ \text{m}) = 5.85\times10^5\ \text{N/C}.$

The ratio is

$\boxed{E_{\text{rod}}/E_{\text{point charge}} = 0.02 = 2\%}$.

24. From symmetry, we know that the electric field will be radially away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius r, centered on the axis. On the ends of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E}\cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. For Gauss' law, we have



$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \rho\pi r^2 L/\varepsilon_0$, which gives

$E = \boxed{\rho r/2\varepsilon_0}$.

25. We choose a Gaussian surface with a top surface just above the ground and a bottom surface below the ground, each of area 40 acres, and the sides perpendicular to the ground. There is no flux through the sides, because $\vec{E}\cdot d\vec{A} = 0$. There is no flux through the bottom, because $\vec{E} = 0$. When we apply Gauss' law to this surface, we get

$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} + \iint_{\text{bottom}} \vec{E}\cdot d\vec{A}$$

$$= -EA_{\text{top}} + 0 + 0 = Q/\varepsilon_0, \text{ which gives}$$

$$Q = -\varepsilon_0 EA$$

$$= -(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(110\ \text{N/C})(60\ \text{acres})(4\times10^3\ \text{m}^2/\text{acre}) = \boxed{-2.3\times10^{-4}\ \text{C}}.$$

26. We assume that the smaller cylinder is positive. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the axis of the cylinders, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius r, centered on the axis. On the ends of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E}\cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$.

For the region inside the smaller cylinder, $r < r_1$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = 0$, because $q = 0$ inside the Gaussian surface, which gives

$E = \boxed{0 \text{ for } r < r_1}$.

For the region between the cylinders, $r_1 < r < r_2$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \lambda L/\varepsilon_0$, because only the smaller cylinder is inside the Gaussian surface, which gives

$E = \boxed{\lambda/2\pi\varepsilon_0 r \text{ radially out for } r_1 < r < r_2}$.

For the region outside the larger cylinder, $r_2 < r$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \lambda L/\varepsilon_0$, because only the smaller cylinder is inside the Gaussian surface, which gives

$E = \boxed{0 \text{ for } r_2 < r}$.

27. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the center of the balloon, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere centered on the balloon with radius $r = 50$ cm. On this surface, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. When we apply Gauss' law, we get

$$\oiint \vec{E}\cdot d\vec{A} = E4\pi r^2 = Q/\varepsilon_0, \text{ which gives}$$

$$E = (1/4\pi\varepsilon_0)(Q/r^2)$$

$$= (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\times10^{-7}\ \text{C})/(0.50\ \text{m})^2$$

$$= 1.8\times10^4\ \text{N/C};$$

$$\vec{E} = \boxed{(1.8\times10^4\ \text{N/C})\ \hat{r}}.$$

If the balloon shrinks, the enclosed charge does not change, so the electric field will be the same:

$$\vec{E} = \boxed{(1.8\times10^4\ \text{N/C})\ \hat{r}}.$$

28. We assume that the wire is positive. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the axis, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius r, centered on the axis. On the ends of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E}\cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. For the region between the wire and the cylinder, $r_1 < r < r_2$, we apply Gauss' law:

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \lambda L/\varepsilon_0$, because only the wire is inside the Gaussian surface, which gives

$$E = \lambda/2\pi\varepsilon_0 r$$
$$= (8.5\times 10^{-9}\ \text{C/cm})(10^2\ \text{cm/m})/2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)r = \boxed{(1.6\times10^4/r)\ \text{N/C radially out}}.$$

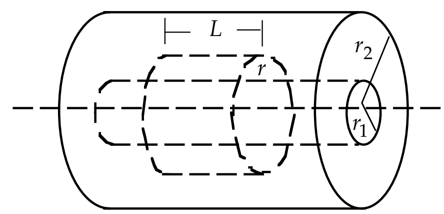
If the radius of the Gaussian surface is reduced to the surface of the wire, λ does not change, so we have

$$E_{\text{wire}} = (1.6\times10^4)/(5.0\times10^{-5}\ \text{m}) = \boxed{3.1\times10^8\ \text{N/C}}.$$

If the radius of the Gaussian surface increases to the inner surface of the cylinder, λ does not change, so

$$E_{\text{cylinder}} = (1.6\times10^4)/(3\times10^{-2}\ \text{m}) = \boxed{5.1\times10^5\ \text{N/C}},$$ much less than that in the wire.

$\boxed{29}$. From the symmetry of the charge distribution, we know that the electric field must be radial, away from the axis of the cylinder, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius r, centered on the axis. On the ends of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E}\cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\,dA$. For the region where $r < r_1$, we apply Gauss' law:



$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = 0$, because there is no charge inside the Gaussian surface, which gives

$$E = \boxed{0 \text{ for } r < r_1}.$$

For the region where $r_1 < r < r_2$, we apply Gauss' law:

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \rho(\pi r^2 - \pi r_1^2)L/\varepsilon_0$, which gives

$$\vec{E} = \boxed{[\rho(r^2 - r_1^2)/2\varepsilon_0 r]\hat{r} \ \text{ for } \ r_1 < r < r_2}.$$

For the region where $r_2 < r$, we apply Gauss' law:

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{ends}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi rL = \rho(\pi r_2^2 - \pi r_1^2)L/\varepsilon_0$, which gives

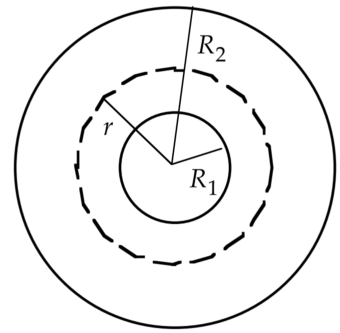
$$\vec{E} = \boxed{[\rho(r_2^2 - r_1^2)/2\varepsilon_0 r]\hat{r} \ \text{ for } \ r_2 < r}.$$

30. The electric field just outside a charged surface is

$E = \sigma/\varepsilon_0$, where σ is the charge per unit area.

Because the charge density is uniform, for the total charge we have

$$q = \sigma A = \varepsilon_0 E(2\pi Rh + 2\pi R^2) = \varepsilon_0 E 2\pi R(h + R)$$
$$= (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\times10^6\ \text{N/C})2\pi(0.40\times10^{-2}\ \text{m})[(7.0\ \text{cm} + 0.40\ \text{cm})(10^{-2}\ \text{m/cm})]$$
$$= \boxed{3.3\times10^{-8}\ \text{C}}.$$



31. From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere of radius r. On this surface, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E}\cdot d\vec{A} = E\, dA$. The charge density is

$\rho = Q/[\frac{4}{3}\pi(R_2{}^3 - R_1{}^3)]$.

For the region where $r < R_1$, we apply Gauss' law:

$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = 0$, because there is no charge inside the Gaussian surface, which gives

$E = \boxed{0 \text{ for } r < R_1}$.

For the region where $R_1 < r < R_2$, we apply Gauss' law:

$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = \rho\frac{4}{3}\pi(r^3 - R_1{}^3)/\varepsilon_0$, which gives

$E = \rho\frac{4}{3}\pi(r^3 - R_1{}^3)/(4\pi\varepsilon_0 r^2) = Q(r^3 - R_1{}^3)/4\pi\varepsilon_0(R_2{}^3 - R_1{}^3)r^2$;

$\vec{E} = \boxed{[Q(r^3 - R_1{}^3)/4\pi\varepsilon_0(R_2{}^3 - R_1{}^3)r^2]\hat{r} \text{ for } R_1 < r < R_2}$.

For the region where $R_2 < r$, we apply Gauss' law:

$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0$;

$E4\pi r^2 = Q/\varepsilon_0$, which gives

$E = (Q/4\pi\varepsilon_0 r^2)$;

$\vec{E} = \boxed{(Q/4\pi\varepsilon_0 r^2)\hat{r} \text{ for } R_2 < r}$.

32. (a) From Gauss' law $\Phi = \oiint \vec{E}\cdot d\vec{A} = EA = E(4\pi r^2) = Q/\varepsilon_0$, so the electric field a distance r from the center of the shell is

$E = Q/4\pi\varepsilon_0 r^2$,

where Q is the net charge enclosed in the spherical region of radius r. If we choose r to be just slightly greater than R, so that the Gaussian surface is inside the metal shell, then $E = 0$, and hence $Q = 0$. But Q is the sum of q and the charge q' on the inner surface of the shell; i.e.,

$Q = q + q' = 0$. Thus $q' = \boxed{-q}$ on the inner surface of the shell.

(b) Since the shell as a whole is charge-neutral, the charge on its outer surface must be $\boxed{+q}$.

(c) For $r < d < R$ the charge enclosed by the Gaussian surface is $Q = q$, so

$\boxed{E = q/4\pi\varepsilon_0 r^2}$, radially outward if $q > 0$ and inward if $q < 0$.

33. (a) The electric field due to a single infinite plane of charge density σ is $\sigma/2\varepsilon_0$, as we learned from the textbook. If we set up an x-axis pointing perpendicularly from the first sheet (with charge density σ_1) to the second sheet (with charge density σ_2), where the first sheet is located at $x = 0$ and the second one at $x = a$, then by superposition

$\vec{E} = \boxed{-(\sigma_1/2\varepsilon_0 + \sigma_2/2\varepsilon_0)\hat{i} \quad (x < 0)}$,

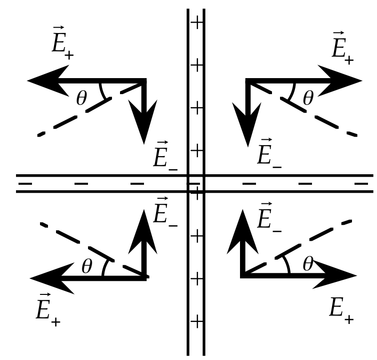
$\vec{E} = \boxed{(\sigma_1/2\varepsilon_0 - \sigma_2/2\varepsilon_0)\hat{i} \quad (0 < x < a)}$,

$\vec{E} = \boxed{(\sigma_1/2\varepsilon_0 + \sigma_2/2\varepsilon_0)\hat{i} \quad (x > a)}$.

(b) Since the metal plate is uncharged, when placed in between the two charged plates the densities of the induced charges on both of its surfaces must have the same magnitude and opposite signs. The net electric field due to this pair of surfaces is zero outside the metal, so $\vec{E}$ remains the same as part (a) above, except for the interior of the metal, where the field of the induced charge is not zero and cancels with that of the two charged plates, resulting in a zero net field.

34. The positive sheet produces an electric field directed away from the sheet with a magnitude
$$E_+ = \sigma_+/2\varepsilon_0 = (5\times10^{-6}\,\mathrm{C/m^2})/2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}) = 2.8\times10^5\,\mathrm{N/C}.$$
The negative sheet produces an electric field directed toward the sheet with a magnitude
$$E_- = \sigma_-/2\varepsilon_0 = (3\times10^{-6}\,\mathrm{C/m^2})/2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}) = 1.7\times10^5\,\mathrm{N/C}.$$
Between the sheets, the two fields are in the same direction, so we have
$$E_{\text{between}} = E_+ + E_- = 2.8\times10^5\,\mathrm{N/C} + 1.7\times10^5\,\mathrm{N/C}$$
$$= \boxed{4.5\times10^5\,\mathrm{N/C}\text{ toward the negative sheet}}.$$
Outside the sheets, the two fields are in opposite directions, so we have
$$E_{\text{outside}} = E_+ - E_- = 2.8\times10^5\,\mathrm{N/C} - 1.7\times10^5\,\mathrm{N/C} = \boxed{1.1\times10^5\,\mathrm{N/C}\text{ away from the sheets}}.$$

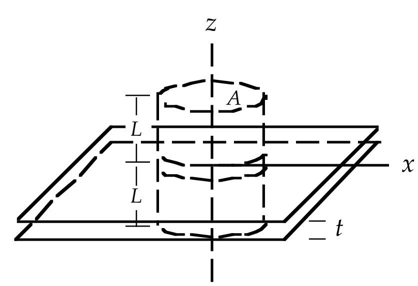
35. The positive sheet produces an electric field directed away from the sheet with a magnitude
$$E_+ = \sigma_+/2\varepsilon_0$$
$$= (5\times10^{-6}\,\mathrm{C/m^2})/2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}) = 2.8\times10^5\,\mathrm{N/C}.$$
The negative sheet produces an electric field directed toward the sheet with a magnitude
$$E_- = \sigma_-/2\varepsilon_0$$
$$= (3\times10^{-6}\,\mathrm{C/m^2})/2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}) = 1.7\times10^5\,\mathrm{N/C}.$$
If we consider the 1st quadrant, because the fields are perpendicular, we have



$$E = (E_+^2 + E_-^2)^{1/2}$$
$$= [(2.8\times10^5\,\mathrm{N/C})^2 + (1.7\times10^5\,\mathrm{N/C})^2]^{1/2} = \boxed{3.3\times10^5\,\mathrm{N/C}}, \quad \text{and}$$
$$\tan\theta = E_-/E_+ = (1.7\times10^5\,\mathrm{N/C})/(2.8\times10^5\,\mathrm{N/C}) = 0.61, \text{ which gives } \theta = \boxed{31^\circ}.$$
From the diagram, we see that the fields in the other quadrants are mirror images of the field in the 1st quadrant:

$\boxed{\text{1st quadrant: } E \text{ at } -\theta}$; $\boxed{\text{2nd quadrant: } E \text{ at } 180^\circ + \theta}$;
$\boxed{\text{3rd quadrant: } E \text{ at } 180^\circ - \theta}$; $\boxed{\text{4th quadrant: } E \text{ at } \theta}$.

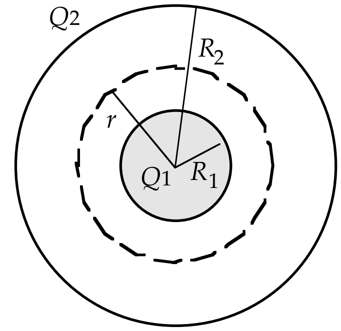
36. Because the slab is infinite, we know from symmetry that the field must be perpendicular to the slab, with a constant magnitude for a constant distance from the slab. If r is positive, the field will be away from the slab. For a Gaussian surface we choose a cylinder of length $2L$ and area A, centered on the axis. On the curved side of this surface, the electric field is not constant but $\vec{E}$ and $d\vec{A}$ are perpen-dicular, so $\vec{E}\cdot d\vec{A} = 0$. On the ends, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel,so $\vec{E}\cdot d\vec{A} = E\,dA$.



To find the field outside the slab, we use the fact that the field will be away from the slab. If we place our Gaussian cylinder so that one end is at $z = L$ and the other end is at $z = -L$, the fields at each end will be directed out of the surface and have the same magnitude. When we apply Gauss' law, we have
$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{bottom}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$
$$= EA + EA + 0 = Q/\varepsilon_0; \quad \text{which gives}$$
$E = \boxed{\rho t/2\varepsilon_0 \text{ away from the slab, where } z > t/2 \text{ or } z < -t/2}$. The field is uniform outside the slab.

To find the field inside the slab, we use the same Gaussian surface, with the ends of the cylinder at $\pm z$, $z < t/2$. The enclosed charge is only that part of the slab inside the cylinder. Apply Gauss' law:
$$\oiint \vec{E}\cdot d\vec{A} = \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{bottom}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$
$$= EA + EA + 0 = Q/\varepsilon_0; \ \text{which gives}$$
$$\vec{E} = \boxed{(\rho z/\varepsilon_0)\hat{k} \quad \text{where} \quad -t/2 < z < t/2}.$$

37. From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. For a Gaussian surface we choose a sphere of radius r. On this surface, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E} \cdot d\vec{A} = E\,dA$. The inner charge density is



$$\rho_1 = Q_1/(\tfrac{4}{3}\pi R_1^3) = 3Q_1/4\pi R_1^3$$
$$= 3(-2\times10^{-6}\text{ C})/4\pi(0.03\text{ m})^3 = -1.77\times10^{-2}\text{ C/m}^3.$$

The outer surface charge density is

$$\sigma_2 = Q_2/4\pi R_2^2 = (5\times10^{-6}\text{ C})/4\pi(0.08\text{ m})^2 = 6.2\times10^{-5}\text{ C/m}^2.$$

For the region where $r < R_1$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0;\ E4\pi r^2 = \rho_1(\tfrac{4}{3}\pi r^3)/\varepsilon_0,\ \text{which gives}$$
$$E = \rho_1 r/3\varepsilon_0 = [(-1.77\times10^{-2}\text{ C/m}^3)/3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)]r = (-6.7\times10^8)r\ \text{N/C with } r \text{ in m};$$
$$\vec{E} = \boxed{(-6.7\times10^8\ \vec{r})\text{N/C with } r \text{ in m}},\ \text{where } r < 3\text{ cm}.$$

For the region where $R_1 < r < R_2$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0;\ E4\pi r^2 = Q_1/\varepsilon_0,\ \text{which gives}$$
$$E = (1/4\pi\varepsilon_0)Q_1/r^2 = (9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-2\times10^{-6}\text{ C})/r^2 = (-1.8\times10^4)/r^2\text{ N/C with } r \text{ in m};$$
$$\vec{E} = \boxed{[(-1.8\times10^4/r^2)\,\hat{r}]\text{ N/C with } r \text{ in m}},\ \text{where } 3\text{ cm} < r < 8\text{ cm}.$$

For the region where $R_2 < r$, we apply Gauss' law:

$$\oiint \vec{E}\cdot d\vec{A} = EA = Q/\varepsilon_0;\ E4\pi r^2 = (Q_1+Q_2)/\varepsilon_0,\ \text{which gives}$$
$$E = (Q_1+Q_2)/4\pi\varepsilon_0 r^2 = (9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)[(-2\times10^{-6}\text{ C}) + (5\times10^{-6}\text{ C})]/r^2)\text{ N/C}$$
$$= (2.7\times10^4)/r^2\text{ N/C with } r \text{ in m};$$
$$\vec{E} = \boxed{[(2.7\times10^4/r^2)\,\hat{r}]\text{ N/C with } r \text{ in m}},\ \text{where } 8\text{ cm} < r.$$

38. The charge within the sphere with $r = a$ is $Q_1 = \rho_0(\tfrac{4}{3}\pi a^3) = \tfrac{4}{3}\pi a^3\rho_0$.
We find the charge within the spherical shell from $r = a$ to $r = R$ by choosing a spherical shell of radius r and thickness dr, and integrating:

$$Q_2 = \int_a^R \rho 4\pi r^2\,dr = \int_a^R \rho_0\left(\frac{r-R}{a-R}\right)4\pi r^2\,dr = \frac{4\pi\rho_0}{(a-R)}\int_a^R (r-R)r^2\,dr$$
$$= \frac{4\pi\rho_0}{(a-R)}\left[\frac{1}{4}\left(R^4-a^4\right)-\frac{R}{3}\left(R^3-a^3\right)\right] = \frac{\pi\rho_0}{3}\frac{\left(4Ra^3-3a^4-R^4\right)}{(a-R)}.$$

The flux through each of the spherical surfaces depends only on the enclosed charge, so we have

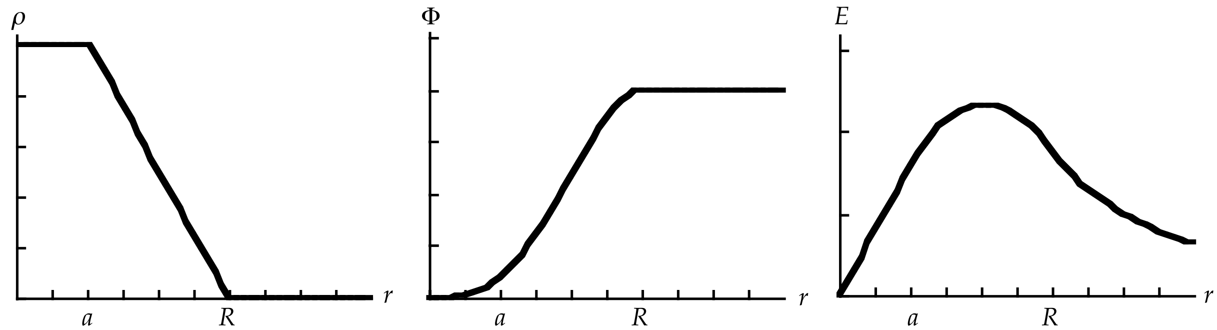
$$\Phi_{r=a} = Q_{\text{enclosed}}/\varepsilon_0 = Q_1/\varepsilon_0 = \boxed{\tfrac{4}{3}\pi a^3\rho_0/\varepsilon_0}.$$
$$\Phi_{r=R} = Q_{\text{enclosed}}/\varepsilon_0 = (Q_1+Q_2)/\varepsilon_0$$
$$= (4\pi a^3\rho_0/3\varepsilon_0) + (\pi\rho_0/3\varepsilon_0)[(4Ra^3-3a^4-R^4)/(a-R)]$$
$$= (\pi\rho_0/3\varepsilon_0)[(4a^4-4a^3R+4Ra^3-3a^4-R^4)/(a-R)]$$
$$= (\pi\rho_0/3\varepsilon_0)[(a^4-R^4)/(a-R)] = \boxed{(\pi\rho_0/3\varepsilon_0)(a^2+R^2)(a+R)}.$$
$$\Phi_{r=10R} = Q_{\text{enclosed}}/\varepsilon_0 = (Q_1+Q_2)/\varepsilon_0 = \Phi_{r=R} = \boxed{(\pi\rho_0/3\varepsilon_0)(a^2+R^2)(a+R)}.$$

From the symmetry of the charge distribution, we know that the electric field must be radial, with a magnitude independent of the direction. On each of the spheres, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have

$$\Phi = \oiint \vec{E}\cdot d\vec{A} = EA,\ \text{ or }\ E = \Phi/A.$$
$$E_{r=a} = \Phi_{r=a}/4\pi a^2 = \boxed{\rho_0 a/3\varepsilon_0\ \text{radial}}.$$
$$E_{r=R} = \Phi_{r=R}/4\pi R^2 = \boxed{(\rho_0/12\varepsilon_0 R^2)(a^2+R^2)(a+R)\ \text{radial}}.$$
$$E_{r=10R} = \Phi_{r=10R}/4\pi(10R)^2 = \boxed{(\rho_0/1200\varepsilon_0 R^2)(a^2+R^2)(a+R)\ \text{radial}}.$$

39. If we apply the solution for Problem 38 to the general point r, we have

	Charge density	Flux	Electric field
$r < a$:	ρ_0	$(4\pi\rho_0/3\varepsilon_0)r^3$	$(\rho_0/3\varepsilon_0)r$
$a < r < R$:	$\rho_0(r - R)/(a - R)$	$(\pi\rho_0/3\varepsilon_0)(a^4 - 4ar^3 - 3r^4)/(a - r)$	$(\rho_0/12\varepsilon_0)(a^4 - 4ar^3 - 3r^4)/(a - r)r^2$
$R < r$:	0	$(\pi\rho_0/3\varepsilon_0)(a^2 + R^2)(a + R)$	$(\rho_0/12\varepsilon_0)(a^2 + R^2)(a + R)/r^2$



40. Each plate produces a downward uniform electric field, so the electric field between the plates is

$$E = (\sigma_+ + \sigma_-)/2\varepsilon_0 = [(6.5 + 4.8) \times 10^{-6}\,\mathrm{C/m^2}]/2\,(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})$$
$$= \boxed{6.4 \times 10^5\ \mathrm{N/C}\quad \text{away from the positive plate}}.$$

41. In the region where $r < R$, we are inside both spherical shells, so we must have

$$r < R:\ \vec{E} = \boxed{0}.$$

In the region where $R < r < 2R$, we are outside the inner shell, so it looks like a point charge; we are inside the outer shell, so it contributes no field:

$$R < r < 2R:\ \vec{E} = \boxed{(q/\,4\pi\varepsilon_0 r^2)\,\hat{r}}.$$

In the region where $2R < r$, we are outside both shells, so each one looks like a point charge, or a net charge of $-q$:

$$2R < r:\ \vec{E} = \boxed{-(q/\,4\pi\varepsilon_0 r^2)\,\hat{r}}.$$

42. The electric field between the plates is

$E = \sigma/\varepsilon_0$, so the charge on each plate is

$$Q = \sigma A = \varepsilon_0 AE$$
$$= (8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(0.1\ \mathrm{m})^2(3 \times 10^6\ \mathrm{N/C}) = \boxed{2.7 \times 10^{-7}\,\mathrm{C}}.$$

43. For a conducting spherical surface, the radial electric field just outside the surface is

$E = \sigma/\varepsilon_0$, so we have

$$\sigma_{\text{max}} = \varepsilon_0 E_{\text{max}} = (8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(3 \times 10^6\ \mathrm{N/C}) = \boxed{2.7 \times 10^{-5}\,\mathrm{C/m^2}}.$$

44. The flux through a Gaussian surface depends on the enclosed charge.

For the surface at a radius of 50 cm, we have

$$\Phi_2 = (Q_{\text{sphere}} + Q_{\text{shell}})/\varepsilon_0.$$

For the surface at a radius of 30 cm, we have

$$\Phi_1 = Q_{\text{sphere}}/\varepsilon_0.$$

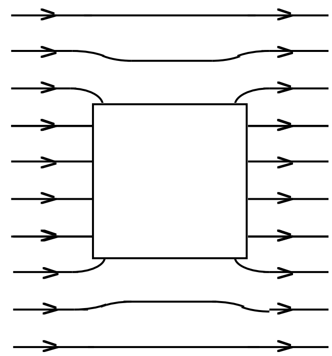
For the ratio we have

$$\Phi_2/\Phi_1 = (Q_{\text{sphere}} + Q_{\text{shell}})/Q_{\text{sphere}} = 1 + Q_{\text{shell}}/Q_{\text{sphere}};$$
$$(1.6 \times 10^{-7}\ \mathrm{N\cdot m^2/C})/(0.8 \times 10^{-7}\ \mathrm{N\cdot m^2/C}) = 1 + Q_{\text{shell}}/Q_{\text{sphere}},\ \text{ which gives } \boxed{Q_{\text{sphere}}/Q_{\text{shell}} = 1}.$$

45. For the ratio of charge densities, we have

$$\sigma_{\text{sphere}}/\sigma_{\text{shell}} = (Q_{\text{sphere}}/Q_{\text{shell}})(R_{\text{shell}}/R_{\text{sphere}})^2 = (1)(35\ \mathrm{cm}/25\ \mathrm{cm})^2 = \boxed{1.96}.$$

46.



47. From Gauss' law, we know that the flux through a Gaussian surface depends on the enclosed charge:

$$\Phi = Q_{\text{enclosed}}/\varepsilon_0.$$

For the region where $a < r < b$, we have $\Phi_1 = Q/\varepsilon_0$, so the enclosed charge is Q. This must be the total charge on the inner sphere. Because the sphere is conducting, the charge is located uniformly on the surface, with the charge density

$$\sigma_{\text{inner sphere}} = \boxed{Q/4\pi a^2}.$$

For the region within the shell, $b < r < R$, the electric flux is 0, so the net enclosed charge is 0. Because there is a charge Q on the inner sphere, there must be a charge $-Q$ on the inner surface of the shell. Because the shell is conducting, the charge is located uniformly on the surface, with the charge density

$$\sigma_{\text{shell, inside}} = \boxed{-Q/4\pi b^2}.$$

For the region outside the shell, $R < r$, we have $\Phi_2 = 2Q/\varepsilon_0$, so the net enclosed charge is $2Q$. Because there is a charge Q on the inner sphere, there must be a charge Q on the shell. Because there is a charge $-Q$ on the inner surface, there must be a charge $+2Q$ on the outer surface, located uniformly on the surface, with the charge density

$$\sigma_{\text{shell, outside}} = +2Q/4\pi R^2 = \boxed{+Q/2\pi R^2}.$$

48. If we choose a Gaussian surface around the earth, we have

$\oiint \vec{E}\cdot d\vec{A} = -EA = Q/\varepsilon_0$, so we have

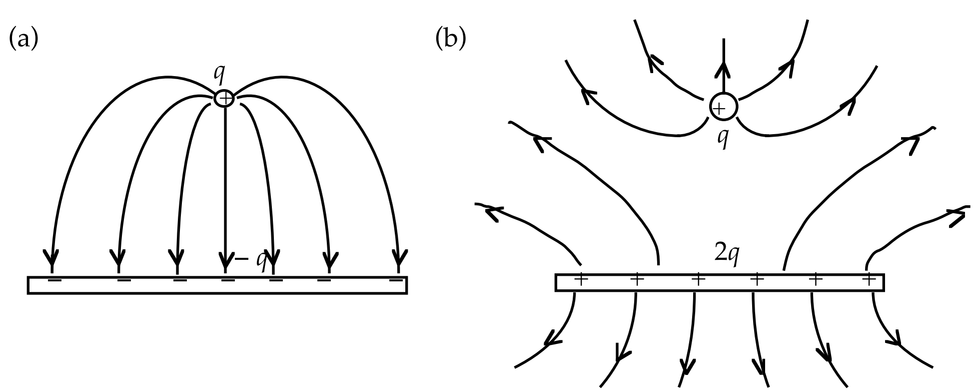
$$Q = -\varepsilon_0 4\pi R^2 E$$

$$= -[1/(9\times10^9\ \mathrm{N\cdot m^2/C^2})](6.37\times10^6\ \mathrm{m})^2(100\ \mathrm{N/C}) = \boxed{-4.5\times10^5\ \mathrm{C,\ on\ the\ surface}}.$$

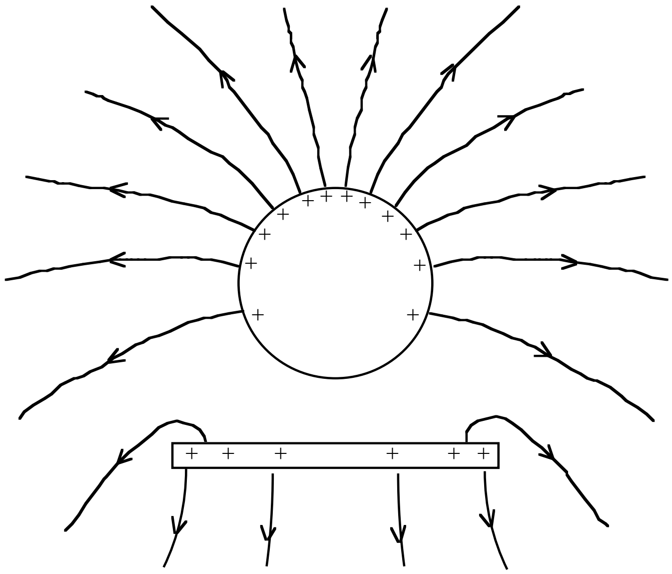
The surface charge density is

$$\sigma = Q/A = \varepsilon_0 E = (8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(-100\ \mathrm{N/C}) = \boxed{-8.9\times10^{-10}\ \mathrm{C/m^2}}.$$

49.



50.



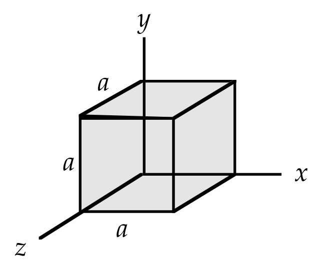
51. We find the flux through a side from

$\Phi = \iint \vec{E} \cdot d\vec{A}$.

For the sides perpendicular to the x-axis, we have

$\Phi_{x=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-\,dA\,\hat{i}) = -\,b(0)^2 a^2 = \boxed{0}$;

$\Phi_{x=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+\,dA\,\hat{i}) = b(a)^2 a^2 = \boxed{ba^4}$.



For the sides parallel to the x-axis, we have

$\Phi_{y=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-\,dA\,\hat{j}) = \boxed{0}$;

$\Phi_{y=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+\,dA\,\hat{j}) = \boxed{0}$;

$\Phi_{z=0} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (-\,dA\,\hat{k}) = \boxed{0}$;

$\Phi_{z=a} = \iint \vec{E} \cdot d\vec{A} = \iint (bx^2 \hat{i}) \cdot (+\,dA\,\hat{k}) = \boxed{0}$.

We use Gauss' law to find the enclosed charge:

$\Phi = \oiint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$;

$b\,a^4 = q/\varepsilon_0$, which gives $q = \boxed{\varepsilon_0 b\,a^4}$.

52. From Example 23–7, we know that the electric field inside the sphere is

$E = (Q/4\pi\varepsilon_0)(r/R^3)$ radial.

Because the charges have opposite signs, the force on the point charge is toward the center of the sphere, with magnitude

$F = qQr/4\pi\varepsilon_0 R^3$,

and is a restoring force proportional to the displacement from the center, as in simple harmonic motion, with an effective force constant of

$k = qQ/4\pi\varepsilon_0 R^3$.

The resulting motion, with $r = R$ at $t = 0$, is

$\boxed{r = R\cos(\omega t)}$, with $\omega = (k/m)^{1/2}$; the motion is simple harmonic.

The period of the motion is

$\tau = 2\pi/\omega = 2\pi(m/k)^{1/2} = \boxed{2\pi(4\pi\varepsilon_0 mR^3/qQ)^{1/2}}$.

The total energy is the initial potential energy:

$E = U = \frac{1}{2}kR^2 = \frac{1}{2}(qQ/4\pi\varepsilon_0 R^3)R^2 = \boxed{qQ/8\pi\varepsilon_0 R}$.

53. (a) Since no charge is present in the region enclosed by the cap and the flat surface, any electric field lines passing through the flat surface will also pass through the cap. So the electric flux through the flat surface is the same as that through the cap. Since the area of the cap is 0.067, or 6.7%, of that of the sphere, the flux Φ through the cap is also 6.7% of Φ_0, the flux through the entire sphere:

$$\Phi = 0.067\ \Phi_0 = \boxed{0.067(Q/\varepsilon_0)},$$

where we noted that $\Phi_0 = Q/\varepsilon_0$, due to Gauss' Law.

(b) Any point on the boundary of the flat surface is also on the Gaussian sphere, so it is a distance R from the charge Q. The magnitude of the electric field there is therefore

$$E = \boxed{Q/4\pi\varepsilon_0 R^2}.$$

54. (a) The total charge Q on the sphere satisfies Gauss' law:

$$\Phi = Q/\varepsilon_0\ ;\ \text{or}$$

$$Q = \Phi\varepsilon_0 = (17.1\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)$$

$$= \boxed{1.51\times10^{-10}\ \text{C}}.$$

(b) The electric flux through the upper hemisphere is greater than that through the lower hemisphere, so the charge distribution is $\boxed{\text{non-uniform}}$.

(c) The sphere is made of $\boxed{\text{insulating material}}$. Otherwise, since it is uniform the charge distribution should also be uniform (as the charges are free to move on the surface of a conductor), and the fluxes through the two hemispheres would be identical.

55. We find the flux through the surface from

$$\Phi = \iint \vec{E}\cdot d\vec{A}.$$

Because $\vec{E}$ and $d\vec{A}$ are constant vectors, we have

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$$

$$= E(L)(L/\cos\theta)(\cos\theta) = EL^2(\cos\theta/\cos\theta) = EL^2.$$

We choose a Gaussian surface by using the sides of the tube and ends at two different angles. Because there is no charge enclosed, the net flux through the surface is 0. There is no flux through the sides of the tube, so the flux that enters one end must be the same as that which exits the other end, and thus independent of angle.

56. (a) The point $r = 0.50$ m is outside the inner sphere, so it is equivalent to a point charge. The point is inside the outer sphere, so it makes no contribution to the electric field. The total field is

$$E_a = (Q_{\text{inner}}/4\pi\varepsilon_0 r_a^2) + 0 = \sigma_{\text{inner}}4\pi r_{\text{inner}}^2/4\pi\varepsilon_0 r_a^2 = \sigma_{\text{inner}} r_{\text{inner}}^2/\varepsilon_0 r_a^2$$

$$= (16\times10^{-6}\ \text{C/m}^2)(0.25\ \text{m})^2/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.50\ \text{m})^2$$

$$= \boxed{4.5\times10^5\ \text{N/C radial}}.$$

(b) The point $r = 0.70$ m is outside the inner sphere, so it is equivalent to a point charge. The point is inside the outer sphere, so it makes no contribution to the electric field. The total field is

$$E_b = (Q_{\text{inner}}/4\pi\varepsilon_0 r_b^2) + 0 = \sigma_{\text{inner}}4\pi r_{\text{inner}}^2/4\pi\varepsilon_0 r_a^2 = \sigma_{\text{inner}} r_{\text{inner}}^2/\varepsilon_0 r_a^2$$

$$= (16\times10^{-6}\ \text{C/m}^2)(0.25\ \text{m})^2/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.70\ \text{m})^2$$

$$= \boxed{2.3\times10^5\ \text{N/C radial}}.$$

(c) Because the outer shell does not contribute to the electric field inside, there will be $\boxed{\text{no change}}$.

(d) The point $r = 1.0$ m is outside both spheres, so each is equivalent to a point charge. The total field is

$$E_d = (Q_1 + Q_2)/4\pi\varepsilon_0 r_d^2 = (\sigma_{\text{inner}}4\pi r_{\text{inner}}^2 + \sigma_{\text{outer}}4\pi r_{\text{outer}}^2)/4\pi\varepsilon_0 r_d^2$$

$$= [(\sigma_{\text{inner}} r_{\text{inner}}^2) + (\sigma_{\text{outer}} r_{\text{outer}}^2)]/\varepsilon_0 r_d^2$$

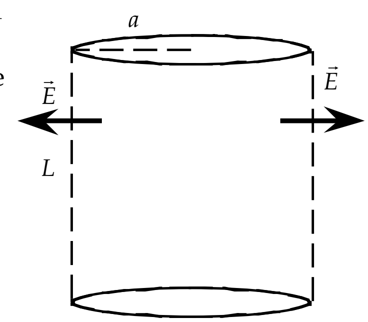
$$= [(16\times10^{-6}\ \text{C/m}^2)(0.25\ \text{m})^2 + (8\times10^{-6}\ \text{C/m}^2)(0.75\ \text{m})^2]/[(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.5\ \text{m})^2]$$

$$= \boxed{2.8\times10^5\ \text{N/C radial}}.$$

57. Because there is no charge enclosed by the tetrahedron, the net flux through all sides is 0:
$\Phi_{net} = \Phi_{upper\ sides} + \Phi_{bottom}$.
Thus we find the flux through the three upper sides from
$$\Phi_{upper\ sides} = -\Phi_{bottom} = -(E\hat{k}) \cdot A(-\hat{k}) = +E(\tfrac{1}{2}L)(L\sin 60°) = \boxed{0.433EL^2}.$$

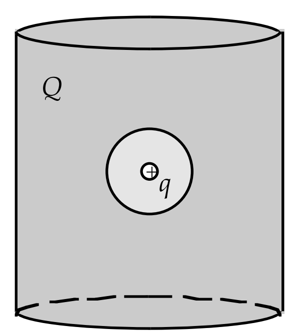
58. If we choose a sphere of radius $r < R$ as a Gaussian surface, we have
$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E4\pi r^2 = Q_{enclosed}/\varepsilon_0, \quad \text{or} \quad Q_{enclosed} = \varepsilon_0 E4\pi r^2.$$
We set up the integral to find the enclosed charge by using a spherical shell of radius r' and thickness dr' for the differential element. We also write the right-hand side as an integral:
$$\int_0^r \rho 4\pi r'^2\, dr' = \varepsilon_0 E4\pi 2\int_0^r r'\, dr'.$$
Comparing the two integrands, we see that $\boxed{\rho \propto 1/r}$.
As $r \to 0$ at the center, $\boxed{\rho \to \infty}$, because the volume of a sphere approaches 0 faster than the area does.

$\boxed{59}$. From the symmetry of the field we construct a Gaussian surface which is a cylinder of length L and radius a with its axis along the axis of the field. Because the field is parallel to the ends of the cylinder, we have



$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E2\pi aL = Q_{enclosed}/\varepsilon_0.$$
If there is a charge distribution $\rho(r)$ within the cylinder, we have
$$Q_{enclosed} = \int_0^a 2\pi rL\rho(r)\, dr. \text{ Thus}$$
$$\varepsilon_0 E2\pi aL = \int_0^a 2\pi rL\rho(r)\, dr, \text{ or } \varepsilon_0 Ea = \int_0^a r\rho(r)\, dr.$$
We can write the left-hand side as an integral to get
$$\varepsilon_0 E\int_0^a dr = \int_0^a r\rho(r)\, dr.$$
Comparing the two integrands, we see that
$\rho(r) = \boxed{\varepsilon_0 E/r}$.
Note that this function diverges when $r \to 0$. The required field can be set up only beginning at some distance r_0 from the axis. Within r_0 only the total charge has to correspond to the required field:
$q/L = \varepsilon_0 E2\pi r_0$, which is finite.

60. (a) For a Gaussian surface within the cylinder just outside the spherical cavity, we have
$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = Q_{enclosed}/\varepsilon_0.$$
Because the field must be 0 inside a conductor, the enclosed charge must be zero. With a charge + 0.12 mC at the center, there must be a charge of $\boxed{-0.12\text{ mC}}$ on the surface of the cavity.



(b) There can be no free static charge inside the conductor. If – 0.12 mC of the total charge of – 0.55 mC on the cylinder resides on the inner surface, the remaining $\boxed{-0.43\text{ mC}}$ must be on the outside surface.

61. We can express the linear electric field as $\vec{E}(x) = bx\hat{i}$. At $x = 0.5$ m, we have

$$\vec{E}(0.5\text{ m}) = (3000\text{ N/C})\hat{i} = b(0.5\text{ m})\hat{i}\text{, which gives } b = 6000\text{ N/C}\cdot\text{m.}$$

We call the area of the surface oriented in the yz-plane A. We choose a Gaussian surface consisting of the boundary of the region parallel to the x-axis and ends of area A at $x = 0$ and $x = x$. Because there is no flux through the sides and the field is constant at each end, we have

$$\Phi = \oint\!\!\oint \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0;$$

$$E(x)\hat{i}\cdot A\hat{i} + E(0)\hat{i}\cdot A(-\hat{i}) = (1/\varepsilon_0)\int \rho A\,dx\text{, which gives}$$

$$\int \rho\,dx = \varepsilon_0(bx) = +b\varepsilon_0 x.$$

Comparing the two sides, we see that

$$\rho = +b\varepsilon_0 = \boxed{+6000\varepsilon_0\text{ C/m}^3\text{ (constant)}}.$$

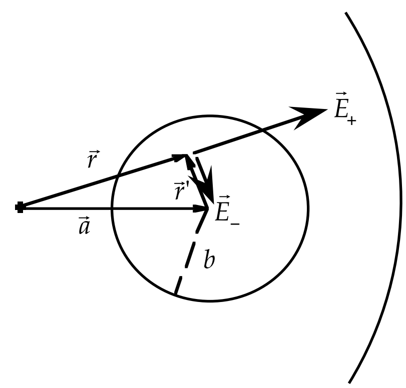
Note that for the field to be 0 at $x = 0$, there must be external charges at $x < 0$.

62. From symmetry, we know that the field inside a uniformly charged sphere must be radial and depends only on the distance from the center. We choose a spherical surface with $r < R$ for a Gaussian surface. Because $\vec{E}$ and $d\vec{A}$ are parallel, we have

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0;$$

$$E(4\pi r^2) = \rho(\tfrac{4}{3}\pi r^3)/\varepsilon_0\text{, which gives } E = \rho r/3\varepsilon_0 \text{ radial, or}$$

$$\vec{E} = (\rho/3\varepsilon_0)\vec{r}.$$



We create the cavity by adding to the original sphere, with density ρ,
a sphere with density $-\rho$ and radius b, centered at $\vec{a}$. Within the cavity, we are inside both spheres, so their fields are

$$\vec{E}_+ = (\rho/3\varepsilon_0)\vec{r} \quad\text{and}\quad \vec{E}_- = (-\rho/3\varepsilon_0)\vec{r}\,',$$

where $\vec{r}\,'$ is the radius vector for the cavity.
From the diagram, we have $\vec{r} = \vec{a} + \vec{r}\,'$, so the total field is

$$\vec{E} = \vec{E}_+ + \vec{E}_- = (\rho/3\varepsilon_0)\vec{r} + (-\rho/3\varepsilon_0)\vec{r}\,' = (\rho\vec{r}/3\varepsilon_0) - [\rho(\vec{r} - \vec{a})/3\varepsilon_0];$$

$$\vec{E} = \boxed{(\rho/3\varepsilon_0)\vec{a}}.$$

Note that the field inside the cavity is uniform.

63. We assume that the positive test charge is at a stable equilibrium point. The electric field there from the other charges is 0. A short distance from the stable equilibrium point, the electric field must be directed toward the point. We choose a small Gaussian surface around the equilibrium point:

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0.$$

Because the field is directed into the surface, we have

$\oint\!\!\oint \vec{E}\cdot d\vec{A} < 0$, which means that there is a negative charge at the equilibrium point. This is a contradiction, because the only charge there is the positive test charge. Thus the test charge cannot be in stable equilibrium.

CHAPTER 24 Electric Potential

Answers to Understanding the Concepts Questions

1. The volt is defined so that 1 N/C = 1 V·m. Cross-multiplying yields 1 V·C = I N·m = 1 J.

2. The electric field due to the charged plane is $E = \sigma/2\varepsilon_0$. If we put a negative test charge q next to the plane then it will experience an electric force of $F = qE = q\sigma/2\varepsilon_0$. Measure F (say, by means of finding the resulting acceleration of the charge toward the plane) and we can obtain the value of σ.

3. If charge is placed inside the hollow space in a spherical metal shell, there will be an electric field. The field lines will join the charges to the inner surface of the shell where induced charges appear so as to yield a net charge inside any surface entirely within the spherical shell. To make a constant field in a small region, insert a small uniformly charged plane within the space; near that plane the field is constant.

4. The reason why the electric field near the surface of Earth points downward is because Earth is negativelky charged. The metal rod, when in contact with the ground, would also be negatively charged as it forms part of an equipotential body with Earth. Since electric charges tend to congregate in sharp corners, we can expect a higher surface charge density near the end of the rod that is exposed to the air, and the electric field just outside that end is expected to be greater than the ambient value of 100 V/m.

5. The question is analogous to asking for the source of the energy which moves a test mass in a gravitational field that arises from a distribution of masses. The test charge — like the test mass — has potential energy by virtue of being in the field of the existing charge distribution (mass distribution) and some of this potential energy is converted to kinetic energy in giving the test charge (test mass) some motion. The source of the potential energy is the work that had to be done to assemble the charge distribution (mass distribution) by bringing in the constituent charges (constituent masses) from infinity.

6. If the kinetic energy of the charge changes, then the work done on the unit test charge would no longer be equal to the change in electric potential. For example, if the kinetic energy of the charge increases by 1 J, then in addition to changing the potential energy of the test charge, which requires a certain amount of work, one must also expend an additional 1 J of work on the charge to cause the increase in its kinetic energy.

7. The net electric force due to the dipole on a point charge is the difference between the force exerted by the positive charge and that exerted by the negative charge. This difference is greater at $\theta = 0$ or π, so the net force due to the dipole is greater at $\theta = 0$ or π than at $\pi/2$.

8. No. They are at the same (high) potential as the Van de Graaff generator itself and are insulated from the ground. In fact if they were grounded (say, by having their bare feet touch the ground) then there would be a potential difference between their hands, which are touching the generator, and their feet, which are grounded. Such potential difference could be dangerous, as it would drive a current through the body.

9. The surface of a conductor will always be an equipotential in equilibrium. If a charge is placed on an electric surface, there is a short time during which it is localized. After that short time interval it distributes itself over the surface so that there is no force on any part of the charge, and equilibrium is achieved. The mention of a "short time" indicates that the statement that a conductor forms an equipotential is specifically true for static fields. The question then becomes more a matter of finding the time scale that distinguishes static from nonstatic fields.

10. According to Eq. (24-29), the electric field equals to the negative value of the gradient of the potential function. If we add a constant term to the potential to shift the location of zero potential to any fixed point
(x_0, y_0, z_0), i.e., change $V(x, y, z)$ to $V(x, y, z) - V(x_0, y_0, z_0)$, whereupon $V = 0$ at (x_0, y_0, z_0), we will not be changing the value of $\vec{E}(x, y, z)$. This is because the gradient of the additional constant term is zero: $\partial V(x_0, y_0, z_0)/\partial x = \partial V(x_0, y_0, z_0)/\partial y = \partial V(x_0, y_0, z_0)/\partial z = 0$.

11. The point of the demonstration is to place charge on the person. Once that happens, the individual hairs share the charge and repel each other much like the leaves of an electroscope. If the person is not on an insulated mat, then the charge from the generator will flow through the person as current, with potentially painful or even fatal results.

12. The zero potential can be defined at any point where the charge density is finite. Even if Earth is negatively charged we can still define its potential as zero. Remember, it's the potential *difference* that's physically relevant, not the potential itself.

13. Knowledge of the potential at a point does not allow us to determine the electric field. The simplest way to see this is to observe that potential energy contains an arbitrary constant. In contrast, if we know the potential at two adjacent points, then we know a potential difference, and this has physical meaning. In fact, the difference in the potential can allow us to find the electric field in the direction of the vector that connects the two adjacent points, as can be seen from Eq. (24-9); the points b and a are taken near each other.

14. The field lines seem to be denser near the sharp edges of the conductors (i.e., the two ends of the rod and the tip of the tear-drop shaped conductor). This is of course expected; see the discussion next to the figure in the textbook.

15. Yes. What matters is the final configuration of the charged systems, not how it was assembled. This is clear from Eq. (24.18).

16. The electric fields are very large at sharp comers of any charged object, and with large fields breakdown becomes much more likely. Smooth spherical surfaces minimize this possibility by minimizing the presence of points.

17. The net work done by an electrostatic force is always zero as we move a test charge along any enclosed path. By definition, then, the electrostatic force is conservative.

18. Nothing of physical significance would change, since potential energy, and thus electric potential, are not specified to within an additive constant.

19. Yes. For example, consider a pair of charges, $+q$ and $-q$, separated from each other by a distance $2r$. If we take $V = 0$ at infinity, then the potential at the midpoint of the line connecting the two charges is $V = kq/r + (-kq/r) = 0$.

20. The potential at a fixed point tells us nothing about the electric field because it is only differences in potentials that give us information about the electric field. (See the discussion of question 13.) If, however, we know the value of the potential in the vicinity of the points where it is zero, we can do better.
21. Yes. For example, take two concentric spherical shells of radii r_1 and r_2, respectively, and charge them uniformly such that $q_1/q_2 = -r_1/r_2$. Then the potential everywhere inside the smaller sphere is $V = k\,q_1/r_1 + kq_2/r_2 = 0$.

22. If we interpret "the easiest way" as the steepest path, then we want to look for a direction in which the lines of constant elevation are the closest from each other. In Fig. 24-10, this is roughly in the "south-east" direction. If you move perpendicularly to the contour s then your elevation drops the fastest. This is what happens to a ball if you let it roll off the top of the peak from rest. (And if you follow a certain contour then your elevation does not change, of course.) If we think of this plot as an equipotential plot for a two-dimensional charge distribution, then each contour represents a certain equipotential, and the electric field lines are always perpendicular to the equipotentials.

Solutions to Problems

1. With the reference level at infinity, the electrostatic potential energy of the two protons is
$U = (1/4\pi\varepsilon_0)(e^2/r)$
$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(1.6 \times 10^{-19}\ \mathrm{C})^2/(5 \times 10^{-15}\ \mathrm{m}) = \boxed{4.6 \times 10^{-14}\ \mathrm{J}}$.

2. With the reference level at infinity, the electrostatic potential energy of the two charges is
$U = (1/4\pi\varepsilon_0)(q_1 q_2/r) = -(1/4\pi\varepsilon_0)(Ze^2/r)$
$= -(9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(92)(1.6 \times 10^{-19}\ \mathrm{C})^2/(3 \times 10^{-12}\ \mathrm{m}) = \boxed{-7.1 \times 10^{-15}\ \mathrm{J}}$.

3. With the reference level at infinity, the potential energy of the two charges is
$U = (1/4\pi\varepsilon_0)(q_1 q_2/r)$
$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(7.0 \times 10^{-7}\ \mathrm{C})(3.0 \times 10^{-6}\ \mathrm{C})/(0.20\ \mathrm{m}) = \boxed{9.5 \times 10^{-2}\ \mathrm{J}}$.

4. The raisin will move directly away from the origin to infinity. Because the raisin starts with no kinetic energy, the initial potential energy becomes its final kinetic energy:
$K_f = U_i = \boxed{9.5 \times 10^{-2}\ \mathrm{J}}$.

5. (a) Because there is no other charge present, no force is required to bring the charge from infinity:
$W = \boxed{0}$.
(b) There is now a potential energy of the two charges, with the reference level at infinity. We find the work done by the electric field from
$W = -\Delta U = (-1/4\pi\varepsilon_0)(q_1 q_2/r_1)$
$= -(9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(3 \times 10^{-6}\ \mathrm{C})(5 \times 10^{-6}\ \mathrm{C})/(0.10\ \mathrm{m}) = \boxed{-1.35\ \mathrm{J}}$.
(c) The work done by the external agent is the negative of the work done by the electric field:
$W_F = -W = \boxed{+1.35\ \mathrm{J}}$.

6. We find the work done by an outside agent from the work-energy theorem:
$W = \Delta K + \Delta U = 0 + U_b - U_a = (1/4\pi\varepsilon_0)q_1 q_2 (1/r_b - 1/r_a)$
$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(+2.0 \times 10^{-5}\ \mathrm{C})(-9.0 \times 10^{-4}\ \mathrm{C})[1/(5 \times 10^{-5}\ \mathrm{m}) - 1/(5 \times 10^{-4}\mathrm{m})] = \boxed{-2.9 \times 10^{-5}\ \mathrm{J}}$.

7. The potential energy is a scalar that depends only on the distance. The distances of the third charge from each of the others are
$r_{a1} = [(30\ \mathrm{cm})^2 + 0 + (50\ \mathrm{cm} - 5\ \mathrm{cm})^2]^{1/2} = 54.1\ \mathrm{cm}$;
$r_{a2} = [(30\ \mathrm{cm})^2 + 0 + (15\ \mathrm{cm} + 5\ \mathrm{cm})^2]^{1/2} = 62.6\ \mathrm{cm}$.
The potential energy is
$U_a = (1/4\pi\varepsilon_0)(q_1 q_3/r_{a1} + q_2 q_3/r_{a2}) = (1/4\pi\varepsilon_0)q_3(q_1/r_{a1} + q_2/r_{a2})$
$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(0.20 \times 10^{-6}\ \mathrm{C})[(3.0 \times 10^{-6}\ \mathrm{C})/(0.541\ \mathrm{m}) + (-3.0 \times 10^{-6}\ \mathrm{C})/(0.626\ \mathrm{m})]$
$= \boxed{1.36 \times 10^{-3}\ \mathrm{J}}$.
When the charge is placed at (30 cm, 0 cm, 0 cm), the distances become
$r_{b1} = r_{b2} = [(30\ \mathrm{cm})^2 + 0 + (5\ \mathrm{cm})^2]^{1/2} = 30.4\ \mathrm{cm}$.
The potential energy is
$U_b = (1/4\pi\varepsilon_0)[(q_1 q_3/r_{b1}) + (q_2 q_3/r_{b2})] = (1/4\pi\varepsilon_0)q_3[(q_1/r_{b1}) + (q_2/r_{b2})]$
$= (1/4\pi\varepsilon_0)(q_3/r_{b1})(q_1 + q_2) = \boxed{0}$, because $q_1 = -q_2$.

8. (a) $U_a = (1/4\pi\varepsilon_0)(q_1q_3/r_{a1} + q_2q_3/r_{a2}) = (1/4\pi\varepsilon_0)q_3(q_1/r_{a1} + q_2/r_{a2})$
$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(-\ 0.20 \times 10^{-6}\ \mathrm{C})[(3.0 \times 10^{-5}\ \mathrm{C})/(0.541\ \mathrm{m}) + (3.0 \times 10^{-5}\ \mathrm{C})/(0.626\ \mathrm{m})]$
$= \boxed{-1.9 \times 10^{-2}\ \mathrm{J}}$.
$U_b = (1/4\pi\varepsilon_0)(q_1q_3/r_{b1} + q_2q_3/r_{b2}) = (1/4\pi\varepsilon_0)q_3(q_1/r_{b1} + q_2/r_{b2})$
$= (1/4\pi\varepsilon_0)(q_3/r_{b1})(q_1 + q_2)$
$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})[(-\ 0.20 \times 10^{-6}\ \mathrm{C})/(0.304\ \mathrm{m})][(3.0 \times 10^{-5}\ \mathrm{C}) + (3.0 \times 10^{-5}\ \mathrm{C})] = \boxed{-3.6 \times 10^{-2}\ \mathrm{J}}$.
(b) Changing the sign of the third charge will change the sign of the potential energy:
$U_a = \boxed{+1.9 \times 10^{-2}\ \mathrm{J}}$, $U_b = \boxed{+3.6 \times 10^{-2}\ \mathrm{J}}$.

9. The distances between the two charges are
$r_i = [(12\ \mathrm{cm} - 12\ \mathrm{cm})^2 + (60\ \mathrm{cm} - 25\ \mathrm{cm})^2 + (-\ 50\ \mathrm{cm} - 0)^2]^{1/2} = 61.0\ \mathrm{cm}$;
$r_f = [(12\ \mathrm{cm} - 12\ \mathrm{cm})^2 + (50\ \mathrm{cm} - 25\ \mathrm{cm})^2 + (25\ \mathrm{cm} - 0)^2]^{1/2} = 35.4\ \mathrm{cm}$.
The work done by an external agent to move the second charge is
$W = \Delta U = (1/4\pi\varepsilon_0)q_1q_2(1/r_f - 1/r_i)$
$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(1.5 \times 10^{-6}\ \mathrm{C})(-\ 3 \times 10^{-6}\ \mathrm{C})\ [1/(0.351\ \mathrm{m}) - 1/(0.610\ \mathrm{m})] = \boxed{-4.8 \times 10^{-2}\ \mathrm{J}}$.

10. A stable radius requires a minimum in the potential energy. With the reference level at infinity, the potential energy of two like charges is positive and increases as r decreases. Thus there will be no minimum and no stable orbit.

11. The electric potential for two charges is
$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2)$.
Because both charges are negative, the only place where V can be 0 is $\boxed{r = \infty}$.

12. The potential is a scalar that depends only on the distance. The potential for two charges is
$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2)$.
If the potential is 0 at a point x, we have
$0 = (1/4\pi\varepsilon_0)[(3 \times 10^{-6}\ \mathrm{C})/|(x - 14\ \mathrm{cm})|\ +\ (-\ 4 \times 10^{-6}\ \mathrm{C})/|(x - 15\ \mathrm{cm})|]$,
which gives $3|x - 15\ \mathrm{cm}| = 4|x - 14\ \mathrm{cm}|$. No position between the two charges gives $V = 0$.
For a point outside the two charges, we have
$3(x_2 - 15\ \mathrm{cm}) = 4(x_2 - 14\ \mathrm{cm})$, which gives $x = \boxed{19\ \mathrm{cm}}$.

13. We find the value of the charge from
$V = (1/4\pi\varepsilon_0)q/r$
$0.12\ \mathrm{V} = (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})q/(2.5 \times 10^{-3}\ \mathrm{m})$, which gives $q = \boxed{3.3 \times 10^{-14}\ \mathrm{C}}$.

14. Because the total energy of the proton is conserved, we have
$\Delta K + \Delta U = 0$;
$\frac{1}{2}m(v_B{}^2 - v_A{}^2) + q(V_B - V_A) = 0$;
$V_B - V_A = -\ \frac{1}{2}(m/q)(v_B{}^2 -\ v_A{}^2)$
$= -\ \frac{1}{2}[(1.67 \times 10^{-27}\ \mathrm{kg})/(1.6 \times 10^{-19}\ \mathrm{C})][(8 \times 10^5\ \mathrm{m/s})^2 - (5 \times 10^4\ \mathrm{m/s})^2] = \boxed{+3.3\ \mathrm{kV}}$.

$\boxed{15}$. We find the work done by an external agent from the work-energy theorem:
$W = \Delta K + \Delta U = 0 + q(V_b - V_a)$
$= (3.0 \times 10^{-7}\ \mathrm{C})[+\ 17\ \mathrm{kV} - (+\ 3.0\ \mathrm{kV})](10^3\ \mathrm{V/kV}) = \boxed{+4.2 \times 10^{-3}\ \mathrm{J}}$.

16. We find the work done by an external agent from the work-energy theorem:
$W = \Delta K + \Delta U = 0 + q(V_b - V_a) = (3 \times 10^{-8}\ \mathrm{C})[+\ 27\ \mathrm{kV} - (+16\ \mathrm{kV})](10^3\ \mathrm{V/kV}) = \boxed{+3.3 \times 10^{-4}\ \mathrm{J}}$.

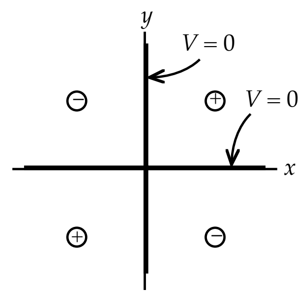
17. We find the potential energy of the system of charges by adding the work required to bring the three charges in from infinity successively:

$$W_1 = q_1 V_0 = 0;$$
$$W_2 = q_2 V_1 = q_2(1/4\pi\varepsilon_0)q_1/r_{12} = (1/4\pi\varepsilon_0)q_1 q_2/r_{12};$$
$$W_3 = q_3 V_2 = q_3(1/4\pi\varepsilon_0)(q_1/r_{13} + q_2/r_{23}) = (1/4\pi\varepsilon_0)(q_1 q_3/r_{13} + q_2 q_3/r_{23}).$$

The total potential energy is

$$U = W_1 + W_2 + W_3 = (1/4\pi\varepsilon_0)(q_1 q_2/r_{12} + q_1 q_3/r_{13} + q_2 q_3/r_{23})$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})[(2\ \mathrm{mC})(0.5\ \mathrm{mC})/(1\ \mathrm{m}) + (2\ \mathrm{mC})(-1.5\ \mathrm{mC})/(0.5\ \mathrm{m}) +$$
$$(0.5\ \mathrm{C})(-1.5\ \mathrm{C})/(1.5\ \mathrm{m})](10^{-3}\ \mathrm{C/mC})^2 = \boxed{-5.0 \times 10^4\ \mathrm{J}}.$$

18. For each positive-negative pair of equal charges, $V = 0$ at points that are equidistant from the charges. From the placement of the charges, we see that the points in the xz-plane and in the yz-plane will have $V = 0$.

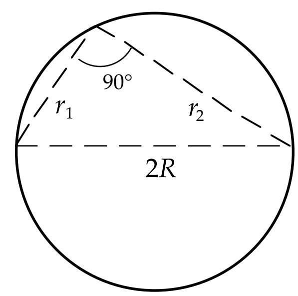


19. (a) The diameter of a circle subtends an angle of 90° at any point on the circle. Thus the distance from the negative charge to the point is

$$r_2 = [(2R)^2 - r_1{}^2]^{1/2} = [(50\ \mathrm{cm})^2 - (30\ \mathrm{cm})^2]^{1/2} = 40\ \mathrm{cm}.$$

The potential at the point is

$$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2)$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})[(24 \times 10^{-8}\ \mathrm{C})/(0.30\ \mathrm{m}) + (-10 \times 10^{-8}\ \mathrm{C})/(0.40\ \mathrm{m})]$$
$$= \boxed{+5.0 \times 10^3\ \mathrm{V}}.$$



(b) The work required is

$$W = q\,\Delta V = (-0.2 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^3\ \mathrm{V} - 0) = \boxed{-1.0 \times 10^{-3}\ \mathrm{J}}.$$

The negative value indicates that the negative charge wants to "fall" to the higher potential.

20. The origin is equidistant from the three charges. If the side of the triangle is L, the distance from the center to a corner is

$$r = (L/2)/\cos 30° = (3\ \mathrm{cm})/(2\cos 30°) = 1.73\ \mathrm{cm}.$$

The potential is

$$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3).$$

Because the charges and distances are the same, we have

$$V = (1/4\pi\varepsilon_0)[3(q_1/r_1)] = (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(3)(0.5 \times 10^{-6}\ \mathrm{C})/(1.73 \times 10^{-2}\ \mathrm{m}) = \boxed{7.8 \times 10^5\ \mathrm{V}}.$$

21. If we let $q = 10^{-6}$ C, the charges are

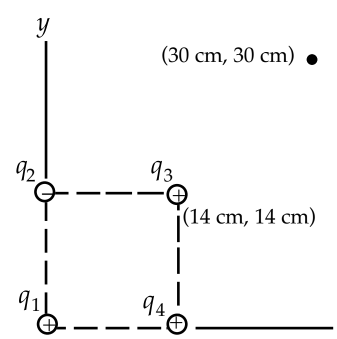
$$q_1 = 2q,\ q_2 = -3q,\ q_3 = 5q,\ q_4 = 3q.$$

The distances from each charge to the point are

$$r_1 = [(30\ \mathrm{cm})^2 + (30\ \mathrm{cm})^2]^{1/2} = 42.4\ \mathrm{cm};$$
$$r_2 = r_4 = [(30\ \mathrm{cm})^2 + (16\ \mathrm{cm})^2]^{1/2} = 34.0\ \mathrm{cm};$$
$$r_3 = [(16\ \mathrm{cm})^2 + (16\ \mathrm{cm})^2]^{1/2} = 22.6\ \mathrm{cm}.$$

The potential at the point is

$$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3 + q_4/r_4)$$
$$= (1/4\pi\varepsilon_0)q[2/r_1 + (-3/r_2) + 5/r_3 + 3/r_4]$$
$$= (1/4\pi\varepsilon_0)q(2/r_1 + 5/r_3)$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(10^{-6}\ \mathrm{C})[2/(0.424\ \mathrm{m}) + 5/(0.226\ \mathrm{m})] = \boxed{+2.4 \times 10^5\ \mathrm{V}}.$$



22. We find the potential energy of the system of charges by adding the work required to bring the three charges in from infinity successively:

$W_1 = q_1 V_0 = 0;$

$W_2 = q_2 V_1 = q_2(1/4\pi\varepsilon_0)q_1/r_{12} = (1/4\pi\varepsilon_0)q_1 q_2/r_{12};$

$W_3 = q_3 V_2 = q_3(1/4\pi\varepsilon_0)(q_1/r_{13} + q_2/r_{23}) = (1/4\pi\varepsilon_0)(q_1 q_3/r_{13} + q_2 q_3/r_{23}).$

The total potential energy is

$U = W_1 + W_2 + W_3 = (1/4\pi\varepsilon_0)[q_1 q_2/r_{12} + q_1 q_3/r_{13} + q_2 q_3/r_{23})$

$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})[(5\ \mu\mathrm{C})(-3\ \mu\mathrm{C})/(10 \times 10^{-3}\ \mathrm{m}) +$

$(3\ \mu\mathrm{C})(-2\ \mu\mathrm{C})/(10 \times 10^{-3}\ \mathrm{m}) + (-3\ \mu\mathrm{C})(-2\ \mu\mathrm{C})/(10\sqrt{2} \times 10^{-3}\ \mathrm{m})](10^{-6}\ \mathrm{C}/\mu\mathrm{C})^2$

$= \boxed{-18.7\ \mathrm{J}}.$

The order in which the charges are brought in does not matter.

23. The distances from each charge to the origin are

$r_1 = [(15\ \mathrm{mm})^2 + (20\ \mathrm{mm})^2]^{1/2} = 25.0\ \mathrm{mm};$

$r_2 = [(15\ \mathrm{mm})^2 + (30\ \mathrm{mm})^2]^{1/2} = 33.5\ \mathrm{mm};$

$r_3 = [(25\ \mathrm{mm})^2 + (20\ \mathrm{mm})^2]^{1/2} = 32.0\ \mathrm{mm}.$

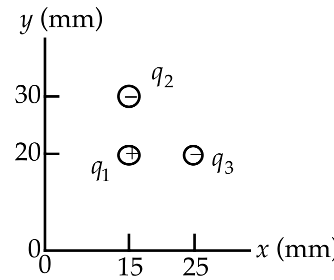
The potential at the point is

$V = (1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2 + q_3/r_3)$

$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(10^{-6}\ \mathrm{C})[5/(0.025\ \mathrm{m}) + (-3)/(0.0335\ \mathrm{m}) +$

$(-2)/(0.032\ \mathrm{m})]$

$= \boxed{+4.3 \times 10^5\ \mathrm{V}}.$



24. The positive charge must be released from the positive plate. We take the negative plate to be the reference level of potential. We find the speed that the pellet has at the negative plate from conservation of energy:

$\Delta K = -\Delta U;$

$\frac{1}{2}mv_f^2 - 0 = -q(0 - V) = qV;$

$\frac{1}{2}(2 \times 10^{-6}\ \mathrm{kg})v_f^2 = (3 \times 10^{-7}\ \mathrm{C})(600\ \mathrm{V})$, which gives

$v_f = \boxed{13\ \mathrm{m/s}}.$

25. (a) Let $q_1 = +12\ \mu\mathrm{C}$, $y_1 = +5.0$ cm, $q_2 = -20\ \mu\mathrm{C}$, and $y_2 = -9.0$ cm. The distance r_1 between q_1 and the point $(x, 0) = (12.0\ \mathrm{cm}, 0)$ is $r_1 = (x^2 + y_1^2)^{1/2} = [(12.0\ \mathrm{cm})^2 + (5.0\ \mathrm{cm})^2]^{1/2} = 13$ cm, while that between q_2 and the same point is $r_2 = (x^2 + y_2^2)^{1/2} = [(12.0\ \mathrm{cm})^2 + (-9.0\ \mathrm{cm})^2]^{1/2} = 15$ cm. The potential at that point due to the two charges is then

$V(x, 0) = (1/4\pi\varepsilon_0)q_1/r_1 + (1/4\pi\varepsilon_0)q_2/r_2$

$= (9.0 \times 10^9\ \mathrm{C^2/N \cdot m^2})[(+12 \times 10^{-6}\ \mathrm{C})/(0.13\ \mathrm{m}) + (-20 \times 10^{-6}\ \mathrm{C})/(0.15\ \mathrm{m})]$

$= \boxed{-3.7 \times 10^5\ \mathrm{V}}.$

(b) The point (0, 0) is a distance $y_1 = 5.0$ cm from q_1 and $|y_2| = 9.0$ cm from q_2. Thus

$V(0, 0) = (1/4\pi\varepsilon_0)q_1/y_1 + (1/4\pi\varepsilon_0)q_2/|y_2|$

$= (9.0 \times 10^9\ \mathrm{C^2/N \cdot m^2})[(+12 \times 10^{-6}\ \mathrm{C})/(0.050\ \mathrm{m}) + (-20 \times 10^{-6}\ \mathrm{C})/(0.090\ \mathrm{m})]$

$= \boxed{+1.6 \times 10^5\ \mathrm{V}};$

$\vec{E}(0, 0) = [(1/4\pi\varepsilon_0)q_1/y_1^2](-\hat{j}) + [(1/4\pi\varepsilon_0)q_2/y_2^2]\hat{j}$

$= (9.0 \times 10^9\ \mathrm{C^2/N \cdot m^2})[-(+12 \times 10^{-6}\ \mathrm{C})/(0.050\ \mathrm{m})^2 + (-20 \times 10^{-6}\ \mathrm{C})/(0.090\ \mathrm{m})^2]\hat{j}$

$= \boxed{-(6.5 \times 10^7\ \mathrm{V/m})\hat{j}}.$

26. Put the origin of the x-axis at $q_1 = 2.5\ \mu$C, and the other charge, $q_2 = 7.5\ \mu$C, is located at $x = L =\ 0.80$ m on the axis. Consider a point with coordinate x $(0 < x < L)$. The electric potential at that point is the sum of those due to the two charges:

$$V(x) = (1/4\pi\varepsilon_0)q_1/x + (1/4\pi\varepsilon_0)q_2/(L - x)$$
$$= (9.0 \times 10^9\ \mathrm{C^2/N\cdot m^2})[(2.5 \times 10^{-6}\ \mathrm{C})/x\ +\ (7.5 \times 10^{-6}\ \mathrm{C})/(0.80\ \mathrm{m} - x)]$$
$$= \boxed{[2.25 \times 10^4/x +\ 6.75 \times 10^4/(0.80 - x)]\ \mathrm{V},\ \text{where } x \text{ is in meters}}.$$

Set $E(x) = -\mathrm{d}V/\mathrm{d}x = -(\mathrm{d}/\mathrm{d}x)[2.25 \times 10^4/x +\ 6.75 \times 10^4/(0.80 - x)]$

$$= 2.25 \times 10^4/x^2 -\ 6.75 \times 10^4/(0.80 - x)^2 = 0$$

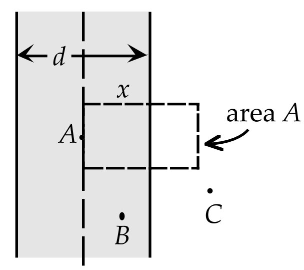
to obtain the position where $E = 0$: $\quad x = \boxed{0.29\ \mathrm{m}}$.

If the test charge q is positive, then as it moves from the equilibrium toward either q_1 or q_2 closely enough, it will be pushed back. So the equilibrium is stable. If the test charge is negative then the equilibrium is unstable.

You can verify that by taking $\mathrm{d}^2V/\mathrm{d}x^2$ at the equilibrium position. It turns out that $\mathrm{d}^2V/\mathrm{d}x^2 > 0$ at the equilibrium position, so the electrostatic energy $U = qV$ is a minimum for $q > 0$, indicating stable equilibrium; and $U = qV$ is a maximum for $q < 0$, indicating unstable equilibrium.

27. (a) From the symmetry of the charge distribution, we see that the electric field is perpendicular to the slab and away from the centerline. This means that

$$E_A = \boxed{0}.$$



To find the field at B, we construct a cylinder of height x with its axis perpendicular to the slab as a Gaussian surface. One end of area A is placed on the centerline, where the field is 0. Because the field is parallel to the sides of the cylinder, there is flux only through the outer end, so we have

$$\oiint \vec{E} \cdot \mathrm{d}\vec{A} = EA = Q_{\text{enclosed}}/\varepsilon_0\ .$$

If the end is at point B, the enclosed charge is ρAx and we have

$$E_B = \rho x/\varepsilon_0 = (10^{-5}\ \mathrm{C/m^3})x/(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})$$
$$= \boxed{(1.13 \times 10^6\ \mathrm{N/C\cdot m})x,\ \ x < 1\ \mathrm{cm}}.$$

If the end is at point C, the enclosed charge is $\rho Ad/2$ and we have

$$E_C = \rho d/2\varepsilon_0 = (10^{-5}\ \mathrm{C/m^3})(2 \times 10^{-2}\ \mathrm{m})/2(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})$$
$$= \boxed{1.13 \times 10^4\ \mathrm{N/C},\ \ x > 1\ \mathrm{cm}}.$$

As expected, the field outside the slab is uniform.

(b) We find the potential from the field by integrating over a path perpendicular to the slab:

$$V_B = V_A - \int_0^x \vec{E}_{\text{inside}} \cdot \mathrm{d}\vec{s} = 0 - (1.13 \times 10^6\ \mathrm{N/C\cdot m})\int_0^x x'\,dx'$$
$$= -(5.65 \times 10^5\ \mathrm{V/m^2})x^2, \qquad x < 1\ \mathrm{cm}.$$

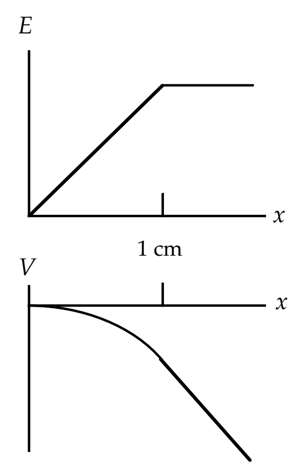
The potential at the edge of the slab is

$$V_{\text{edge}} = -(5.65 \times 10^5\ \mathrm{V/m^2})(0.01\ \mathrm{m})^2 = -56.5\ \mathrm{V}.$$

For the potential at point C we have

$$V_C = V_{\text{edge}} - \int_{\text{edge}}^x \vec{E}_{\text{outside}} \cdot \mathrm{d}\vec{s} = -(56.5\ \mathrm{V}) - (1.13 \times 10^4\ \mathrm{N/C})\int_{0.01\ \mathrm{m}}^x \mathrm{d}x'$$
$$= -56.5\ \mathrm{V} - (1.13 \times 10^4\ \mathrm{V/m})(x - 0.01\ \mathrm{m}), \qquad x > 1\ \mathrm{cm}$$
$$= +56.5\ \mathrm{V} - (1.13 \times 10^4\ \mathrm{V/m})x\,, \qquad x > 1\ \mathrm{cm}.$$

(c)



28. We need to find the potential energy stored in the charge Q distributed uniformly over the spherical shell. We do this by successively bringing a differential charge in from infinity. The total potential energy is the sum (integral) of the differential work done. As we bring in a differential charge, the charge q already on the shell appears to be a point charge, so the work required to bring in the next differential charge is

$$dW = (1/4\pi\varepsilon_0)(q/r)\, dq.$$

The potential energy is the total work required:

$$U_1 = W_1 = \frac{1}{4\pi\varepsilon_0}\int_0^Q \frac{q}{R}\, dq = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}\right).$$

For a shell with half the radius, we have

$$U_2 = \tfrac{1}{2}Q^2/4\pi\varepsilon_0(R/2) = 2U_1.$$

The work required to move the charges is

$$W = \Delta U = 2U_1 - U_1 = U_1 = \boxed{\tfrac{1}{2}Q^2/4\pi\varepsilon_0 R}.$$

29. We consider the sphere to consist of an infinite number of spherical shells with thickness dr and charge $dq = \rho 4\pi r^2\, dr$, where the density of charge is

$$\rho = 3Q/4\pi R^3.$$

We choose the potential reference level at infinity.
At a point outside the sphere, all of the shells, and thus the sphere, are equivalent to point charges:

$$V_{\text{outside}} = \boxed{Q/4\pi\varepsilon_0 r, \text{ when } r > R}.$$

At a point inside the sphere, $r < R$, there are two contributions to the potential.
All of the shells with radius less than r are equivalent to point charges:

$$V_1 = q/4\pi\varepsilon_0 r = (\rho 4\pi r^3/3)/4\pi\varepsilon_0 r = \rho r^2/3\varepsilon_0.$$

For a shell with radius greater than r, the potential anywhere inside is constant and equal to the potential on the shell:

$$dV = dq/4\pi\varepsilon_0 r = \rho 4\pi r^2\, dr/4\pi\varepsilon_0 r = \rho r\, dr/\varepsilon_0.$$

We find the potential contribution from all of the shells with $r < r' < R$ by integrating:

$$V_2 = \frac{\rho}{\varepsilon_0}\int_r^R r'\, dr' = \frac{\rho}{\varepsilon_0}\left(\frac{r'^2}{2}\right)\Bigg|_r^R = \frac{\rho}{2\varepsilon_0}\left(R^2 - r^2\right).$$

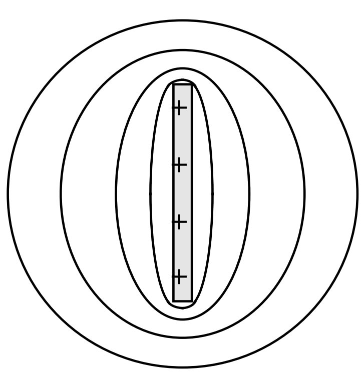
The total potential is

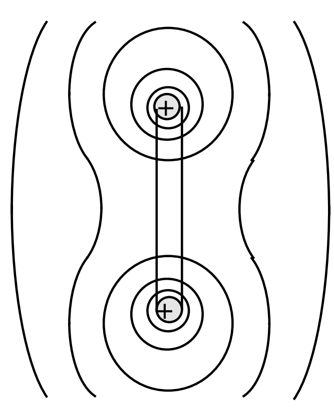
$$\begin{aligned} V_{\text{inside}} &= V_1 + V_2 = (\rho r^2/3\varepsilon_0) + [\rho(R^2 - r^2)/2\varepsilon_0] \\ &= (\rho/\varepsilon_0)[(R^2/2) - (r^2/6)] = (3Q/4\pi\varepsilon_0 R^3)[(R^2/2) - (r^2/6)] \\ &= \boxed{(Q/8\pi\varepsilon_0 R)[3 - (r/R)^2], \text{ when } r < R}. \end{aligned}$$

If we compare the values at $r = R$, we get

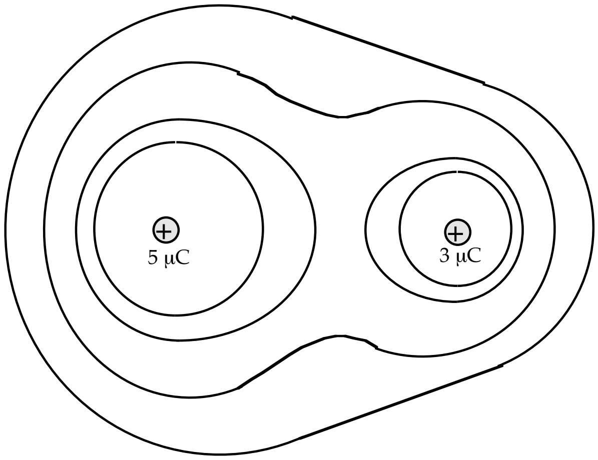
$$V_{\text{outside}} = Q/4\pi\varepsilon_0 R \quad\text{and}\quad V_{\text{inside}} = (Q/8\pi\varepsilon_0 R)(3 - 1) = Q/4\pi\varepsilon_0 R = V_{\text{outside}}.$$

30. (a) (b)

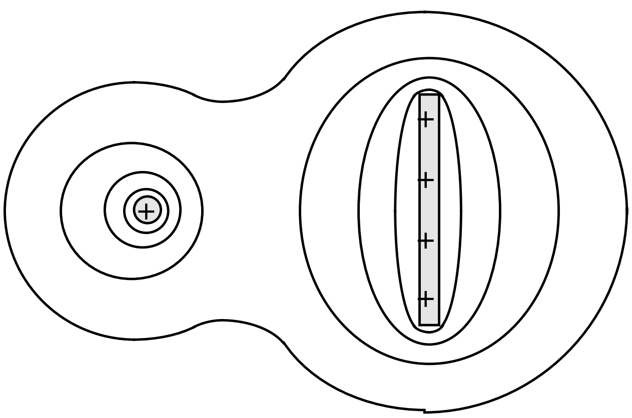




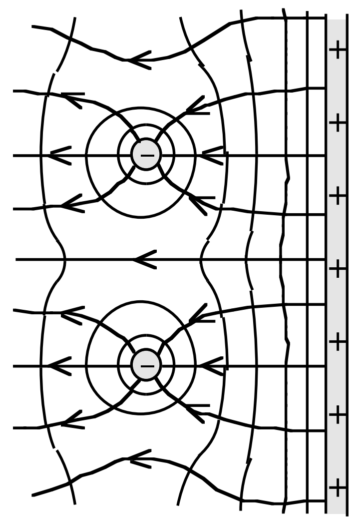
31.



32.



$\boxed{33}$.



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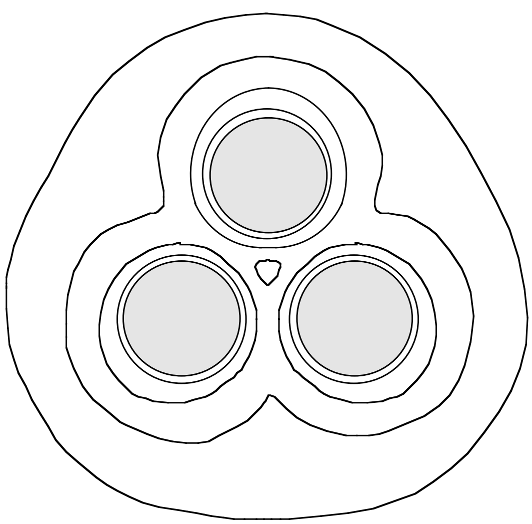

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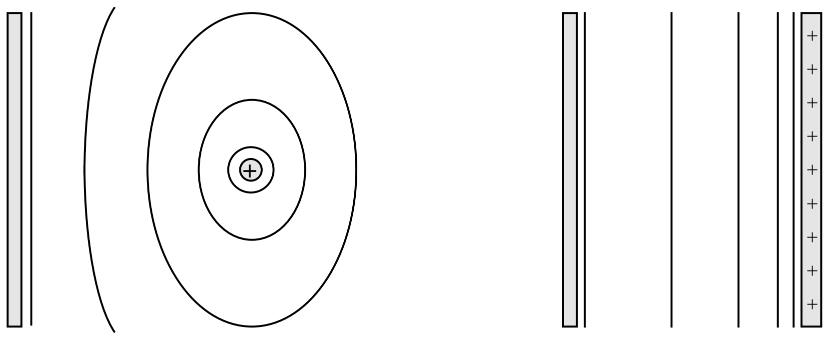

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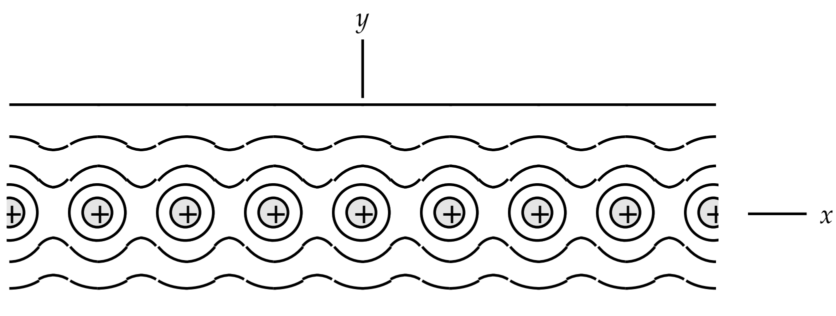

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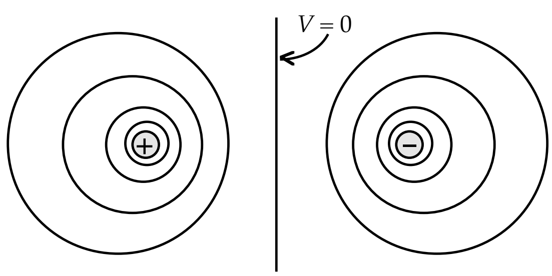
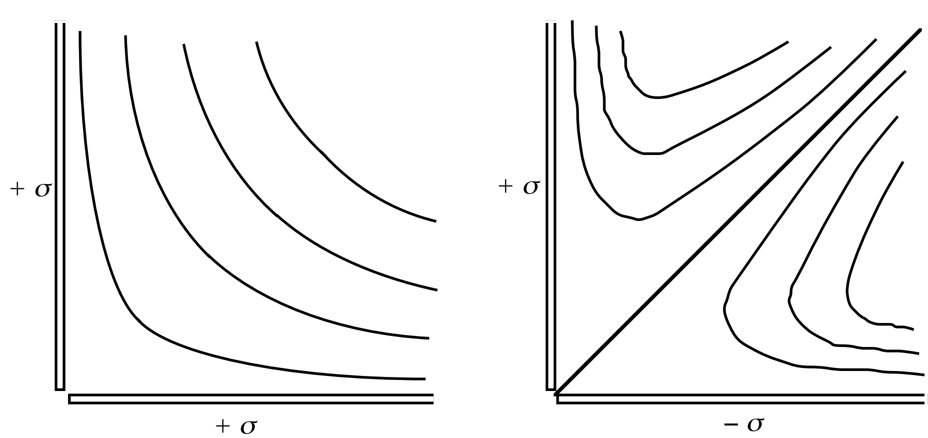


38.



39. From the spatial dependence of the electric potential, $V(x, y, z) = Q/4\pi\varepsilon_0 x$, we find the components of the electric field from the partial derivatives of V:

$E_x = -\,\partial V/\partial x = Q/4\pi\varepsilon_0 x^2$;

$E_y = -\,\partial V/\partial y = 0$;

$E_z = -\,\partial V/\partial z = 0$.

We can write the electric field: $\vec{E} = \boxed{(Q/4\pi\varepsilon_0 x^2)\,\hat{i}}$.

40. From the spatial dependence of the electric potential, $V(x, y, z) = Ax^2y^2 + Byz^2 + C$, we find the components of the electric field from the partial derivatives of V:

$E_x = -\,\partial V/\partial x = -\,2Axy^2$;

$E_y = -\,\partial V/\partial y = -\,2Ax^2y - Bz^2$;

$E_z = -\,\partial V/\partial z = -\,2Byz$.

We can write the electric field: $\vec{E} = \boxed{-(2Axy^2)\hat{i} - (2Ax^2y + Bz^2)\hat{j} - (2Byz)\hat{k}}$.

41. From the spatial dependence of the electric potential, $V(x) = a_0 + a_1x$, the electric field will have only an x-component, which we find from the partial derivative of V:

$E_x = -\,\partial V/\partial x = -\,a_1 = -\,(-\,6.68\text{ V/m}) = +\,6.68\text{ V/m}$.

We can write the electric field: $\vec{E} = \boxed{(6.68\text{ V/m})\,\hat{i}}$.

42. Because the electric field is along the x-axis, we find the the field from

$\vec{E} = -\,(\partial V/\partial x)\hat{i}$

$= -\,(\partial/\partial x)[Q/4\pi\varepsilon_0(R^2 + x^2)^{1/2}]\hat{i} = (-\,Q/4\pi\varepsilon_0)(-\,\tfrac{1}{2})[2x/(R^2 + x^2)^{3/2}]\hat{i}$;

$\vec{E} = \boxed{Qx/[4\pi\varepsilon_0(R^2 + x^2)^{3/2}]\,\hat{i}}$.

43. With the dipole pointing in the x-direction, the potential is

$V = (p\cos\theta)/4\pi\varepsilon_0 r^2 = px/4\pi\varepsilon_0 r^3$.

We find the components of the electric field from the partial derivatives of V. For the x-component, we have

$E_x = -\,\partial V/\partial x = -\,(p/4\pi\varepsilon_0 r^3) - [-\,(3px/4\pi\varepsilon_0 r^4)(\partial r/\partial x)]$.

From $r^2 = x^2 + y^2 + z^2$, we have

$2r(\partial r/\partial x) = 2x$, or $\partial r/\partial x = x/r$, so we get

$E_x = -\,(p/4\pi\varepsilon_0 r^3) + (3px^2/4\pi\varepsilon_0 r^5)$.

Similarly, we have

$E_y = -\,\partial V/\partial y = -\,[-\,(3px/4\pi\varepsilon_0 r^4)(\partial r/\partial y)]$

$= +\,3pxy/4\pi\varepsilon_0 r^5$;

$E_z = -\,\partial V/\partial z = -\,[-\,(3px/4\pi\varepsilon_0 r^4)(\partial r/\partial z)]$

$= +\,3pxz/4\pi\varepsilon_0 r^5$.

Along the bisector (the y-axis), $x = 0$, so we have

$\vec{E} = \boxed{-(p/4\pi\varepsilon_0 r^3)\,\hat{i}}$.

Note that the symmetry along the y-axis shows us that the field there has only an x-component.

44. From the symmetry of the charge distribution, we know that the electric field is radial. From the spatial dependence of the electric potential, $V(r) = (Q/2\pi\varepsilon_0)[A(r/R) + B(r/R)^2 + C]$, we find the electric field from

$E_r = -\,\partial V/\partial r = \boxed{-(Q/2\pi\varepsilon_0)[(A/R) + (2Br/R^2)]\ \text{radial}}$.

If the potential is zero at the surface, we have

$V(R) = 0 = (Q/2\pi\varepsilon_0)(A + B + C)$, which gives $\boxed{C = -\,A - B}$.

45. From the symmetry of the charge distribution, we know that the electric field is radial, so we find the electric field from

$E_r = -\,\partial V/\partial r.$

For $r < R$, we have

$V_{r<R} = (Q/4\pi\varepsilon_0 R)[-2 + 3(r/R)^2];$

$E_{r<R} = -(Q/4\pi\varepsilon_0 R)(+6r/R^2) = -(Q/4\pi\varepsilon_0)(6r/R^3)$, or

$\vec{E}_{r<R} = \boxed{(Q/4\pi\varepsilon_0)(6r/R^3)\,\hat{r}}.$

For $r > R$, we have

$V_{r>R} = Q/4\pi\varepsilon_0 r;$

$E_{r>R} = -(-Q/4\pi\varepsilon_0 r^2)$, or

$\vec{E}_{r>R} = \boxed{(Q/4\pi\varepsilon_0)\,\hat{r}}.$

46. If we choose a sphere of radius $r < R$ as a Gaussian surface, we have

$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E4\pi r^2 = Q_{\text{enclosed}}/\varepsilon_0$, or

$Q_{\text{enclosed}} = -\,\varepsilon_0(Q/4\pi\varepsilon_0)(6r/R^3)4\pi r^2 = -\,6Qr^3/R^3.$

We set up the integral to find the enclosed charge, by using a spherical shell of radius r' and thickness dr' for the differential element. We also write the right-hand side as an integral:

$$\int_0^r \rho 4\pi r'^2\,dr' = \left(-18Q/R^3\right)\int_0^r r'^2\,dr'.$$

Comparing the two integrands, we see that $\boxed{\text{for } r < R,\ \rho = -\,4.5Q/\pi R^3,\ \text{ a constant}}$.

If the Gaussian surface is just inside $r = R$, the total enclosed charge is $-\,6Q$.

If we choose a sphere with a radius r just slightly greater than R as a Gaussian surface, we have

$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E4\pi R^2 = Q_{\text{enclosed}}/\varepsilon_0$, or

$Q_{\text{enclosed}} = \varepsilon_0(Q/4\pi\varepsilon_0 R^2)4\pi R^2 = Q.$

Because there is a charge of $-\,6Q$ inside the sphere, there must be a charge of $+\,7Q$ on the surface of the sphere to give a net charge of Q:

$\boxed{r = R,\ q = +\,7Q}.$

If we choose a sphere with a radius $r > R$ as a Gaussian surface, we have

$\oint\!\!\oint \vec{E}\cdot d\vec{A} = E4\pi r^2 = Q_{\text{enclosed}}/\varepsilon_0$, or

$Q_{\text{enclosed}} = \varepsilon_0(Q/4\pi\varepsilon_0 r^2)4\pi r^2 = Q.$

Because this is the net charge on the sphere, we have $\boxed{r > R,\ \rho = 0}$.

47. From the spatial dependence of the electric potential,

$$V(x, y) = (Q/4\pi\varepsilon_0 L)\{\tan^{-1}[y/(x - a_0)] - 2\tan^{-1}(y/x) + \tan^{-1}[y/(x + a_0)]\},$$

we find the components of the electric field from the partial derivatives of V:

$$E_x = -\frac{\partial V}{\partial x} = \frac{-Q}{4\pi\varepsilon_0 L}\left\{\left[\frac{1}{1 + \left(\frac{y}{x - a_0}\right)^2}\right]\left[\frac{-y}{(x - a_0)^2}\right] - 2\left[\frac{1}{1 + \left(\frac{y}{x}\right)^2}\right]\left(\frac{-y}{x^2}\right) + \left[\frac{1}{1 + \left(\frac{y}{x + a_0}\right)^2}\right]\left[\frac{-y}{(x + a_0)^2}\right]\right\}$$

$$= \frac{-Q}{4\pi\varepsilon_0 L}\left\{-\left[\frac{y}{(x - a_0)^2 + y^2}\right] + \left(\frac{2y}{x^2 + y^2}\right) - \left[\frac{y}{(x + a_0)^2 + y^2}\right]\right\}.$$

We expand the denominators and use the approximation, $1/(1 \pm z) \simeq 1 \mp z$, when $z << 1$:

$$E_x = \frac{-Q}{4\pi\varepsilon_0 L}\left[-\left(\frac{y}{x^2 + y^2 - 2a_0 x + a_0^2}\right) + \left(\frac{2y}{x^2 + y^2}\right) - \left(\frac{y}{x^2 + y^2 + 2a_0 x + a_0^2}\right)\right]$$

$$= \frac{Qy}{4\pi\varepsilon_0 L(x^2 + y^2)}\left\{\left[\frac{1}{1 - (2a_0 x - a_0^2)/(x^2 + y^2)}\right] - 2 + \left[\frac{1}{1 + (2a_0 x - a_0^2)/(x^2 + y^2)}\right]\right\}$$

$$\simeq \frac{Qy}{4\pi\varepsilon_0 L(x^2 + y^2)}\left(1 + \frac{2a_0 x - a_0^2}{x^2 + y^2} - 2 + 1 - \frac{2a_0 x - a_0^2}{x^2 + y^2}\right) = \frac{-2Qya_0^2}{4\pi\varepsilon_0 L(x^2 + y^2)^2}.$$

We use the same approximation for the y-component:

$$E_y = -\frac{\partial V}{\partial y} = \frac{-Q}{4\pi\varepsilon_0 L}\left(\left\{\frac{1}{1 + [y/(x - a_0)]^2}\right\}\left(\frac{1}{x - a_0}\right) - 2\left[\frac{1}{1 + (y/x)^2}\right]\left(\frac{1}{x}\right) + \left\{\frac{1}{1 + [y/(x + a_0)]^2}\right\}\left(\frac{1}{x + a_0}\right)\right)$$

$$= \frac{-Q}{4\pi\varepsilon_0 L}\left\{\left[\frac{x - a_0}{(x - a_0)^2 + y^2}\right] - \left(\frac{2x}{x^2 + y^2}\right) + \left[\frac{x + a_0}{(x + a_0)^2 + y^2}\right]\right\}$$

$$= \frac{-Q}{4\pi\varepsilon_0 L}\left[\left(\frac{x - a_0}{x^2 + y^2 - 2a_0 x + a_0^2}\right) - \left(\frac{2x}{x^2 + y^2}\right) + \left(\frac{x + a_0}{x^2 + y^2 + 2a_0 x + a_0^2}\right)\right]$$

$$= \frac{-Qx}{4\pi\varepsilon_0 L(x^2 + y^2)}\left\{\left[\frac{1 - (a_0/x)}{1 - (2a_0 x - a_0^2)/(x^2 + y^2)}\right] - 2 + \left[\frac{1 + (a_0/x)}{1 + (2a_0 x - a_0^2)/(x^2 + y^2)}\right]\right\}$$

$$\simeq \frac{-Qx}{4\pi\varepsilon_0 L(x^2 + y^2)}\left[\left(1 - \frac{a_0}{x}\right)\left(1 + \frac{2a_0 x - a_0^2}{x^2 + y^2}\right) - 2 + \left(1 + \frac{a_0}{x}\right)\left(1 - \frac{2a_0 x - a_0^2}{x^2 + y^2}\right)\right] = \frac{-6Qxa_0^2}{4\pi\varepsilon_0 L(x^2 + y^2)^2}.$$

The total electric field is

$$\vec{E} = \frac{2Qa_0^2}{4\pi\varepsilon_0 L(x^2 + y^2)^2}\left(-y\,\hat{i} - 3\,x\,\hat{j}\right) = \frac{2Qa_0^2}{4\pi\varepsilon_0 Lr^4}\left(-r\sin\theta\,\hat{i} - 3\,r\cos\theta\,\hat{j}\right)$$

$$= \frac{2Qa_0^2}{4\pi\varepsilon_0 Lr^3}\left(-\sin\theta\,\hat{i} - 3\cos\theta\,\hat{j}\right).$$

48. For two large, parallel plates, we have

$E = -\Delta V/\frac{1}{4}d$;

7×10^3 V/m = (200 V)/d, which gives d = 0.029 m = $\boxed{2.9 \text{ cm}}$.

49. We find the charge from the expression for the potential on the axis of the ring:

$V = (1/4\pi\varepsilon_0)Q/(R^2 + x^2)^{1/2}$;

5 V = $(9 \times 10^9 \text{ C}^2/\text{N}\cdot\text{m}^2)Q/[(0{,}10 \text{ m})^2 + (0.15 \text{ m})^2]^{1/2}$, which gives $Q = \boxed{1.0 \times 10^{-10} \text{ C}}$.

50. (a) Choosing y up as positive, we find the potential from the field by integrating:

$V = -\int \vec{E} \cdot d\vec{s} = -(-E)\int dy = \boxed{Ey + (\text{a constant})}$.

(b) The most convenient reference point is $\boxed{V = 0 \text{ at surface}}$.

(c) The electric potential energy is $U_e = \boxed{qE\,h}$.

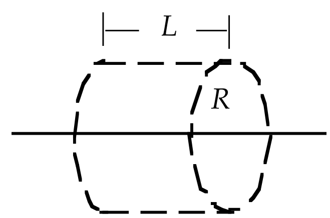
The gravitational potential energy is $U_g = \boxed{mg\,h}$, so they have the same form.

(d) For the electric force to balance the force of gravity, we need a negative charge with magnitude given by

$qE = mg$;

$q(100 \text{ N/C}) = (50 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{4.9 \text{ C of negative charge}}$.

51. From symmetry, we know that the electric field will be radially away from the line charge, with a magnitude independent of the direction. For a Gaussian surface we choose a cylinder of length L and radius R, centered on the line. On the ends of this surface, the electric field is not constant, but $\vec{E}$ and $d\vec{A}$ are perpendicular, so we have $\vec{E} \cdot d\vec{A} = 0$. On the curved side, the field has a constant magnitude and $\vec{E}$ and $d\vec{A}$ are parallel, so we have $\vec{E} \cdot d\vec{A} = E\,dA$. For Gauss' law, we have



$$\oiint \vec{E} \cdot d\vec{A} = \iint_{\text{ends}} \vec{E} \cdot d\vec{A} + \iint_{\text{side}} \vec{E} \cdot d\vec{A} = 0 + EA_{\text{side}} = Q/\varepsilon_0;$$

$E2\pi RL = \lambda L/\varepsilon_0$, which gives

$E = 2\lambda/4\pi\varepsilon_0 R$ radial.

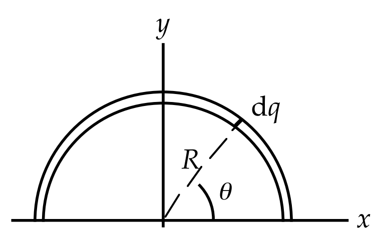
We find the potential by integrating along a radial line:

$V = -\int \vec{E} \cdot d\vec{R} = -(2\lambda/4\pi\varepsilon_0)\int dR/R = \boxed{-(2\lambda/4\pi\varepsilon_0)\ln(R) + (\text{a constant})}$.

52. The potential at the origin from a differential element of the charge is

$dV = (1/4\pi\varepsilon_0)(dq/R)$.

To find the potential at the origin, we add (integrate) the contributions from all elements:

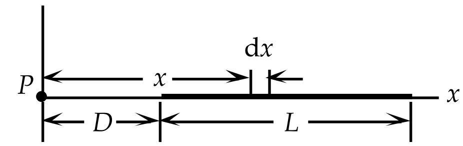


$V = \int (1/4\pi\varepsilon_0)(dq/R)$

$= (1/4\pi\varepsilon_0 R)\int dq = q/4\pi\varepsilon_0 R = \lambda\pi R/4\pi\varepsilon_0 R$

$= \boxed{\lambda/4\varepsilon_0}$.

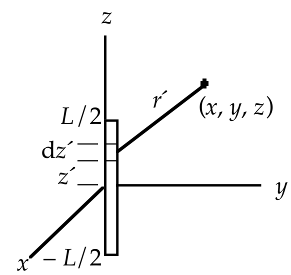
53. We choose the point P as the origin. The potential at P from a differential element of the rod, which has a charge $dq = (q/L)\,dx$, is
$dV = (1/4\pi\varepsilon_0)(dq/x)$.
To find the potential at P, we add (integrate) the contributions from all elements:



$$V = \frac{1}{4\pi\varepsilon_0}\int_D^{D+L}\left(\frac{q}{L}\right)\frac{dx}{x} = \frac{q}{4\pi\varepsilon_0 L}\ln\left(\frac{D+L}{D}\right)$$

$$= (9\times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(2\times 10^{-6}\ \mathrm{C})}{0.2\ \mathrm{m}}\ln\left(\frac{0.1\ \mathrm{m}+0.2\ \mathrm{m}}{0.1\ \mathrm{m}}\right) = \quad 9.9\times 10^4\ \mathrm{V}.$$

54. We choose a differential element of the rod at position z', length dz', and charge $\lambda\,dz'$. From the diagram, we see that
$r^2 = x^2 + y^2 + z^2$ and $r'^2 = x^2 + y^2 + (z - z')^2$.
The potential from the differential element at the point (x, y, z) is
$dV = (1/4\pi\varepsilon_0)\,dq/r' = (\lambda/4\pi\varepsilon_0)\,dz'/r'$.
The potential from the rod is



$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{z'=-L/2}^{z'=L/2}\frac{dz'}{r'} = \frac{\lambda}{4\pi\varepsilon_0}\int_{z'=-L/2}^{z'=L/2}\frac{dz'}{\sqrt{x^2+y^2+(z-z')^2}}$$

$$= \frac{-\lambda}{4\pi\varepsilon_0}\ln\left[z - z' + \sqrt{x^2+y^2+(z-z')^2}\right]\Bigg|_{z'=-L/2}^{z'=L/2} = \frac{-\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z-(L/2)+\sqrt{x^2+y^2+\left[z-(L/2)\right]^2}}{z+(L/2)+\sqrt{x^2+y^2+\left[z+(L/2)\right]^2}}\right\}$$

$$= \quad \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z+(L/2)+\sqrt{x^2+y^2+\left[z+(L/2)\right]^2}}{z-(L/2)+\sqrt{x^2+y^2+\left[z-(L/2)\right]^2}}\right\}.$$

To find the potential when $r \gg L$, we use the approximations $(1\pm u)^{1/2} \simeq 1 \pm (u/2)$, $1/(1\pm u) \simeq 1 \mp u$, and $\ln(1+u) \simeq u$, when $u \ll 1$:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z+(L/2)+\sqrt{x^2+y^2+\left[z+(L/2)\right]^2}}{z-(L/2)+\sqrt{x^2+y^2+\left[z-(L/2)\right]^2}}\right\}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{z+(L/2)+\sqrt{r^2+zL+(L/2)^2}}{z-(L/2)+\sqrt{r^2-zL+(L/2)^2}}\right] \simeq \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{z+(L/2)+r\sqrt{1+(zL/r^2)}}{z-(L/2)+r\sqrt{1-(zL/r^2)}}\right]$$

$$\simeq \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{z+(L/2)+r\left[1+(zL/2r^2)\right]}{z-(L/2)+r\left[1-(zL/2r^2)\right]}\right\} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\frac{(z+r)\left[1+(L/2r)\right]}{(z+r)\left[1-(L/2r)\right]}\right\}$$

$$\simeq \frac{\lambda}{4\pi\varepsilon_0}\ln\left\{\left[1+(L/2r)\right]^2\right\} = \frac{2\lambda}{4\pi\varepsilon_0}\ln\left[1+(L/2r)\right] \simeq \frac{2\lambda}{4\pi\varepsilon_0}\frac{L}{2r} = \frac{\lambda L}{4\pi\varepsilon_0 r}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\text{, where } Q = \lambda L \text{ and } r \gg L.$$

55. (a) The potential for the two charges is the sum:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{3q_0}{x - x_0}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q_0}{x + \frac{x_0}{2}}\right) = \frac{q_0}{4\pi\varepsilon_0}\left[\left(\frac{3}{x - x_0}\right) - \left(\frac{1}{x + \frac{x_0}{2}}\right)\right].$$

(b) When $x >> x_0$, we use the approximation $1/(1 \pm u) \simeq 1 \mp u + u^2 \mp u^3 + \ldots$, when $u << 1$:

$$V = \frac{q_0}{4\pi\varepsilon_0 x}\left[\left(\frac{3}{1 - \frac{x_0}{x}}\right) - \left(\frac{1}{1 + \frac{x_0}{2x}}\right)\right]$$

$$= \frac{q_0}{4\pi\varepsilon_0}\left\{\frac{3}{x}\left[1 + \frac{x_0}{x} + \left(\frac{x_0}{x}\right)^2 + \left(\frac{x_0}{x}\right)^3 + \ldots\right] - \frac{1}{x}\left[1 - \frac{x_0}{2x} + \left(\frac{x_0}{2x}\right)^2 - \left(\frac{x_0}{2x}\right)^3 + \ldots\right]\right\}$$

$$= \frac{q_0}{4\pi\varepsilon_0}\left[\frac{2}{x} + \frac{7}{2}\frac{x_0}{x^2} + \frac{11}{4}\frac{x_0^2}{x^3} + \frac{25}{8}\frac{x_0^3}{x^4} + \ldots\right].$$

(c) At large values of x, the contribution of each term to the total decreases.
The first term from part (b) is the potential of a point charge, with $\boxed{q_{\text{net}} = 2q_0}$.
The second term is the potential of a dipole, with $\boxed{p = 7q_0x_0/2}$.

(d) For the point charge plus the dipole to be within 1% of the exact answer, we have

$$\frac{2}{x} + \frac{7}{2}\frac{x_0}{x^2} = 0.99\left[\frac{3}{x - x_0} - \frac{1}{x + (x_0/2)}\right] = \frac{0.99}{x}\left[\frac{3}{1 - (x_0/x)} - \frac{2}{2 + (x_0/x)}\right].$$

If we let $x_0/x = y$, we have

$$2 + \frac{7}{2}y = 0.99\left[\frac{3(2 + y) - 2(1 - y)}{(1 - y)(2 + y)}\right] = 0.99\left(\frac{4 + 5y}{2 + y - y^2}\right).$$

A numerical solution gives $y = -0.0825, +0.0875$. The values of x are $-12.1x_0, +11.4x_0$.
Thus if $\boxed{|x| > 12.1x_0}$, the point charge plus the dipole will be within 1% of the exact answer.

56. The minimum work brings the charge to the point with no kinetic energy. We use the expression for the potential on the axis of a disk:

$$W_{\infty \to a} = q(V_a - V_\infty) = q(Q/2\pi\varepsilon_0 R^2)[(R^2 + x^2)^{1/2} - x]$$
$$= [(3.2 \times 10^{-7}\ \text{C})(6.0 \times 10^{-8}\ \text{C})(2)(9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)/(0.028\ \text{m})^2]\{[(0.088\ \text{m})^2 + (0.028\ \text{m})^2]^{1/2} - 0.088\ \text{m}\}$$
$$= \boxed{1.8 \times 10^{-3}\ \text{J}}.$$

57. There is no change in the kinetic energy. We use the expression for the potential on the axis of a ring:

$$W_{a \to b} = q(V_b - V_a) = q(Q/4\pi\varepsilon_0)[(R^2 + x_b^2)^{-1/2} - (R^2 + x_a^2)^{-1/2}]$$
$$= (-8.5 \times 10^{-8}\ \text{C})(3.5 \times 10^{-7}\ \text{C})(9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2) \times \{[(0.24\ \text{m})^2 + (0.85\ \text{m})^2]^{-1/2} - [(0.24\ \text{m})^2 + (0.28\ \text{m})^2]^{-1/2}\}$$
$$= \boxed{+4.2 \times 10^{-4}\ \text{J}}.$$

58. At the surface of a sphere, we have

$V = Q/4\pi\varepsilon_0 R$ and $E = Q/4\pi\varepsilon_0 R^2$, which gives

$V = ER = (2.8 \times 10^6\ \text{V/m})(0.03\ \text{m}) = 8.4 \times 10^4\ \text{V} = \boxed{84\ \text{kV}}$.

59. After the connection, the two spheres must have the same potential:

$$V = (1/4\pi\varepsilon_0)(q_1'/r_1) = (1/4\pi\varepsilon_0)(q_2'/r_2), \text{ or } \quad q_1' = (r_1/r_2)q_2'.$$

Because charge is conserved we have

$$q_1 + q_2 = q_1' + q_2' .$$

When we combine these two equations, we get

$$q_2' = (q_1 + q_2)[r_2/(r_1 + r_2)].$$

The amount of charge that moves between the two spheres is

$$\Delta q_2 = q_2' - q_2 = \boxed{(q_1 r_2 - q_2 r_1)/(r_1 + r_2)}.$$

60. The electric field that causes air to ionize is $E = 2.8 \times 10^6$ V/m. Thus

$$\Delta V = E\,\Delta x = (2.8 \times 10^6 \text{ V/m})(0.002 \text{ m}) = 5.6 \times 10^3 \text{ V} \approx \boxed{6 \text{ kV}}.$$

61. The potential of the dome of radius R carrying a charge Q is

$V = Q/4\pi\varepsilon_0 R$. Solve for Q:

$$Q = 4\pi\varepsilon_0 RV = (0.61 \text{ m})(5.5 \times 10^6 \text{ V})/(9.0 \times 10^9 \text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{3.7 \times 10^{-4} \text{ C}}.$$

The energy gained by a proton (charge e) after being accelerated by this potential is

$$eV = e(5.5 \text{ MV}) = \boxed{5.5 \text{ MeV}}, \text{ or } (5.5 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \boxed{8.8 \times 10^{-13} \text{ J}}.$$

To find the resulting speed v of the proton, let

$$eV = \tfrac{1}{2} mv^2, \text{ or } \quad v = (2eV/m)^{1/2} = [2(8.8 \times 10^{-13} \text{ J})/(1.67 \times 10^{-27} \text{ kg})]^{1/2} = \boxed{3.2 \times 10^7 \text{ m/s}}.$$

62. We call the radius of the initial drops R_1 and the radius of the combined drop R_2. Because the volume of the mercury does not change, we have

$$\tfrac{4}{3}\pi R_2^3 = 2(\tfrac{4}{3}\pi R_1^3), \text{ which gives } R_1/R_2 = \left(\tfrac{1}{2}\right)^{1/3}.$$

We use the potential for a spherical charge:

$$V_1 = q/4\pi\varepsilon_0 R_1, \text{ and } V_2 = 2q/4\pi\varepsilon_0 R_2, \text{ so we have}$$

$$V_2/V_1 = 2R_1/R_2 = 2\left(\tfrac{1}{2}\right)^{1/3}; \quad V_2 = 2\left(\tfrac{1}{2}\right)^{1/3}(70 \text{ V}) = \boxed{1.1 \times 10^2 \text{ V}}.$$

63. We label the larger sphere 1 and the smaller sphere 2. The wire connecting the spheres means that the potentials of the spheres are the same:

$$Q_1/4\pi\varepsilon_0 R_1 = Q_2/4\pi\varepsilon_0 R_2, \text{ which gives } Q_1 = (R_1/R_2)Q_2 = 3Q_2.$$

We combine this with the conservation of charge:

$$Q_1 + Q_2 = Q, \text{ to get } \quad Q_2 = \boxed{Q/4} \quad \text{and } Q_1 = \boxed{3Q/4}.$$

64. (a) Inside the inner shell, there is no charge, so we have

$$E = \boxed{0,\ r < R}.$$

Between the two shells, the electric field is that of the inner shell:

$$E = \boxed{q/4\pi\varepsilon_0 r^2 \text{ radial, } R < r < 1.5R}.$$

Outside the two shells, the two shells look like a point charge with $Q = q - 3q = -2q$:

$$E = -2q/4\pi\varepsilon_0 r^2 \text{ radial, } 1.5R < r.$$

(b) We add the potentials from the two shells at each location. Because the potential inside a spherical shell is constant and equal to the potential on the surface, we have

$$V_R = (q/4\pi\varepsilon_0 R) + (-3q/4\pi\varepsilon_0 1.5R) = -q/4\pi\varepsilon_0 R$$

$$V_{1.5R} = (q/4\pi\varepsilon_0 1.5R) + (-3q/4\pi\varepsilon_0 1.5R) = -4q/4\pi\varepsilon_0 3R.$$

The potential difference is

$$V_{1.5R} - V_R = \boxed{-q/12\pi\varepsilon_0 R}.$$

(c) When the two shells are connected, the potential difference between the two shells must be 0. The system can be considered as one conductor, which can have no charge inside.

$\boxed{\text{All of the charge moves to the outer shell, with } q_{\text{net}} = -2q}$.

$\boxed{65}$. The number of raindrops per unit volume is $n = 1.2 \times 10^{10}/\text{m}^3$. Each raindrop carries a charge $q = 16 \times 10^{-19}$ C, so the total charge Q on a spherical piece of cloud of volume V is

$Q = nqV = nq\,(\frac{4}{3}\pi R^3)$, where R is the radius of the cloud. The electric field E outside the cloud is then

$E = Q/4\pi\varepsilon_0 R^2 = nq\,(\frac{4}{3}\pi R^3)/4\pi\varepsilon_0 R^2 = nqR/3\varepsilon_0$. Equate this to $E_{\max} = 3.2 \times 10^6$ V/m to obtain the radius at which electrical breakdown occurs:

$R = 3\varepsilon_0 E_{\max}/nq$

$= 3(8.85 \times 10^{-12}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.2 \times 10^6\ \text{V/m})/[(1.2 \times 10^{10}/\text{m}^3)(16 \times 10^{-19}\ \text{C})]$

$= 4.4 \times 10^3\ \text{m} = \boxed{4.4\ \text{km}}$.

66. Let the charge on the sphere of radius $R_1 = 0.05$ m be Q_1 and that on the other one be Q_2. Then

$Q_1 + Q_2 = Q = 40\ \mu\text{C}$.

Also, Since the spheres are connected by a metal wire they must be at the same potential:

$V_1 = V_2$;

$Q_1/4\pi\varepsilon_0 R_1 = Q_2/4\pi\varepsilon_0 R_2$. Thus

$Q_1 = Q/(1 + R_2/R_1) = 40\ \mu\text{C}/(1 + 0.05\ \text{m}/\ 0.08\ \text{m}) = \boxed{25\ \mu\text{C}}$ and

$Q_2 = Q - Q_1 = 40\ \mu\text{C} - 25\ \mu\text{C} = \boxed{15\ \mu\text{C}}$.

67. (a) The wire connecting the spheres means that the potentials of the spheres are the same:

$q_1/4\pi\varepsilon_0 R_1 = q_2/4\pi\varepsilon_0 R_2$;

$q_1 = (R_1/R_2)q_2 = [(20\ \text{mm})/(100\ \text{mm})]q_2$, which gives $q_2 = 5q_1$.

The Coulomb force is

$F = (1/4\pi\varepsilon_0)(q_1 q_2/r^2)$;

$3.5\ \text{N} = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)(5{q_1}^2)/(0.25\ \text{m})^2$, which gives

$q_1 = 2.2 \times 10^{-6}\ \text{C} = \boxed{2.2\ \mu\text{C}}$ and $q_2 = 5q_1 = \boxed{11\ \mu\text{C}}$.

(b) The electric fields at the surfaces of the spheres are

$E_1 = (1/4\pi\varepsilon_0)(q_1/{R_1}^2) = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.2 \times 10^{-6}\ \text{C})/(0.020\ \text{m})^2 = \boxed{4.95 \times 10^7\ \text{V/m, radial}}$.

$E_2 = (1/4\pi\varepsilon_0)(q_2/{R_2}^2) = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)(11 \times 10^{-6}\ \text{C})/(0.100\ \text{m})^2 = \boxed{9.90 \times 10^6\ \text{V/m, radial}}$.

68. (a) At the surface of the balloon, we have

$V_1 = (1/4\pi\varepsilon_0)(q/R_1) = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.5 \times 10^{-5}\ \text{C})/(4.30\ \text{m}) = \boxed{3.1 \times 10^4\ \text{V}}$.

(b) Because the charge is conserved, we have

$V_2 = (1/4\pi\varepsilon_0)(q/R_2) = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.5 \times 10^{-5}\ \text{C})/(3.10\ \text{m}) = \boxed{4.4 \times 10^4\ \text{V}}$.

(c) The energy increases because the outside pressure compresses the balloon and therefore does positive work. This increases the charge density on the surface, which means that the positive charges are forced closer.

69. (a) The increase in kinetic energy comes from the decrease in potential energy, which means the proton must go from high to low potential:

$\Delta K = K - 0 = -\Delta U = -q\,\Delta V$;

$K = -(+1\ \text{e})(-5.5 \times 10^6\ \text{V}) = \boxed{+5.5 \times 10^6\ \text{eV}}$.

To convert units, we have

$K = (+5.5 \times 10^6\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV}) = \boxed{8.8 \times 10^{-13}\ \text{J}}$.

(b) We find the final speed from

$K = \frac{1}{2}mv^2$;

$8.8 \times 10^{-13}\ \text{J} = \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})v^2$, which gives

$v = \boxed{3.3 \times 10^7\ \text{m/s}}$.

70. The maximum potential is reached when the charge on the sphere creates an electric field large enough to break down the air. At the surface of a sphere, we have

$E = (1/4\pi\varepsilon_0)(Q/R^2)$ and $V = (1/4\pi\varepsilon_0)(Q/R)$, or

$V = ER = (2.8 \times 10^6\ \text{V/m})(0.41\ \text{m}) = \boxed{1.1 \times 10^6\ \text{V}}$.

We find the charge on the sphere from

$V = (1/4\pi\varepsilon_0)(Q/R)$;

$1.1 \times 10^6\ \text{V} = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)Q/(0.41\ \text{m})$, which gives $Q = \boxed{5.2 \times 10^{-5}\ \text{C}}$.

71. (a) The maximum potential is reached when the charge on the sphere creates an electric field large enough to break down the air. At the surface of a sphere, we have

$E = (1/4\pi\varepsilon_0)(Q/R^2)$ and $V = (1/4\pi\varepsilon_0)(Q/R)$, or

$V = ER = (3 \times 10^6\ \text{V/m})(1.3\ \text{m}) = \boxed{3.9 \times 10^6\ \text{V}}$.

(b) The increase in kinetic energy comes from the decrease in potential energy, which means the proton must go from high to low potential:

$\Delta K = K - 0 = -\Delta U = -q\,\Delta V$;

$K = -(+1\ \text{e})(-3.9 \times 10^6\ \text{V}) = +3.9 \times 10^6\ \text{eV} = \boxed{3.9\ \text{MeV}\ \ (6.2 \times 10^{-13}\ \text{J})}$.

(c) We find the charge on the sphere from

$V = (1/4\pi\varepsilon_0)(Q/R)$;

$3.9 \times 10^6\ \text{V} = (9 \times 10^9\ \text{C}^2/\text{N}\cdot\text{m}^2)Q/(1.3\ \text{m})$, which gives $Q = \boxed{5.6 \times 10^{-4}\ \text{C}}$.

72. For parallel plates, we have

$\Delta V = Ed$;

$5 \times 10^3\ \text{V} = (3 \times 10^6\ \text{V/m})d$, which gives $d = 1.7 \times 10^{-3}\ \text{m} = \boxed{1.7\ \text{mm}}$.

73. The potential at a point on the x-axis between the disks is

$$V = \frac{Q}{2\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + (a+x)^2} - (a+x) + \sqrt{R^2 + (a-x)^2} - (a-x)\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left[\sqrt{R^2 + (a+x)^2} + \sqrt{R^2 + (a-x)^2} - 2a\right].$$

74. The potential of the two rings at a point on the x-axis is

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2 + (x-a)^2}} - \frac{1}{\sqrt{R^2 + (x+a)^2}}\right].$$

75. When $x >> a$ and $x >> R$, we write the solution for the two rings as

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x\sqrt{\left(\frac{R}{x}\right)^2 + \left(1 - \frac{a}{x}\right)^2}} - \frac{1}{x\sqrt{\left(\frac{R}{x}\right)^2 + \left(1 + \frac{a}{x}\right)^2}}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 x}\left(\frac{1}{\sqrt{1 - \frac{2a}{x} + \frac{a^2 + R^2}{x^2}}} - \frac{1}{\sqrt{1 + \frac{2a}{x} + \frac{a^2 + R^2}{x^2}}}\right).$$

Because $x >> a$ and $x >> R$, we use the approximation $(1 + u)^{-1/2} \simeq 1 - \frac{1}{2}u$ to get

$$V \simeq \frac{Q}{4\pi\varepsilon_0 x}\left[1 + \frac{a}{x} - \frac{1}{2}\left(\frac{a^2 + R^2}{x^2}\right) - 1 + \frac{a}{x} + \frac{1}{2}\left(\frac{a^2 + R^2}{x^2}\right)\right] \simeq \frac{Q}{4\pi\varepsilon_0 x}\left(\frac{2a}{x}\right) \simeq \frac{Qa}{2\pi\varepsilon_0 x^2}.$$

We see that the potential is that of a dipole; from far away the disks approximate two point charges separated by $2a$.

76. If the second charge is stationary, the total energy is potential, which we find by considering one charge to be at the potential created by the other charge:

$$U = (-q)(1/4\pi\varepsilon_0)(Q/r) = \boxed{-qQ/4\pi\varepsilon_0 r}.$$

For the circular motion, the Coulomb force must provide the centripetal acceleration:

$F = qQ/4\pi\varepsilon_0 r^2 = mv^2/r$, which gives $mv^2 = qQ/4\pi\varepsilon_0 r$.

The total energy is

$$E = K + U$$
$$= \tfrac{1}{2}mv^2 + (-qQ/4\pi\varepsilon_0 r) = (\tfrac{1}{2} - 1)(qQ/4\pi\varepsilon_0 r) = \boxed{-qQ/8\pi\varepsilon_0 r}.$$

Because the force is a central force, the angular momentum must be conserved; thus the angular velocity is constant.

77. From Table 24–1, with $V = 0$ at $r = \infty$, we have the potential inside a nonconducting sphere:

$$V = (Q/8\pi\varepsilon_0 R)[3 - (r^2/R^2)].$$

The potential energy of a charge $-q$ is

$$U = -qV = \boxed{-(qQ/8\pi\varepsilon_0 R)(3 - r^2/R^2)}.$$

The variable part of the potential energy has the form of the elastic potential energy of a spring:

$$U = \tfrac{1}{2}kr^2,$$

so the motion can be an oscillation, like the mass on a spring.

Comparing the coefficients, we have

$$k = \boxed{qQ/4\pi\varepsilon_0 R^3}.$$

78. With all electrons at infinity, which is the reference level, no work is required to place the first electron at $x = -6\ \mu$m:

$$U_1 = W_1 = \boxed{0\text{, first electron at } x = -6\ \mu\text{m}}.$$

To place the second electron at $x = +6\ \mu$m, we have

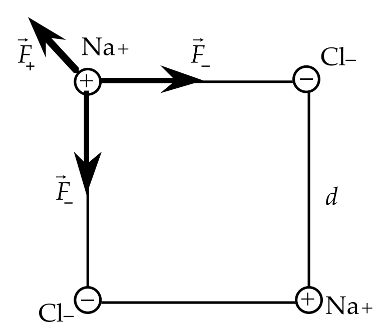
$$U_2 = W_2 = qV_a = (-e)(1/4\pi\varepsilon_0)(-e/r_a) = (1/4\pi\varepsilon_0)(e^2/r)$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(1.6 \times 10^{-19}\ \mathrm{C})^2/(6\times 10^{-6}\ \mathrm{m})$$
$$= \boxed{3.9 \times 10^{-23}\ \mathrm{J,\ second\ electron\ at\ } x = +6\ \mu\mathrm{m}}.$$

To place the third electron at $x = 0$ nm, we have

$$U_3 = W_3 = qV_b = (-e)(1/4\pi\varepsilon_0)[(-e/r_b) + (-e/r_b)] = (1/4\pi\varepsilon_0)(2e^2/r_b)$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})2(1.6 \times 10^{-19}\ \mathrm{C})^2/(6 \times 10^{-6}\ \mathrm{m})$$
$$= \boxed{5.9 \times 10^{-23}\ \mathrm{J,\ third\ electron\ at\ } x = 0}.$$

The order in which the electrons are moved will affect the individual terms but not the total energy of 9.8×10^{-23} J (6.0×10^{-4} eV).

79. If we consider the sodium ion in the upper left corner, we see from symmetry that the net force must be along the diagonal:



$$F_{\text{net}} = 2F_- \cos 45° - F_+ = (e^2/4\pi\varepsilon_0)\{(2 \cos 45°/d^2) - [1/(d\sqrt{2})^2]\}$$
$$= (e^2/4\pi\varepsilon_0 d^2)(\sqrt{2} - \tfrac{1}{2})$$
$$= [(1.6 \times 10^{-19}\ \mathrm{C})^2(9 \times 10^9\ \mathrm{C^2/N\cdot m^2})/(2.5 \times 10^{-19}\ \mathrm{m})^2](\sqrt{2} - \tfrac{1}{2})$$
$$= \boxed{3.4 \times 10^{-9}\ \mathrm{N\ toward\ the\ other\ Na^+}}.$$

We find the work required from the potential energy change:

$$W = \Delta U = e(V_\infty - V_{\text{corner}}) = (e/4\pi\varepsilon_0)\{0 - [-(2e/d) + (e/d\sqrt{2})]\}$$
$$= (e^2/4\pi\varepsilon_0 d)[2 - (1/\sqrt{2})]$$
$$= [(1.6 \times 10^{-19}\ \mathrm{C})^2(9 \times 10^9\ \mathrm{C^2/N\cdot m^2})/(2.5 \times 10^{-19}\ \mathrm{m})][2 - (1/\sqrt{2})]$$
$$= \boxed{1.2 \times 10^{-18}\ \mathrm{J}\ (7.4\ \mathrm{eV})}.$$

80. Because the point is not far away from the dipole, we find the potential from the sum of the potentials of two point charges:

$$V = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q}{\left[x^2 + (y-a)^2\right]^{1/2}} + \frac{-q}{\left[x^2 + (y+a)^2\right]^{1/2}}\right\}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{\left[x^2 + (y-a)^2\right]^{1/2}} - \frac{1}{\left[x^2 + (y+a)^2\right]^{1/2}}\right\}.$$

We find the components of the electric field from the partial derivatives of V:

$$E_x = -\frac{\partial V}{\partial x} = \frac{-q}{4\pi\varepsilon_0}\left\{\frac{-x}{\left[x^2 + (y-a)^2\right]^{3/2}} - \frac{-x}{\left[x^2 + (y+a)^2\right]^{3/2}}\right\}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{x}{\left[x^2 + (y-a)^2\right]^{3/2}} - \frac{x}{\left[x^2 + (y+a)^2\right]^{3/2}}\right\};$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{-q}{4\pi\varepsilon_0}\left\{\frac{-(y-a)}{\left[x^2 + (y-a)^2\right]^{3/2}} - \frac{-(y+a)}{\left[x^2 + (y+a)^2\right]^{3/2}}\right\}$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{y-a}{\left[x^2 + (y-a)^2\right]^{3/2}} - \frac{y+a}{\left[x^2 + (y+a)^2\right]^{3/2}}\right\}.$$

The electric field at the point (x, y) is

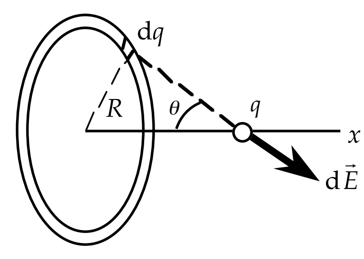
$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\left\{\frac{x}{[x^2+(y-a)^2]^{3/2}} - \frac{x}{[x^2+(y+a)^2]^{3/2}}\right\}\hat{i} + \left\{\frac{y-a}{[x^2+(y-a)^2]^{3/2}} - \frac{y+a}{[x^2+(y+a)^2]^{3/2}}\right\}\hat{j}\right].$$

At the point P, $r = 3a$ 45° from the y-axis, and $x = y = 3a \cos 45° = 2.12a$. Thus

$V_P = \boxed{0.152q/4\pi\varepsilon_0 a}$, and $\vec{E}_P = \boxed{(q/4\pi\varepsilon_0 a^2)(0.114\hat{i} + 0.023\hat{j})}$.

81. From Example 24–9, we know the potential on the axis of a ring is

$V = Q/4\pi\varepsilon_0(R^2 + x^2)^{1/2}$.



From symmetry, the electric field is along the x-axis, which we find from

$$\vec{E} = -(\partial V/\partial x)\hat{i}$$
$$= -(\partial/\partial x)[Q/4\pi\varepsilon_0(R^2 + x^2)^{1/2}]\hat{i}$$
$$= (-Q/4\pi\varepsilon_0)(-\tfrac{1}{2})(2x)/(R^2 + x^2)^{3/2}\hat{i} = \boxed{[(Qx/4\pi\varepsilon_0(R^2 + x^2)^{3/2}]\hat{i}}.$$

To find the field from direct integration, we use the diagram. Choosing a differential element of the ring, we see that the symmetry of the charge distribution means that we need to integrate the x-component:

$$E = \int dE_x = \int (1/4\pi\varepsilon_0)[dq/(R^2 + x^2)] \cos\theta.$$

To perform the integration, we must reduce the integrand to one variable.

If we compare the two ways, we see that the direct integration method requires the selection of differential elements and the use of symmetry to handle the vector components, plus the actual integration. Because potential is a scalar, finding the field by differentiating V is generally easier.

82. For the positron, the increase in kinetic energy comes from the decrease in potential energy:

$$K_f - K_i = -(U_f - U_i) = -e(V_f - V_i);$$
$$K_f - 0 = -e(1/4\pi\varepsilon_0)(0 - e/r_i);$$
$$K_f = (e^2/4\pi\varepsilon_0)(1/r_0) = (1.60 \times 10^{-19}\ \mathrm{C})^2(9 \times 10^9\ \mathrm{C^2/N\cdot m^2})/(6.5 \times 10^{-10}\ \mathrm{m}) = \boxed{3.5 \times 10^{-19}\ \mathrm{J}\ (2.2\ \mathrm{eV})}.$$

83. We find the change in potential energy from

$$\Delta U = -e\,\Delta V = -e(Ze/4\pi\varepsilon_0)(1/r_2 - 1/r_1) = -(Ze^2/4\pi\varepsilon_0)(1/r_2 - 1/r_1)$$
$$= -(9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(2)(1.60 \times 10^{-19}\ \mathrm{C})^2(1/2 - 1/3)/(10^{-10}\ \mathrm{m}) = \boxed{-7.68 \times 10^{-19}\ \mathrm{J}}.$$

For the electron orbiting the nucleus, the attractive Coulomb force provides the centripetal acceleration:

$$Ze^2/4\pi\varepsilon_0 r^2 = mv^2/r, \text{ which gives } mv^2 = Ze^2/4\pi\varepsilon_0 r.$$

The change in kinetic energy is

$$\Delta K = \tfrac{1}{2}\Delta(mv^2) = \tfrac{1}{2}(Ze^2/4\pi\varepsilon_0)(1/r_2 - 1/r_1)$$
$$= (9 \times 10^9\ \mathrm{C^2/N\cdot m^2})(2)(1.60 \times 10^{-19}\ \mathrm{C})^2\,(1/2 - 1/3)/(10^{-10}\ \mathrm{m}) = \boxed{+3.84 \times 10^{-19}\ \mathrm{J}}.$$

The change in total energy is

$$\Delta E = \Delta K + \Delta U = +3.84 \times 10^{-19}\ \mathrm{J} + (-7.68 \times 10^{-19}\ \mathrm{J}) = \boxed{-3.84 \times 10^{-19}\ \mathrm{J}}.$$

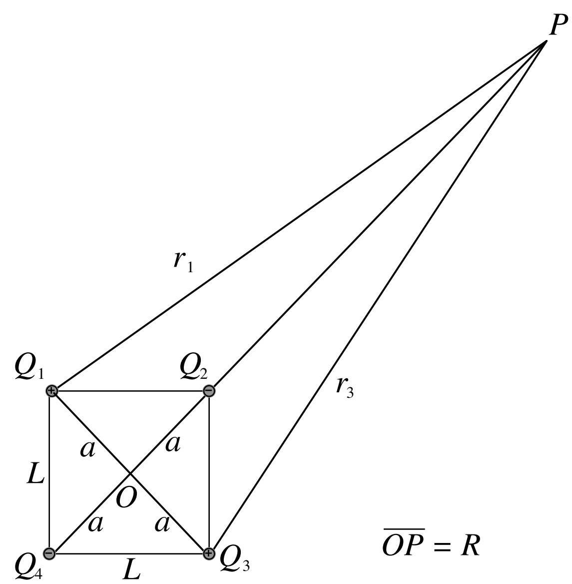
We see that the energy decreases as the electron gets closer to the nucleus; the energy is carried off by light emitted by the electron.

84. In the diagram shown to the right, the distances between each charge, labeled 1 through 4, to the point P of interest, are given by

$$r_1 = r_3 = (R^2 + a^2)^{1/2} = (R^2 + \tfrac{1}{2}L^2)^{1/2},$$
$$r_2 = R - a = R - L/\sqrt{2}, \text{ and}$$
$$r_4 = R + a = R + L/\sqrt{2}.$$

The potential at point P is then

$$V = (1/4\pi\varepsilon_0)(Q_1/r_1 + Q_2/r_2 + Q_3/r_3 + Q_4/r_4)$$
$$= (1/4\pi\varepsilon_0)[Q/(R^2 + \tfrac{1}{2}L^2)^{-1/2} - Q/(R - L/\sqrt{2}) + Q/(R^2 + \tfrac{1}{2}L^2)^{1/2} - Q/(R + L/\sqrt{2})]$$
$$= (2Q/4\pi\varepsilon_0 R)[(1 + L^2/2R^2)^{-1/2} - (1 - L^2/2R^2)^{-1}]$$
$$\approx (2Q/4\pi\varepsilon_0 R)[(1 - L^2/4R^2) - (1 + L^2/2R^2)]$$
$$= \boxed{-3QL^2/(8\pi\varepsilon_0 R^3)}.$$



Note that here we made use of the approximation $(1 + x)^n \approx nx$, for $x = L^2/2R^2 << 1$. Also, we assumed that point P is aligned with the two negative charges ($Q_2 = Q_4 = -Q$). Otherwise V will differ by a negative sign.

85. Assume that $r_1 > r_2$. Place the origin of the coordinate system at the center of both shells. For $r > r_2$ the electric field is identical to that of a point charge, $Q = q_1 + q_2$, at the origin. So

$$V = (1/4\pi\varepsilon_0)Q/r = \boxed{(1/4\pi\varepsilon_0)(q_1 + q_2)/r \quad (r_2 < r)}.$$

Between r_1 and r_2, the E-field produced by q_1 is zero, so the potential due to q_1 remains the same as its value at r_1, i.e., $(1/4\pi\varepsilon_0)q_1/r_1$. For q_1, the E-field is still equivalent to that of a point charge q_1 at the origin, so the contribution to V due to q_1 is $(1/4\pi\varepsilon_0)q_1/r$. Add both contributions up to obtain

$$V = \boxed{(1/4\pi\varepsilon_0)(q_2/r_2 + q_1/r) \quad (r_1 < r < r_2)}.$$

Once $r < r_1$, there is no electric field, so the potential no longer changes once it reaches its value at r_1. Thus

$$V = \boxed{(1/4\pi\varepsilon_0)(q_1/r_1 + q_2/r_2) \quad (r < r_1)}.$$

86. Because the work done by the electric field of the dipole is independent of the path, we have

$$W_{a \to b} = q_0(V_b - V_a).$$

The initial and final points are not far from the dipole, so we find the potentials for two point charges:

$$W_{a \to b} = q_0[(1/4\pi\varepsilon_0)(q/r_{b2} - q/r_{b1}) - (1/4\pi\varepsilon_0)(q/r_{a2} - q/r_{a1})]$$
$$= (1/4\pi\varepsilon_0)(q_0 q)(1/r_{b2} - 1/r_{b1} - 1/r_{a2} + 1/r_{a1})$$
$$= (9 \times 10^9\ \mathrm{C^2/N \cdot m^2})(3.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})[1/(0.2\ \mathrm{m}) - 1/(0.6\ \mathrm{m}) - 1/(0.8\ \mathrm{m}) + 1/(0.4\ \mathrm{m})]$$
$$= \boxed{+0.62\ \mathrm{J}}.$$

87. (a) From the force diagram, we apply $\sum \vec{F} = 0$:

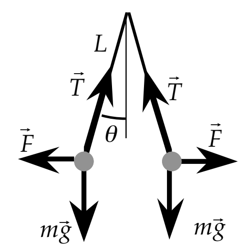
horizontal: $T \sin\theta = F = kqq/r^2$;

vertical: $T \cos\theta = mg$.

If we divide the two equations, we get

$$\tan\theta = F/mg = kq^2/r^2 mg = kq^2/(2L\sin\theta)^2 mg$$
$$\tan 30° = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(2.0 \times 10^{-6}\ \mathrm{C})^2/[2(0.80\ \mathrm{m})\sin 30°]^2 m(9.8\ \mathrm{m/s^2}),$$

which gives $m = \boxed{9.9 \times 10^{-3}\ \mathrm{kg}}$.



(b) With the electric potential reference level at infinity and the gravitational potential reference level at $\theta = 0°$, we have

$$U = qV + mgy = (1/4\pi\varepsilon_0)(q^2/2L\sin\theta) + mg(L - L\cos\theta)$$
$$= [(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(2 \times 10^{-6}\ \mathrm{C})^2/2(0.80\ \mathrm{m})(\sin\theta)] + (9.9 \times 10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(0.80\ \mathrm{m})(1 - \cos\theta)$$
$$= \boxed{0.023/\sin\theta + 0.078(1 - \cos\theta)}.$$

88. If we neglect end effects, the electric field (the potential gradient) of each plane is uniform. We find the work required to move the second plane from

$$W = q_2\,\Delta V = q_2\,(-E_1\,\Delta x)$$
$$= -\sigma_2 L^2(\sigma_1/2\varepsilon_0)[(a - (x_2 - x_1)] = \boxed{(\sigma_1\sigma_2 L^2/2\varepsilon_0)(x_2 - x_1 - a)}.$$

89. We use the analogy to the charged spherical shell. When we are outside a charged cylindrical shell of radius r', the potential is that of a line charge: $V = -(\lambda/2\pi\varepsilon_0)\ln(r/a)$ with $V = 0$ at $r = a$. When we are inside a charged cylindrical shell of radius r', the potential is the potential on the surface: $V = -(\lambda/2\pi\varepsilon_0)\ln(r'/a)$. For a point inside the cylinder, $r < R$, the potential has two contributions: the sum (integral) of the shells inside r and the sum (integral) of the shells outside r. For a shell of radius r', the linear charge density is $d\lambda = \rho 2\pi r'\,dr'$. We find the potential from

$$V = \int_0^r -\frac{\rho 2\pi r'\,dr'}{2\pi\varepsilon_0}\ln\left(\frac{r}{a}\right) + \int_r^R -\frac{\rho 2\pi r'\,dr'}{2\pi\varepsilon_0}\ln\left(\frac{r'}{a}\right)$$
$$= -\frac{\rho}{\varepsilon_0}\ln\left(\frac{r}{a}\right)\int_0^r r'\,dr' - \frac{\rho}{\varepsilon_0}\int_r^R r'\ln\left(\frac{r'}{a}\right)dr'$$
$$= -\frac{\rho}{\varepsilon_0}\ln\left(\frac{r}{a}\right)\frac{r^2}{2} - \frac{\rho}{\varepsilon_0}\left\{\frac{r'^2}{2}\left[\ln\left(\frac{r'}{a}\right) - \frac{1}{2}\right]\right\}\Bigg|_r^R$$
$$= -\frac{\rho r^2}{2\varepsilon_0}\ln\left(\frac{r}{a}\right) - \frac{\rho R^2}{2\varepsilon_0}\left[\ln\left(\frac{R}{a}\right) - \frac{1}{2}\right] + \frac{\rho r^2}{2\varepsilon_0}\left[\ln\left(\frac{r}{a}\right) - \frac{1}{2}\right]$$
$$= -\frac{\rho r^2}{4\varepsilon_0} - \frac{\rho R^2}{2\varepsilon_0}\left[\ln\left(\frac{R}{a}\right) - \frac{1}{2}\right],\quad r < R.$$

For a point outside the cylinder, $r > R$, all of the cylindrical shells appear to be line charges:

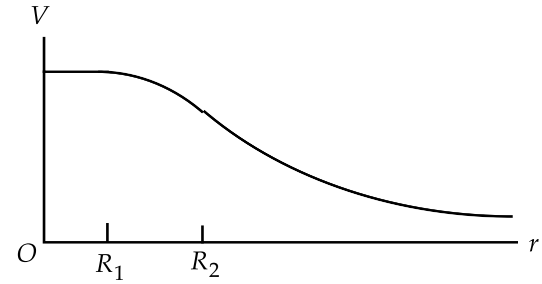
$$V = \int_0^R -\frac{\rho 2\pi r'\,dr'}{2\pi\varepsilon_0}\ln\left(\frac{r}{a}\right) = -\frac{\rho}{\varepsilon_0}\ln\left(\frac{r}{a}\right)\int_0^R r'\,dr'$$
$$= -\frac{\rho R^2}{2\varepsilon_0}\ln\left(\frac{r}{a}\right),\quad r > R.$$

90. The charge density of the dielectric shell is

$\rho = Q/[\frac{4}{3}\pi(R_2{}^3 - R_1{}^3)]$

$= (5 \times 10^{-6}\text{ C})/\{\frac{4}{3}\pi[(0.45\text{ m})^3 - (0.16\text{ m})^3]\}$

$= 1.4 \times 10^{-5}\text{ C/m}^3$.



In the region where $r < R_1 = 16$ cm, there is no electric field, so the potential is constant. Inside the dielectric shell, $R_1 < r < R_2 = 45$ cm, we see from Table 24–1 that the potential is proportional to $-r^2$. Outside the dielectric shell, $r > R_2$, the shell is equivalent to a point charge at the center, so the potential is proportional to $1/r$.
We find the potential at $r = 0$ by adding (integrating) the potentials of the differential spherical shells between R_1 and R_2:

$V = k\int(\rho 4\pi r^2\,\mathrm{d}r)/r = 2\pi k\rho\,(R_2{}^2 - R_1{}^2)$

$= 2\pi(9 \times 10^9\text{ C}^2/\text{N}\cdot\text{m}^2)(1.4 \times 10^{-5}\text{ C/m}^3)[(0.45\text{ m})^2 - (0.16\text{ m})^2]$

$= \boxed{1.4 \times 10^5\text{ V},\ r = 0}$.

Because the potential is constant from $r = 0$ to $r = R_1$, the potential at the inner radius is

$\boxed{1.4 \times 10^5\text{ V},\quad r = R_1}$.

Because the potential outside the shell is the same as that of a point charge, we find the potential at $r = R_2$ from

$V = (1/4\pi\varepsilon_0)(Q/R_2)$

$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C})(5 \times 10^{-6}\text{ C})/(0.45\text{ m}) = \boxed{1.0 \times 10^5\text{ V},\quad r = R_2}$.

91. To find the total potential energy of the sphere, we consider it to be made up of differential shells and add (integrate) the work required to bring each shell in from infinity.
If a sphere of radius $r < R$ has been formed, the potential at the surface is

$V = (1/4\pi\varepsilon_0)(q/r) = (1/4\pi\varepsilon_0)(\rho\frac{4}{3}\pi r^3/r) = \rho r^2/3\varepsilon_0$.

The work to bring the charge of the next shell, $\mathrm{d}q = \rho 4\pi r^2\,\mathrm{d}r$, in from infinity is

$\mathrm{d}W = \mathrm{d}q\,V = (\rho 4\pi r^2\,\mathrm{d}r)(\rho r^2/3\varepsilon_0)$.

The total work and thus the total potential energy stored is

$$W = \int_0^R \frac{\rho^2 4\pi r^4\,\mathrm{d}r}{3\varepsilon_0} = \frac{4\pi\rho^2}{3\varepsilon_0}\int_0^R r^4\,\mathrm{d}r = \frac{4\pi\rho^2 R^5}{15\varepsilon_0}.$$

CHAPTER 25 Capacitors and Dielectrics

Answers to Understanding the Concepts Questions

1. The fact that there are two ways of writing the units of permittivity changes nothing. The flexibility may allow for ease in the cancellation of units in different equations, but this is purely a matter of convenience.

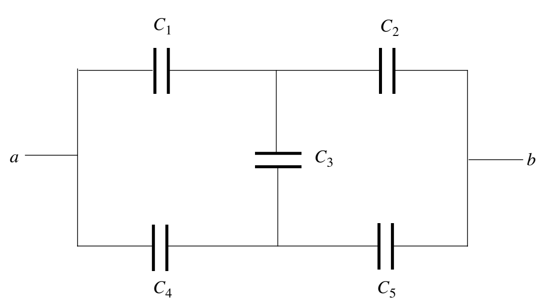
2. Form a parallel-plate capacitor with the two parallel metal plates. Charge the capacitor by hooking it up to the battery, and measure the potential difference V across the capacitor with the voltmeter. The charge on the capacitor must satisfy $Q = CV$. Next, disconnect the battery from the capacitor, so that Q cannot change anymore. Now insert the plastic to completely fill the air gap in between the two metal plate. As a result the capacitance of the capacitor becomes $C' = \kappa C$, with κ the dielectric constant of the plastic. Since Q cannot change, the voltage difference V', which again can be measured by the voltmeter, must satisfy $Q = C'V' = \kappa CV'$. Thus $CV = \kappa CV'$, and $\kappa = V/V'$.

3. The reminder provides the proof: Consider a closed path which consists of a segment between the plates, leading from one to the other, and a segment which closes the path outside of the plates. If there is a voltage drop in the interior region, there must be a voltage rise outside that region. This would be impossible if the electric field vanished outside.

4. By definition $C = Q/V$. If $C = 0$, then $Q = 0$, meaning that no charge can be stored on the capacitor without introducing an infinite V. This can happen, for example, as the cross-sectional area of a parallel-plate capacitor approaches zero.

5. The reason the capacitance per unit length goes to zero is that in the limit under consideration, the potential difference becomes infinitely large. It takes infinite work to concentrate an infinite amount of charge along an infinitely long line.

6. An example is shown below.



7. A capacitor consists of two separate metal pieces that are insulated from one another. When a voltage difference is applied across the two metal pieces, one piece is charged to $+Q$ while the other is charged to $-Q$. The net charge on the entire capacitor, including both metal pieces, is therefore always zero.

8. The fact that unlike charges attract and like charges repel forces the dielectric constant to be larger than or equal to unity.

9. Since the plates are disconnected from the battery Q cannot change, and neither can the electric field E in between the plates (as it is proportional to the surface charge density on each plate). If the two plates are pushed closer together, the potential difference between them, $V = Ed$, must therefore decrease along with d, the plate separation). Thus $C = Q/V$ must increase. (This is also clear from the expression for the capacitance of the parallel-plate capacitor). Meanwhile, the electrostatic energy store in the capacitor, $Q^2/2C$, must decrease as Q is fixed whiled C increases. This is also understood from the fact that, while the energy density in between the two plates remains the same (as it is proportional to E^2, which does not change), the volume of the region in between the two plates decreases.

10. When V is held fixed, Q is proportional to C. When the plates are pushed together, C increases, and so must Q. Another way to see this is to note that if the potential difference between the plates is fixed as the distance between the plates decreases, then the electric field must grow. This can only happen if the surface charge density, and thus the total charge, grows. What about energy? For fixed V, the energy is proportional to C (or Q). Thus pushing the plates together increases the energy, and positive work must be done to push the plates together. Alternatively, note that the energy is proportional to E^2 times the interior volume. The volume decreases as the distance between the plates decreases, but the electric field grows in the same way. Thus E^2 grows quadratically with the separation, and the total energy is proportional to the separation.

11. The capacitance of a spherical capacitor is proportional to its radius, which is fixed if the surface area A is fixed. There is not much we can do to adjust its capacitance. If we form a parallel-plate capacitor or a concentric-cylinder capacitor, then theoretically C can be made arbitrarily large by making the separation between the two metal pieces arbitrarily small. Note, however, that as the separation between the two concentric cylinders become very small, the capacitance of the concentric-cylinder arrangement approaches that of the parallel-plate capacitor (for the same plate area). So either of them can be used to make a capacitor of large capacitance by reducing the plate separation.

12. The two plates of a large charged capacitor carry charges Q and $-Q$; these charges may be large. When a wire connects the plates the charge will flow through the wire, generally in a very short time. This could be dangerous if the person making the connection is careless and allows some of the charge to flow through him or her. More generally, a large amount of energy has been stored in the capacitor and is dissipated when the charges flow out. It is always a potential danger when a large amount of energy is dissipated over a short time interval.

13. Consider a pair of plates carrying charges Q and $-Q$, respectively. Without the fringe effect, the electric filed would abruptly reduce to zero at the edge of the plates. The fringe effect causes the electric field to "leak" outside. Since the total electric flux depends on the charge on each plate and cannot change, the electric field inside must be somewhat diluted as a result. Therefore, for the same Q, the potential difference V between the two plates is weaker due to fringe effect, and the capacitance, $C = Q/V$, increases as a result.

14. Two oppositely charged nonconductors will give rise to an electric field between them, with the field lines going from the positive to the negative charges. This configuration will store electrical energy and act in that sense just like a capacitor. The difficulty is that it is hard to put a large charge on insulators, and it is also hard to discharge them. That is why conductors are much more useful.

15. According to the calculation in Example 25-5, the battery contains some 600 million times the energy it takes to charge up a single capacitor. So it can certainly be used to charge more than one of them. In fact, it would only take 1/600,000 of the energy of the battery to charge 1000 such capacitors.

16. If the plates are not shorted, they could accumulate charges and produce a voltage across them, and that can be hazardous should your candle such a capacitor improperly and it gets discharged by driving a current through your body.

17. The question is whether the vacuum can be polarized; that is, whether positive and negative charges can be separated from it. In terms of what we know, there are no charges in the vacuum, and therefore an electric field applied to it will not induce a charge separation. When you study quantum mechanics you will learn that the full answer to this question is quite different from the answer given here.

18. The air-filled capacitors are usually operated at a high frequency, i.e., they are hooked up to a rapidly alternating voltage source that quickly charges and discharges the capacitor, so the slow leakage of charges through the air gap does not present a significant problem.

Solutions to Problems

1. (a) For a parallel-plate capacitor, we have

$$C = \varepsilon_0 A/d$$
$$= (8.85 \times 10^{-12}\ \mathrm{F/m})(50 \times 10^{-4}\mathrm{m}^2)/(1 \times 10^{-3}\ \mathrm{m}) = 4.4 \times 10^{-11}\ \mathrm{F} = \boxed{44\ \mathrm{pF}}.$$

(b) For a sphere, we have

$$C = 4\pi\varepsilon_0 R;$$
$$4.4 \times 10^{-11}\ \mathrm{F} = 4\pi(8.85 \times 10^{-12}\ \mathrm{F/m})R, \text{ which gives } R = 0.40\ \mathrm{m} = \boxed{40\ \mathrm{cm}}.$$

2. For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(R_2/R_1)$$
$$= (1.0 \times 10^3\ \mathrm{m})2\pi(8.85 \times 10^{-12}\ \mathrm{F/m})/\ln[(1.2\ \mathrm{cm})/(0.8\ \mathrm{cm})] = 1.4 \times 10^{-7}\ \mathrm{F} = \boxed{0.14\ \mu\mathrm{F}}.$$

3. From $V = Q/C$, we have

(a) $V = (4\ \mu\mathrm{C})/(4\ \mu\mathrm{F}) = \boxed{1\ \mathrm{V}}$.

(b) $V = (10\ \mu\mathrm{C})/(4\ \mu\mathrm{F}) = \boxed{2.5\ \mathrm{V}}$.

(c) $V = (1 \times 10^{-3}\ \mathrm{C})/(4 \times 10^{-6}\ \mathrm{F}) = \boxed{250\ \mathrm{V}}$.

4. From $Q = CV$, we have

(a) $Q = (1\ \mu\mathrm{F})(2\ \mathrm{V}) = \boxed{2\ \mu\mathrm{C}}$.

(b) $Q = (1\ \mu\mathrm{F})(12\ \mathrm{V}) = \boxed{12\ \mu\mathrm{C}}$.

5. For a parallel-plate capacitor, we have

$$C = Q/V = \varepsilon_0 A/d;$$
$$(2 \times 10^{-6}\ \mathrm{C})/(3000\ \mathrm{V}) = (8.85 \times 10^{-12}\ \mathrm{F/m})(2.5 \times 10^{-2}\mathrm{m}^2)/d, \text{ which gives } d = 3.3 \times 10^{-4}\ \mathrm{m} = \boxed{0.33\ \mathrm{mm}}.$$

Because V is the maximum of the power supply, this is the maximum separation. Note that

$$E = V/d = (3000\ \mathrm{V})/(3.3 \times 10^{-4}\ \mathrm{m}) = 9 \times 10^6\ \mathrm{V/m},$$

which is greater than the dielectric strength of air; the plates must be evacuated.

6. For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(R_2/R_1)$$
$$= (1.8\ \mathrm{m})2\pi(8.85 \times 10^{-12}\ \mathrm{F/m})/\ln[(1.5\ \mathrm{cm})/(1.0\ \mathrm{cm})] = 2.5 \times 10^{-10}\ \mathrm{F} = \boxed{0.25\ \mathrm{nF}}.$$

7. Because the potential from the outer conductor is constant inside, the potential difference between the two conductors is due to the inner conductor, which is equivalent to a point charge:

$$V = (Q/4\pi\varepsilon_0)[(1/r) - (1/R)], \text{ so the capacitance is}$$
$$C = Q/V = 4\pi\varepsilon_0 rR/(R - r).$$

(a) When r is finite and $R \to \infty$, we have

$$C \to \boxed{4\pi\varepsilon_0 r}, \text{ which is the capacitance of the inner sphere.}$$

(b) When $(R - r) << r$, we have $R \to r$, so

$$C \to 4\pi\varepsilon_0 r^2/(R - r) = \boxed{\varepsilon_0 A/d}, \text{ which is the capacitance of parallel plates with separation } d.$$

8. From Problem 7, we have

$$C = 4\pi\varepsilon_0 rR/(R - r) \quad \text{and} \quad V = Q/C = Q(R - r)/4\pi\varepsilon_0 rR;$$
$$V = (1.4 \times 10^{-7}\ \mathrm{C})(15 \times 10^{-2}\ \mathrm{m} - 3.0 \times 10^{-2}\ \mathrm{m})/[4\pi(8.85 \times 10^{-12}\ \mathrm{F/m})(3.0 \times 10^{-2}\ \mathrm{m})(15 \times 10^{-2}\ \mathrm{m})]$$
$$= 3.4 \times 10^4\ \mathrm{V} = \boxed{34\ \mathrm{kV}}.$$

9. For a parallel-plate capacitor, we have

$$C = q/V = \varepsilon_0 A/d;$$

$$d = \varepsilon_0 AV/q = (8.85 \times 10^{-12}\text{ F/m})(4.0 \times 10^{-2}\text{m}^2)[50.0\text{ mV} + (0.10\text{ mV/s})t](10^{-3}\text{ V/mV})/(4.0 \times 10^{-8}\text{ C})$$

$$= \boxed{(4.43 \times 10^{-7}\text{ m}) + (8.85 \times 10^{-10}\text{ m/s})t}.$$

10. For a parallel-plate capacitor, we have

$$E = \sigma/\varepsilon_0 = V/d;$$

$$\sigma = \varepsilon_0 V/d$$

$$= (8.85 \times 10^{-12}\text{ F/m})(3\text{V})/(0.3 \times 10^{-3}\text{ m}) = \boxed{8.9 \times 10^{-8}\text{ C/m}^2.}$$

The total charge on each plate is

$$Q = \sigma A = (8.9 \times 10^{-8}\text{ C/m}^2)(0.06\text{ m})^2 = 3.2 \times 10^{-10}\text{ C} = \boxed{0.3\text{ nC}}.$$

11. When the capacitor is isolated, the charge must be constant, so we have

$$Q = C_{\min}V_{\max} = C_{\max}V_{\min};$$

$$V_{\max} = (C_{\max}/C_{\min})V_{\min}$$

$$= [(0.2\ \mu\text{F})/(0.01\ \mu\text{F})](300\text{ V}) = 6 \times 10^3\text{ V} = \boxed{6\text{ kV}}.$$

12. (a) The capacitance of the system is

$$C = Q/V$$

$$= (900\text{ C})/(90 \times 10^6\text{ V}) = 10 \times 10^{-6}\text{ F} = \boxed{10\ \mu\text{F}}.$$

(b) The energy stored in the system is

$$U = \tfrac{1}{2}QV$$

$$= \tfrac{1}{2}(900\text{ C})(90 \times 10^6\text{ V}) = \boxed{4.1 \times 10^{10}\text{ J}}.$$

13. The energy stored in the capacitor is

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}(0.068\text{ C})(2900\text{ V}) = \boxed{99\text{ J}}.$$

14. The energy stored in the capacitor is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(0.7\text{ pF})(2\text{ V})^2 = 0.98\text{ pJ} = \boxed{1\text{ pJ}}.$$

15. We find the capacitance from the energy stored in the capacitor:

$$U = \tfrac{1}{2}CV^2;$$

$27\text{ J} = \tfrac{1}{2}C(300\text{ V})^2$, which gives $C = 6.0 \times 10^{-4}\text{ F} = \boxed{600\ \mu\text{F}}$.

16. For a sphere, we have $C = 4\pi\varepsilon_0 R$, so the energy stored is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2/4\pi\varepsilon_0 R$$

$$= \tfrac{1}{2}(3.0 \times 10^{-5}\text{ C})^2/4\pi(8.85 \times 10^{-12}\text{ F/m})(35 \times 10^{-2}\text{ m}) = \boxed{12\text{ J}}.$$

17. (a) For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(b/a)$$

$$= (10\text{ m})2\pi(8.85 \times 10^{-12}\text{ F/m})/\ln[(8\text{ mm})/(3\text{ mm})] = \boxed{5.67 \times 10^{-10}\text{ F}}.$$

(b) The energy stored in 10 m of cable is

$$U_1 = \tfrac{1}{2}CV^2$$

$$= \tfrac{1}{2}(5.67 \times 10^{-10}\text{ F})(10^3\text{ V})^2 = \boxed{2.83 \times 10^{-4}\text{ J}}.$$

Because the capacitance is directly proportional to the length, the energy stored in 1 km of cable is

$$U_2 = [(10^3\text{ m})/(10\text{ m})]U_1 = \boxed{2.83 \times 10^{-2}\text{ J}}.$$

18. From Problem 7, we have $C = 4\pi\varepsilon_0 rR/(R - r)$ so the stored energy is
$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2(R-r)/4\pi\varepsilon_0 rR$$
$$= \tfrac{1}{2}(6.0 \times 10^{-8}\ \text{C})^2\ (12\ \text{cm} - 4\ \text{cm})(10^{-2}\ \text{m/cm})/[4\pi(8.85 \times 10^{-12}\ \text{F/m})(12\ \text{cm})(4\ \text{cm})(10^{-2}\ \text{m/cm})^2]$$
$$= 2.7 \times 10^{-4}\ \text{J} = \boxed{0.27\ \text{mJ}}.$$

19. (a) When the plates are pulled apart, the charge does not change, while both the capacitance and the potential difference will change. The initial capacitance and charge are
$$C_0 = \varepsilon_0 A/d_0\ ;\ \ Q_0 = C_0 V_0 = \varepsilon_0 A\, V_0/d_0 = Q.$$
If we express the energy stored in the capacitor as
$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2 d/\varepsilon_0 A,$$
the change in stored energy is
$$\Delta U = \boxed{\tfrac{1}{2}(Q_0{}^2/\varepsilon_0 A)(d_1 - d_0) = (\varepsilon_0 A\, V_0{}^2/2d_0{}^2)(d_1 - d_0)}.$$
(b) Because the external force is the only interaction with the capacitor, we have
$$W_F = \Delta U = \boxed{(\varepsilon_0 A\, V_0{}^2/2d_0{}^2)(d_1 - d_0)}.$$
(c) If the plates stay connected to the battery, the potential difference does not change, while both the charge and capacitance will change. The change in stored energy is
$$\Delta U = \tfrac{1}{2}V_0{}^2(C - C_0) = (\varepsilon_0 A\, V_0{}^2/2)[(1/d_1) - (1/d_0)] = \boxed{(\varepsilon_0 A\, V_0{}^2/2d_1 d_0)(d_0 - d_1)}.$$
(d) Even though there must still be work done by the external force to separate the opposite charges on the plates, the energy stored in the capacitor decreases ($d_0 < d_1$). The charge on the plates has decreased and $\boxed{\text{energy has been stored in the battery}}$.

20. The energy stored in the electric field is
$$U_1 = \tfrac{1}{2}\varepsilon_0 E^2\ (\text{volume})_1$$
$$= \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{F/m})(1.25 \times 10^5\ \text{V/m})^2(1\ \text{m}^3) = \boxed{6.91 \times 10^{-2}\ \text{J}}.$$
$$U_2 = \tfrac{1}{2}\varepsilon_0 E^2\ (\text{volume})_2$$
$$= \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{F/m})(1.25 \times 10^5\ \text{V/m})^2(10^3\ \text{m})^3 = \boxed{6.91 \times 10^7\ \text{J}}.$$

21. The energy density around the long wire is
$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(\lambda/2\pi\varepsilon_0 r)^2 = \boxed{\lambda^2/8\pi^2\varepsilon_0 r^2}.$$

22. For a sphere, we have $C = 4\pi\varepsilon_0 R$. The energy stored in the electric field is the energy stored in the capacitor:
$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(4\pi\varepsilon_0 R)V^2 = 2\pi\varepsilon_0 R\, V^2$$
$$= 2\pi(8.85 \times 10^{-12}\ \text{F/m})(0.75\ \text{m})(2.0 \times 10^4\ \text{V})^2 = 1.7 \times 10^{-2}\ \text{J} = \boxed{17\ \text{mJ}.}$$

23. Because the cube is small compared to the distance from the point charge, we approximate the field in the cube from the point charge as constant:
$$E_{\text{av}} = (1/4\pi\varepsilon_0)(q/r^2) = (9 \times 10^9\ \text{m/F})[(5 \times 10^{-4}\ \text{C})/(1\ \text{m})^2] = 45 \times 10^5\ \text{V/m};$$
$$U = \tfrac{1}{2}\varepsilon_0 E_{\text{av}}{}^2 L^3 = \tfrac{1}{2}\varepsilon_0[(1/4\pi\varepsilon_0)(q/r^2)]^2 L^3 = (1/4\pi\varepsilon_0)q^2 L^3/8\pi r^4$$
$$= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5 \times 10^{-4}\ \text{C})^2(5 \times 10^{-2}\ \text{m})^3/[8\pi(1\ \text{m})^4]$$
$$= \boxed{1.1 \times 10^{-2}\ \text{J}}.$$

24. For the energy density of the uniform field we have
$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(V/d)^2;$$
$10^{-6}\ \text{J/m}^3 = \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)V^2/(1 \times 10^{-2}\ \text{m})^2$, which gives
$$V = \boxed{48\ \text{V}}.$$

$\boxed{25}$. For a conducting sphere, we have

$V = (1/4\pi\varepsilon_0)(q/r)$:

$8.3 \times 10^3\ \text{V} = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q/(18 \times 10^{-2}\ \text{m})$, which gives

$q = \boxed{1.7 \times 10^{-7}\ \text{C}}$.

The energy density outside the sphere is

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0[(1/4\pi\varepsilon_0)(q/r^2)]^2$$
$$= \tfrac{1}{2}(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)[(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.7 \times 10^{-7}\ \text{C})/r^2]^2$$
$$= \boxed{1.0 \times 10^{-5}/r^4\ \text{J/m}^3,\text{ with } r \text{ in m}}.$$

Because there is no field inside the sphere, we find the total energy in the electric field by adding the energies in spherical shells with radius r and thickness dr:

$$U = \int_R^\infty \frac{1.0\times 10^{-5}\ \text{J}\cdot\text{m}}{r^4}4\pi r^2\ \text{d}r = \left(4.0\pi\times 10^{-5}\ \text{J}\cdot\text{m}\right)\int_R^\infty \frac{\text{d}r}{r^2} = \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-\frac{1}{r}\right)\Big|_R^\infty$$
$$= \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-\frac{1}{\infty}+\frac{1}{R}\right) = \left(4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}\right)\left(-0+\frac{1}{R}\right) = \frac{4.0\pi \times 10^{-5}\ \text{J}\cdot\text{m}}{0.18\ \text{m}} = \quad 7.0\times 10^{-4}\ \text{J}.$$

Note that this is $\tfrac{1}{2}qV$.

26. Because the sphere is conducting, there is no field inside. The field outside is

$E_{\text{outside}} = Q/4\pi\varepsilon_0 r^2$.

Using the technique of Problem 25, we find the energy in the spherical region between $R = 25$ cm and $r =$ 50 cm:

$$U = \int(\varepsilon_0/2)(Q/4\pi\varepsilon_0 r^2)^2 4\pi r^2\text{d}r$$
$$= (Q^2/8\pi\varepsilon_0)\int \text{d}r/r^2$$
$$= (Q^2/8\pi\varepsilon_0)(-1/r)$$
$$= (Q^2/8\pi\varepsilon_0)(-1/r + 1/R)$$
$$= [(6.0 \times 10^{-7}\ \text{C})^2/8\pi(8.85 \times 10^{-12}\ \text{F/m})][-1/(0.50\ \text{m}) + 1/(0.25\ \text{m})]$$
$$= 3.2\times 10^{-3}\ \text{J} = \boxed{3.2\ \text{mJ}}.$$

27. (a) For a coaxial cable, we have

$$C = L2\pi\varepsilon_0/\ln(b/a)$$
$$= (0.15\ \text{m})2\pi(8.85 \times 10^{-12}\ \text{F/m})/\ln[(2\ \text{cm}/0.02\ \text{cm})]$$
$$= 1.8 \times 10^{-12}\ \text{F} = \boxed{1.8\ \text{pF}}.$$

(b) The energy that recharges the tube is stored in the capacitor:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(1.8 \times 10^{-12}\ \text{F})(5 \times 10^2\ \text{V})^2 = \boxed{2.3 \times 10^{-7}\ \text{J}}.$$

28. From Chapter 24, we have the electric field for a uniformly charged nonconducting sphere:

$E_{\text{inside}} = Qr/4\pi\varepsilon_0 R^3$ and $E_{\text{outside}} = Q/4\pi\varepsilon_0 r^2$.

Using the technique of Problem 25, we find the energy in the spherical region of radius $r = 0.25$ m

$$U = \int(\varepsilon_0/2)(Qr/4\pi\varepsilon_0 R^3)^2 4\pi r^2\text{d}r + \int(\varepsilon_0/2)(Q/4\pi\varepsilon_0 r^2)^2 4\pi r^2\text{d}r$$
$$= (Q^2/8\pi\varepsilon_0 R^6)\int r^4\ \text{d}r + (Q^2/8\pi\varepsilon_0)\int \text{d}r/r^2$$
$$= (Q^2/8\pi\varepsilon_0 R^6)(r^3/5) + (Q^2/8\pi\varepsilon_0)(-1/r)$$
$$= (Q^2/8\pi\varepsilon_0)(1/5R - 1/r + 1/R)$$
$$= (Q^2/8\pi\varepsilon_0)(6/5R - 1/r)$$
$$= [(7.3\times 10^{-6}\ \text{C})^2/8\pi(8.85 \times 10^{-12}\ \text{F/m})](1.2/0.07 - 1/0.25)$$
$$= \boxed{3.1\ \text{J}}.$$

29. (a) We find the electric field between two parallel plates from

$E = V/d = (1500\text{ V})/(0.5 \times 10^{-2}\text{ m}) = \boxed{3.00 \times 10^5\text{ V/m}}$.

(b) In terms of the charge density on the plates, the field is $E = \sigma/\varepsilon_0$, which gives

$Q = \sigma A = \varepsilon_0 EA = (8.85 \times 10^{-12}\text{ F/m})(3.00 \times 10^5\text{ V/m})(400 \times 10^{-4}\text{m}^2) = \boxed{1.06 \times 10^{-7}\text{ C}}$.

(c) We have to remember that the field between the plates is produced by both plates, but the field that one plate produces at the other is one-half of this. The force exerted on one plate is

$F = \frac{1}{2}QE = \frac{1}{2}(1.06 \times 10^{-7}\text{ C})(3.00 \times 10^5\text{ V/m}) = \boxed{1.59 \times 10^{-2}\text{ N}}$.

(d) When the plates are pulled apart, the charge does not change, while both the capacitance and the potential difference will change. If we express the energy stored in the capacitor as

$U = \frac{1}{2}Q^2/C = \frac{1}{2}Q^2d/\varepsilon_0 A$,

the change in stored energy is

$\Delta U = \frac{1}{2}(Q^2/\varepsilon_0 A)(d_2 - d_1)$

$= \frac{1}{2}[(1.06 \times 10^{-7}\text{ C})^2/(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(400 \times 10^{-4}\text{m}^2)](0.20)(0.5 \times 10^{-2}\text{ m}) = 1.59 \times 10^{-5}\text{ J}.$

To pull the plates apart requires a force to balance the attractive force from part (c).
The work done by this force is

$W = F\,\Delta d = (1.59 \times 10^{-2}\text{ N})(0.2)(0.5 \times 10^{-2}\text{ m}) = \boxed{1.59 \times 10^{-5}\text{ J}}$, consistent with part (c).

30. (a) The electric field outside a charged sphere is that of a point charge:

$E = \boxed{e/4\pi\varepsilon_0 r^2,\ r > R}$.

(b) We can use the result of Problem 26, with $a \to \infty$, to find the energy stored in the electric field:

$U = e^2/8\pi\varepsilon_0 R$.

We could also find the energy from

$U = \frac{1}{2}eV = \frac{1}{2}e(e/4\pi\varepsilon_0 R) = e^2/8\pi\varepsilon_0 R$

$= (1.6 \times 10^{-19}\text{ C})^2/8\pi(8.85 \times 10^{-12}\text{ F/m})R = \boxed{(1.15 \times 10^{-28}\text{ J}\cdot\text{m})/R}$.

(c) If the energy stored in the electric field is the rest energy, we have

$e^2/8\pi\varepsilon_0 R = mc^2$;

$(1.6 \times 10^{-19}\text{ C})^2/8\pi(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)R = (0.9 \times 10^{-30}\text{ kg})(3 \times 10^8\text{ m/s})^2$, which gives

$R = \boxed{1.4 \times 10^{-15}\text{m}}$.

31. For the two combinations we have

$C_p = C_1 + C_2$; $6.5\ \mu\text{F} = C_1 + C_2$, and

$1/C_s = 1/C_1 + 1/C_2$ or $C_s = C_1C_2/(C_1 + C_2) = C_1C_2/C_p$; $1.4\ \mu\text{F} = C_1C_2/(6.5\ \mu\text{F})$.

When we combine these two equations, we get a quadratic equation for C_1:

$C_1^2 - (6.5\ \mu\text{F})C_1 + 9.1\ \mu\text{F}^2 = 0$.

The two solutions are $C_1 = 2.04\ \mu$F and $4.46\ \mu$F. Thus the two capacitors are $\boxed{2.04\ \mu\text{F},\ \ 4.46\ \mu\text{F}}$.

32. The voltage must be the same across both the top and bottom sections, so we have

$E_1 = V/\ell_1 = \sigma_1/\varepsilon_0$,

which gives the charge on the top section: $Q_1 = \varepsilon_0(V/\ell_1)A_1$;

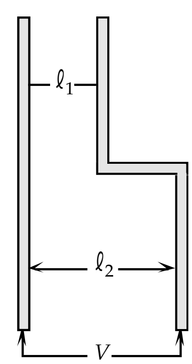
$E_2 = V/\ell_2 = \sigma_2/\varepsilon_0$,

which gives the charge on the bottom section: $Q_2 = \varepsilon_0(V/\ell_2)A_2$.

The areas are equal, $A_1 = A_2 = A/2$, and the total charge on the capacitor is

$Q = Q_1 + Q_2 = \varepsilon_0 V(A_1/\ell_1 + A_2/\ell_2)$

$= \varepsilon_0 V(A/2)(1/\ell_1 + 1/\ell_2) = \varepsilon_0 VA(\ell_1 + \ell_2)/2\ell_1\ell_2$.



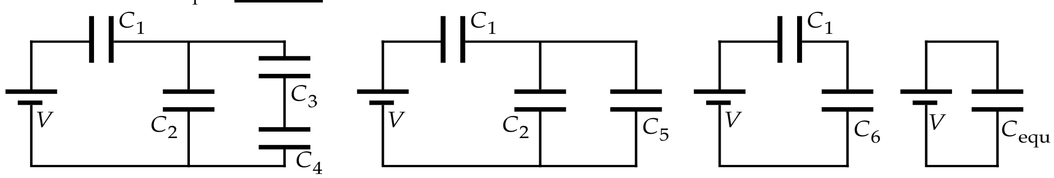
From the definition of capacitance, we have

$C = Q/V = \boxed{\varepsilon_0 A(\ell_1 + \ell_2)/2\ell_1\ell_2}$.

This is the equivalent capacitance for 2 capacitors in parallel:

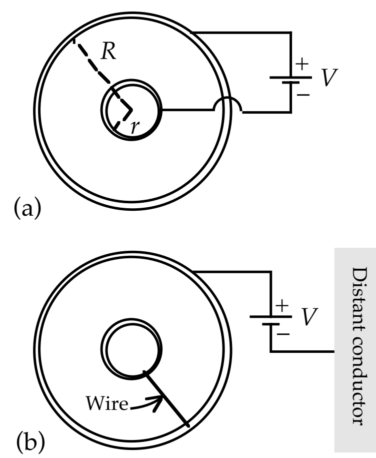
$C = (\varepsilon_0 A/2\ell_1) + (\varepsilon_0 A/2\ell_2) = C_1 + C_2$.

33. From the redrawn circuit, we see that C_3 and C_4 are in series.
We find their equivalent capacitance from
$1/C_5 = 1/C_3 + 1/C_4 = 1/(2\ \mu\text{F}) + 1/(5\ \mu\text{F})$, which gives $C_5 = 1.43\ \mu\text{F}$.
We find the equivalent capacitance of C_2 and C_5, which are in parallel:
$C_6 = C_2 + C_5 = 4\ \mu\text{F} + 1.43\ \mu\text{F} = 5.43\ \mu\text{F}$.
Finally, we find the equivalent capacitance of C_1 and C_6, which are in series:
$1/C_{\text{equ}} = 1/C_1 + 1/C_6 = 1/(3\ \mu\text{F}) + 1/(5.43\ \mu\text{F})$,
which gives $C_{\text{equ}} = \boxed{1.93\ \mu\text{F}}$.

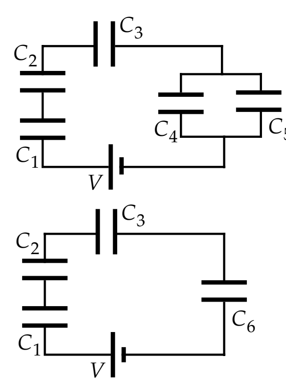


34. When the uncharged plate is placed between the two charged plates, charges will separate so that there is a charge $+Q$ on the side facing the negative plate and a charge $-Q$ on the side facing the positive plate. Thus we have two capacitors in series, with an equivalent capacitance:
$1/C = 1/C_1 + 1/C_2 = 1/(\varepsilon_0 A/x) + 1/[\varepsilon_0 A/(D - x - d)] = (x + D - x - d)/\varepsilon_0 A$, which gives
$C = \boxed{\varepsilon_0 A/(D - d),\ \text{independent of } x}$.
Note that this is a parallel-plate capacitor with separation $D - d$.

35. Because the potential from the outer conductor is constant inside, the potential difference between the two conductors is due to the inner conductor, which is equivalent to a point charge:
$V = (Q/4\pi\varepsilon_0)(1/r - 1/R)$, so the capacitance is
$C = Q/V = 4\pi\varepsilon_0 rR/(R - r)$
$= 4\pi(8.85 \times 10^{-12}\ \text{F/m})(3 \times 10^{-3}\ \text{m})(12 \times 10^{-3}\ \text{m})/[(12 - 3) \times 10^{-3}\ \text{m}]$
$= 4.5 \times 10^{-13}\ \text{F} = \boxed{0.45\ \text{pF}}$.
When a wire connects the two spheres, they must be at the same potential and all the charge will be on the outer sphere. The potential of the charged outer sphere is
$V = Q/4\pi\varepsilon_0 R$, so the capacitance is
$C = Q/V = 4\pi\varepsilon_0 R$
$= 4\pi(8.85 \times 10^{-12}\ \text{F/m})(12 \times 10^{-3}\ \text{m})$
$= 1.33 \times 10^{-12}\ \text{F} = \boxed{1.33\ \text{pF}}$.



36. From the circuit, we see that C_4 and C_5 are in parallel, with an equivalent capacitance
$C_6 = C_4 + C_5 = 18\ \mu\text{F} + 18\ \mu\text{F} = 36\ \mu\text{F}$.
We now have four capacitors in series:
$1/C_{\text{equ}} = 1/C_1 + 1/C_2 + 1/C_3 + 1/C_6$
$= 1/(18\ \mu\text{F}) + 1/(18\ \mu\text{F}) + 1/(18\ \mu\text{F}) + 1/(36\ \mu\text{F})$,
which gives
$C_{\text{equ}} = \boxed{5.1\ \mu\text{F}}$.

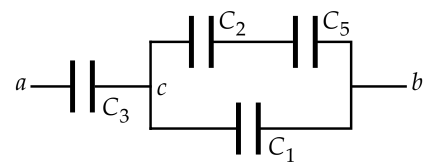


$\boxed{37}$. (a) From the circuit, we see that C_2 and C_5 are in series and find their equivalent capacitance from

$1/C_6 = 1/C_2 + 1/C_5$

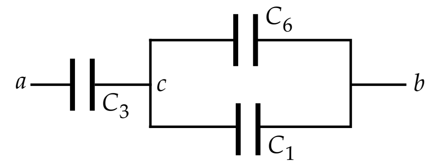
$= 1/(2\ \mu\text{F}) + 1/(5\ \mu\text{F})$, which gives

$C_6 = 1.43\ \mu\text{F}$.



From the new circuit, we see that C_1 and C_6 are in parallel, with an equivalent capacitance

$C_7 = C_1 + C_6 = 1\ \mu\text{F} + 1.43\ \mu\text{F} = 2.43\ \mu\text{F}$.

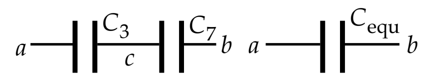


From the new circuit, we see that C_3 and C_7 are in series and find their equivalent capacitance from

$1/C_{\text{equ}} = 1/C_3 + 1/C_7$

$= 1/(3\ \mu\text{F}) + 1/(2.43\ \mu\text{F})$, which gives

$C_{\text{equ}} = \boxed{1.34\ \mu\text{F}}$.



(b) The charge on the equivalent capacitor is also the charge on C_3 and C_7:

$Q_{\text{equ}} = Q_3 = Q_7 = C_{\text{equ}}V_{ab} = (1.34\ \mu\text{F})(300\ \text{V}) = \boxed{402\ \mu\text{C}}$.

We find the potential difference between c and b from

$V_{cb} = Q_7/C_7 = (402\ \mu\text{C})/(2.43\ \mu\text{F}) = 165\ \text{V}$.

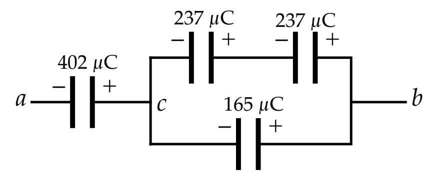
The charge on C_6 is also the charge on C_2 and C_5:

$Q_6 = Q_2 = Q_5 = C_6V_{cb} = (1.43\ \mu\text{F})(165\ \text{V}) = \boxed{237\ \mu\text{C}}$.

The charge on C_1 is

$Q_1 = C_1V_{cb} = (1\ \mu\text{F})(165\ \text{V}) = \boxed{165\ \mu\text{C}}$.

Because point b is at the higher potential, the charges are as shown in the diagram.



38. Although there are no apparent series or parallel combinations in the circuit that can be reduced, we use symmetry to simplify the circuit. The top and bottom paths from a to b are equivalent, so we have

$V_c = V_d$,

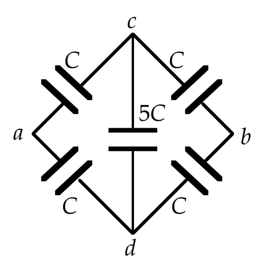
which means there is no potential difference across and no charge on the 5C capacitor. The circuit will not change if we replace the 5C capacitor with a wire. The left and right sides have two capacitors in parallel, with equivalent capacitance

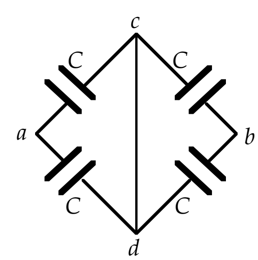
$C_1 = C + C = 2C$.

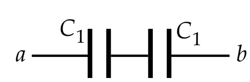
We combine these two capacitors in series to find the equivalent capacitance of the circuit:

$1/C_{\text{equ}} = 1/2C + 1/2C$, which gives

$C_{\text{equ}} = \boxed{C}$.







39. To find the equivalent capacitance between a and b, we redraw the circuit, and see that there are two capacitors in series in the right branch:

$1/C_1 = 1/C + 1/C$, which gives $C_1 = \frac{1}{2}C$.

For the two capacitors in parallel between d and b we have

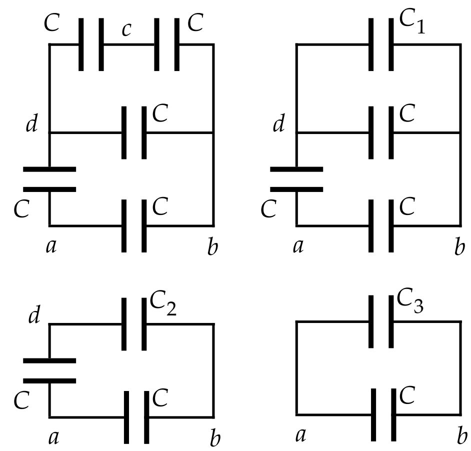
$C_2 = C_1 + C = \frac{1}{2}C + C = \frac{3}{2}C$.

For the two capacitors in series between a and b we have

$1/C_3 = 1/C + 1/C_2$, which gives $C_3 = 0.6C$.

For the equivalent capacitance, we have

$C_{\text{equ},ab} = C + C_3 = C + 0.6C = \boxed{1.6C}$.



To find the equivalent capacitance between a and c, we redraw the circuit, and use symmetry to simplify the circuit. The top and bottom paths from a to c are equivalent, so we have

$V_b = V_d$,

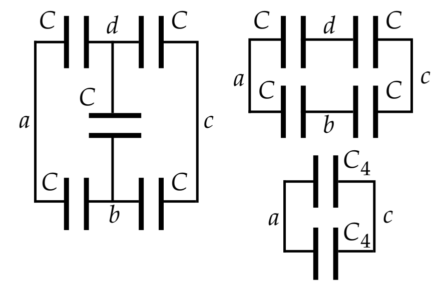
which means there is no potential difference across and no charge on the middle capacitor. The circuit will not change if we remove it.

The top and bottom branches have two capacitors in series:

$1/C_4 = 1/C + 1/C$, which gives $C_4 = \frac{1}{2}C$.

We combine these two capacitors in parallel to find the equivalent capacitance of the circuit:

$C_{\text{equ},ac} = C_4 + C_4 = \frac{1}{2}C + \frac{1}{2}C = \boxed{C}$.

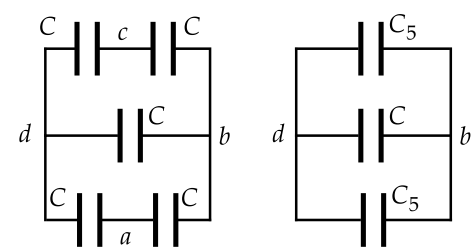


To find the equivalent capacitance between b and d, we redraw the circuit, and see that the top and bottom paths have two capacitors in series:

$1/C_5 = 1/C + 1/C$, which gives $C_5 = \frac{1}{2}C$.

We now have three capacitors in parallel, with equivalent capacitance

$C_{\text{equ},bd} = C_5 + C_5 + C = \frac{1}{2}C + \frac{1}{2}C + C = \boxed{2C}$.



40. (a) For capacitor C_1 , we have

$U_1 = \frac{1}{2}q_1{}^2/C_1 = \frac{1}{2}(4\ \mu\text{C})^2/(175\ \mu\text{F}) = 0.046\ \mu\text{J}$.

For capacitor C_2 , we have

$U_2 = \frac{1}{2}C_2V_2{}^2 = \frac{1}{2}(18\ \mu\text{F})(3\ \text{V})^2 = 81\ \mu\text{J}$.

The total energy stored in the two capacitors is

$U = U_1 + U_2 = 0.046\ \mu\text{J} + 81\ \mu\text{J} = \boxed{81\ \mu\text{J}}$.

(b) When the negatively charged plate of C_1 is connected to the positively charged plate of C_2 we have a single equivalent capacitor, with

$q_{\text{net}} = |\,q_2 - q_1\,| = |\,(18\ \mu\text{F})(3\ \text{V}) - 4\ \mu\text{C}\,| = \boxed{50\ \mu\text{C}}$,

$C_{\text{equ}} = C_1 + C_2 = 175\ \mu\text{F} + 18\ \mu\text{F} = \boxed{193\ \mu\text{F}}$, and

$V = q_{\text{net}}\,/\,C_{\text{equ}} = 50\ \mu\text{C}/193\ \mu\text{F} = \boxed{0.26\ \text{V}}$.

The total energy stored in the system becomes

$U = \frac{1}{2}q_{\text{net}}{}^2/C_{\text{equ}} = \frac{1}{2}(50\ \mu\text{C})^2/(193\ \mu\text{F})^2 = \boxed{6.5\ \mu\text{J}}$.

41. (a) We find the capacitance C_2 from

$$C_2 = Q_2/V_2 = (25\ \mu\text{C})/(5\ \text{V}) = \boxed{5.0\ \mu\text{F}}.$$

(b) Because $Q_1 = Q_2$, the capacitors can be considered to be in series. The charge on the equivalent capacitor is

$$Q = Q_1 = Q_2 = \boxed{25\ \mu\text{C}}.$$

42. Because the equivalent capacitance for a parallel combination is the sum, we see that there must be some series combination as well. Because the required result of 4.968 μF is greater than 4 μF, we try the 4-μF capacitor in parallel with a series combination of the others. The equivalent capacitance of some of the combinations are

2-μF & 3-μF: $(2\ \mu\text{F})(3\ \mu\text{F})/(2\ \mu\text{F} + 3\ \mu\text{F}) = 1.2\ \mu\text{F}$;
2-μF & 5-μF: $(2\ \mu\text{F})(5\ \mu\text{F})/(2\ \mu\text{F} + 5\ \mu\text{F}) = 1.4\ \mu\text{F}$;
2-μF & 3-μF & 5-μF: $(1.2\ \mu\text{F})(5\ \mu\text{F})/(1.2\ \mu\text{F} + 5\ \mu\text{F}) = 0.97\ \mu\text{F}$.

We get the desired result by putting the 4-μF capacitor in parallel with a series combination of the 2-μF, 3-μF, and 5-μF capacitors.

43. Because the charge is constant, we have

$$Q = C_{\text{teflon}}V_{\text{teflon}} = C_{\text{plexiglas}}V_{\text{plexiglas}}, \quad \text{or} \quad V_{\text{plexiglas}}/V_{\text{teflon}} = C_{\text{teflon}}/C_{\text{plexiglas}} = \kappa_{\text{teflon}}/\kappa_{\text{plexiglas}};$$

$V_{\text{plexiglas}}/(600\ \text{V}) = 2.1/3.4$, which gives $V_{\text{plexiglas}} = \boxed{370\ \text{V}}$.

44. We find the dielectric constant from

$$\kappa = C/C_0 = (Q/V)/(Q/V_0) = V_0/V = (4\ \text{V})/(3.6\ \text{V}) = \boxed{1.1\text{V}}.$$

45. For the same energy stored at the same potential difference, we have

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(\kappa\varepsilon_0 A/d)V^2;$$

$4 \times 10^6\ \text{J} = \tfrac{1}{2}[(3)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)A/(1 \times 10^{-3}\ \text{m})](12\ \text{V})^2$, which gives

$A = \boxed{2.1 \times 10^{12}\ \text{m}^2}$.

46. (a) Using the results from Problem 17, we have

$$C = L2\pi\kappa\varepsilon_0/\ln(b/a) = \kappa C_0 = (2.5)(5.67 \times 10^{-10}\ \text{F}) = \boxed{1.42 \times 10^{-9}\ \text{F}}.$$

(b) The energy stored in 10 m of cable is

$$U_1 = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\kappa C_0 V^2 = \kappa U_0 = (2.5)(2.83 \times 10^{-4}\ \text{J}) = \boxed{7.08 \times 10^{-4}\ \text{J}}.$$

Because the capacitance is directly proportional to the length, the energy stored in 1 km of cable is

$$U_2 = [(10^3\ \text{m})/(10\ \text{m})]U_1 = \boxed{7.08 \times 10^{-2}\ \text{J}}.$$

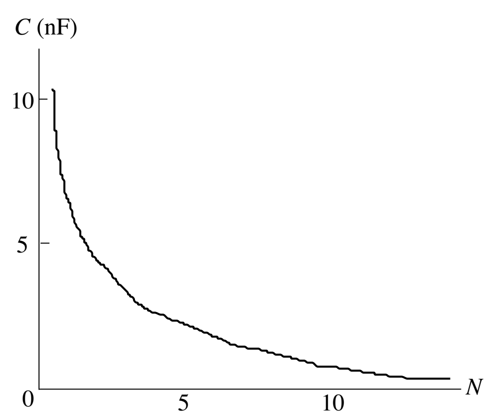
47. (a) The area of the parallel-pate capacitor is
$A = (0.20\ \text{m})(0.15\ \text{m}) = 0.030\ \text{m}^2$, and its plate Separation is $d = 7.7\ \text{mm}/100 = 7.7 \times 10^{-5}\ \text{m}$.
Its capacitance is then

$$C = \kappa\varepsilon_0 A/d$$
$$= (2.9)(8.85 \times 10^{-12}\ \text{F/m})(0.030\ \text{m}^2)/(7.7 \times 10^{-5}\ \text{m})$$
$$= 1.0 \times 10^{-8}\ \text{F} = \boxed{10\ \text{nF}}.$$

(b) Since C is proportional to κ, the actual value of κ is

$$\kappa = (4.6\ \text{nF}/\ 10\ \text{nF})(2.9) = \boxed{1.3}.$$

(c)



48. With $A = 2\text{ cm}^2 = 2\times10^{-4}\text{m}^2$, $d = 0.1\text{ nm} = 1\times10^{-10}$ m, and $\kappa = 10$, the capacitance is
$C = \kappa\varepsilon_0 A/d = (10)(8.85\times10^{-12}\text{ F/m})(2\times10^{-4}\text{m}^2)/(1\times10^{-10}\text{ m}) = \boxed{2\times10^{-4}\text{ F}}$.

49. The capacitance of an isolated sphere in air is $C_0 = 4\pi\varepsilon_0 R$. When it is embedded in a dielectric, we have
$C = \kappa C_0$, so the change is
$C - C_0 = (\kappa - 1)C_0 = \boxed{(\kappa-1)4\pi\varepsilon_0 R}$.
The sign of the induced charge on the surface of the dielectric will be opposite to that of the original charge. If $E_0 = \sigma/\varepsilon_0$ is the original electric field just outside the sphere, we have
$E = E_0/\kappa = E_0 + E_{\text{ind}}$;
$\sigma/\kappa\varepsilon_0 = \sigma/\varepsilon_0 + \sigma_{\text{ind}}/\varepsilon_0$, which gives $\sigma_{\text{ind}}/\sigma = \boxed{(\kappa-1)/\kappa}$.

50. We find the initial separation from
$E_0 = V/d$;
$0.90\times10^6\text{ V/m} = (12\times10^3\text{ V})/d$, which gives $d = 0.013$ m.
Because the capacitance does not change, we have
$C = \varepsilon_0 A/d = \kappa\varepsilon_0 A/d'$, which gives
$d' = \kappa d = 1.5(0.013\text{ m}) = \boxed{0.020\text{ m}}$.

51. If we write the energies as $U_0 = \frac{1}{2}q_0^2/C_0$ and $U = \frac{1}{2}q^2/C$, the ratio is
$U/U_0 = (q/q_0)^2(C_0/C) = (q/q_0)^2(1/\kappa)$;
$3 = (q/q_0)^2(1/1.8)$, which gives $q = \boxed{2.3q_0}$.

52. (a) For a coaxial cable, we have
$C = L2\pi\kappa\varepsilon_0/\ln(b/a)$
$= (100\text{ m})(2\pi)(2.2)(8.85\times10^{-12}\text{ F/m})/\ln[(5.0\text{ mm})/(3.5\text{ mm})] = \boxed{3.4\times10^{-8}\text{ F}}$.
(b) The charge on the inner (and the outer) conductor is
$Q = CV$
$= (3.4\times10^{-8}\text{ F})(5.0\times10^2\text{ V}) = \boxed{1.7\times10^{-5}\text{ C}}$.
The energy stored is
$U = \frac{1}{2}CV^2$
$= \frac{1}{2}(3.4\times10^{-8}\text{ F})(5.0\times10^2\text{ V})^2 = \boxed{4.3\times10^{-3}\text{ J}}$.

53. We can consider the system to be two capacitors in series:
$1/C = 1/C_1 + 1/C_2$
$= (D-d)/\varepsilon_0 A + d/\kappa\varepsilon_0 A$
$= (D - d + d/\kappa)]/\varepsilon_0 A$, which gives
$C = \boxed{\kappa\varepsilon_0 A/[d + \kappa(D-d)]}$.

54. From Problem 53, we have
$C = \kappa\varepsilon_0 A/[d + \kappa(D-d)]$.
The charge on the plates is
$Q = CV = \kappa\varepsilon_0 AV/[d + \kappa(D-d)]$, so the electric field in the empty space is
$E_0 = \sigma/\varepsilon_0 = Q/\varepsilon_0 A = \kappa V/[(d + \kappa(D-d)]$
$= (1.8)(600\text{ V})/[0.6\times10^{-2}\text{ m} + (1.8)(1.6\times10^{-2}\text{ m} - 0.6\times10^{-2}\text{ m})] = \boxed{4.5\times10^4\text{ V/m}}$.
The electric field in the dielectric is
$E = E_0/\kappa = (4.5\times10^4\text{ V/m})/1.8 = \boxed{2.5\times10^4\text{ V/m}}$.

$\boxed{55}$. The free charge on the capacitor is

$$Q = CV = (\kappa\varepsilon_0 A/d)V = \kappa\varepsilon_0 AV/d$$
$$= \kappa(8.85 \times 10^{-12}\ \text{F/m})(10 \times 10^{-4}\,\text{m}^2)(300\ \text{V})/(5 \times 10^{-3}\ \text{m}) = 5.3 \times 10^{-10}\kappa\ \text{C}.$$

For the materials, we have

air, $\kappa = 1$; $Q = \boxed{5.3 \times 10^{-10}\ \text{C}}$;

paper, $\kappa = 3.7$; $Q = \boxed{2.0 \times 10^{-9}\ \text{C}}$;

neoprene, $\kappa = 6.7$; $Q = \boxed{3.6 \times 10^{-9}\ \text{C}}$;

Bakelite, $\kappa = 4.9$; $Q = \boxed{2.6 \times 10^{-9}\ \text{C}}$;

strontium titanate, $\kappa = 332$; $Q = \boxed{1.8 \times 10^{-7}\ \text{C}}$.

56. Using the dielectric strength of plexiglas, we find the separation of the plates:

$$E = V/d;$$
$$d = V_{\max}/E_{\max} = (6 \times 10^3\ \text{V})/(2.8 \times 10^6\ \text{V/m}) = 2.14 \times 10^{-3}\ \text{m}.$$

When the plexiglas is removed, the capacitance is

$$C_0 = \varepsilon_0 A/d$$
$$= (8.85 \times 10^{-12}\ \text{F/m})(0.80 \times 10^{-4}\,\text{m}^2)/(2.14 \times 10^{-3}\ \text{m}) = 3.3\times 10^{-13}\ \text{F}.$$

The maximum voltage with air between the plates is

$$V_0 = E_{0\max}d$$
$$= (3 \times 10^6\ \text{V/m})(2.14 \times 10^{-3}\ \text{m}) = 6.4 \times 10^3\ \text{V}.$$

The maximum charge the plates can hold now is

$$Q_0 = C_0V_0$$
$$= (3.3 \times 10^{-13}\ \text{C})(6.4 \times 10^3\ \text{V}) = \boxed{2.1 \times 10^{-9}\ \text{C}}.$$

57. Using the result of Problem 35, we know that the capacitance is

$$C = \kappa C_0 = \kappa 4\pi\varepsilon_0 r_1 r_2/(r_2 - r_1).$$

With air between the shells, the energy is

$$U_0 = \tfrac{1}{2}Q^2/C_0.$$

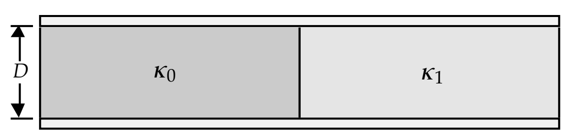
When the dielectric is added, the charge does not change, so the energy is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2/\kappa C_0.$$

The change in energy is

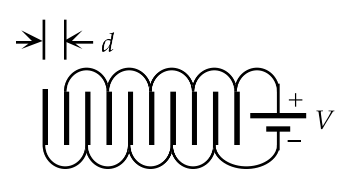
$$U - U_0 = \tfrac{1}{2}(Q^2/C_0)(1/\kappa - 1) = \tfrac{1}{2}(Q^2/\kappa C_0)(1 - \kappa) = \boxed{(1 - \kappa)Q^2/2C\ \ \text{(a decrease)}}.$$

58. Because $D << L$, we can ignore fringing fields. The potential difference must be the same on each half of the space, so we can treat the system as two capacitors in parallel:



$$C = C_1 + C_2 = \kappa_0\varepsilon_0(\tfrac{1}{2}L^2)/d + \kappa_1\varepsilon_0(\tfrac{1}{2}L^2)/d$$
$$= (\varepsilon_0\tfrac{1}{2}L^2/d)(\kappa_0 + \kappa_1) = \boxed{\tfrac{1}{2}(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/d)}.$$

59. From the diagram, we see that the arrangement is equivalent to 9 capacitors in parallel:



$$C = 9C_1 = 9(\varepsilon_0 A/d)$$
$$= 9(8.85 \times 10^{-12}\ \text{F/m})(6.0 \times 10^{-2}\ \text{m})(8.0 \times 10^{-2}\ \text{m})/(1.2 \times 10^{-3}\ \text{m})$$
$$= 3.2 \times 10^{-10}\ \text{F} = \boxed{0.32\ \text{nF}}.$$

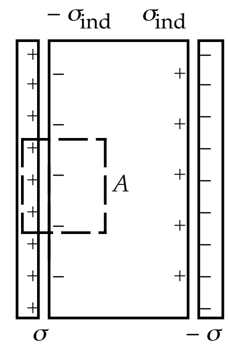
If the region is filled with a dielectric, we have

$$C' = \kappa C = 2.8(0.32\ \text{nF}) = \boxed{0.90\ \text{nF}}.$$

60. We find the induced charge from

$$Q_{\text{ind}} = Q(1 - 1/\kappa) = (18\ \mu\text{C})(1 - 1/4.5) = \boxed{14\ \mu\text{C}}.$$

61. We choose a cylinder with one end inside the conducting plate and the other end in the dielectric for a Gaussian surface. Because there is no field inside the plate and the field is parallel to the sides, the only part of the cylinder with flux through it is the end in the dielectric:



$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint E\, dA = Q_{encl}/\varepsilon_0 ;$$
$$EA = (\sigma - \sigma_{ind})A/\varepsilon_0 , \text{ which gives}$$
$$E = E_0 - E_{ind} = \sigma/\varepsilon_0 - \sigma_{ind}/\varepsilon_0 .$$

Because $E_0 = \sigma/\varepsilon_0$, we have

$$E_{ind} = \sigma_{ind}/\varepsilon_0.$$

62. We find the induced surface charge density from

$$\sigma_{ind} = \sigma - \sigma/\kappa = \sigma(1 - 1/\kappa) = (\kappa - 1)(Q/L^2\kappa).$$

The induced surface charge is

$$Q_{ind} = \sigma_{ind}L^2 = (\kappa - 1)(Q/\kappa)$$
$$= (3.5 - 1)(0.3 \times 10^{-6}\text{ C})/3.5 = \boxed{2.1 \times 10^{-7}\text{ C}}.$$

The field in the dielectric is

$$E = E_0/\kappa = \sigma/\kappa\varepsilon_0 = Q/L^2\kappa\varepsilon_0$$
$$= (0.3 \times 10^{-6}\text{ C})/[(0.22\text{ m})^2(3.5)(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)] = \boxed{2.2 \times 10^5\text{ V/m}}.$$

The energy stored in the capacitor is

$$U = \tfrac{1}{2}Q^2/C = \tfrac{1}{2}Q^2 d/\kappa\varepsilon_0 L^2$$
$$= \tfrac{1}{2}(0.3 \times 10^{-6}\text{ C})^2(1.8\times 10^{-3}\text{ m})/[(3.5)(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.22\text{ m})^2] = \boxed{5.4 \times 10^{-5}\text{ J}}.$$

63. For a polar dielectric we have

$$\kappa = 1 + a/T, \text{ so}$$
$$C = \kappa\varepsilon_0 A/d = \kappa C_0 = (1 + a/T)C_0.$$

With the given data we have

$$3.2\ \mu\text{F} = (1 + a/296\text{ K})C_0 \quad\text{and}\quad 2.65\ \mu\text{F} = (1 + a/360\text{ K})C_0 .$$

When we solve these two equations, we get

$$a = 8640\text{ K} \text{ and } C_0 = 0.106\ \mu\text{F}.$$

At a temperature of 48°C, we have

$$C = (1 + 8640\text{ K}/321\text{ K})(0.106\ \mu\text{F}) = \boxed{2.96\ \mu\text{F}}.$$

64. $\boxed{\text{The uncharged plate will be sucked in}}$. The charges induced on the surfaces of the inserted plate will be opposite to those on the charged plates. If we consider one of the charged plates, the induced charge of the opposite sign will be closer than the induced charge of the same sign and thus the force of attraction will be greater than the force of repulsion. The same will be true for the other charged plate.

65. (a) Since the two capacitors are in series they must have the same charge:

$$q = C_1V_1 = C_2V_2 .$$

Also, the sum of their voltages is V:

$$V = V_1 + V_2.$$

Combine these two equations to obtain

$$V_1 = \boxed{C_2V/(C_1 + C_2)}, \quad V_2 = \boxed{C_1V/(C_1 + C_2)}.$$

(b) Since

$$V_1 = C_2V/(C_1 + C_2) = (3\text{ nF})V/(2\text{ nF} + 3\text{ nF}) = 3V/5 < V_{1max} = 10\text{ V, we have } V < 17\text{ V. Also,}$$
$$V_2 = C_1V/(C_1 + C_2) = (2\text{ nF})V/(2\text{ nF} + 3\text{ nF}) = 2V/5 < V_{2max} = 30\text{ V, we have } V < 75\text{ V.}$$

To satisfy both inequalities we must have $V < 17$ V. So $\boxed{V_{max} = 17\text{ V}}$.

66. (a) Two capacitors connected in parallel must have the same potential difference, so

$$V_2 = \boxed{V}.$$

(b) The single equivalent capacitor is still subject to the same voltage difference, $\boxed{V}$.

(c) Since the voltage difference across the two capacitors is the same, and the maximum voltage difference across C_1 is 10 V, less than that across C_2, the maximum voltage difference across the combination is

$$\boxed{V_{max} = V_{1max} = 10 \text{ V}}.$$

67. We take a radius R of 0.5 m for the sphere and a spark distance d of 0.5 cm. Because the spark distance is small, we assume the breakdown field is constant, so the required potential is $V = Ed$. From the potential of a sphere, we have

$$V = (1/4\pi\varepsilon_0)(Q/R) = E_{max}d;$$

$$(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)Q/(0.5 \text{ m}) = (3 \times 10^6 \text{ V/m})(0.5 \times 10^{-2} \text{ m}), \text{ which gives } \boxed{Q \simeq 10^{-6} \text{ C}}.$$

68. (a) Because a single potential is available, from $Q = CV$ we see that the maximum charge will be produced by the maximum capacitance. For a parallel-plate capacitor, $C = \kappa\varepsilon_0 A/d$. We need a system with maximum area and minimum separation. The minimum separation is 5 mm, and the maximum area possible is 150 cm^2. (Note that if we make a number of smaller capacitors, they will be connected in parallel to produce the maximum capacitance. This is the same as a single capacitor.) The system consists of 2 aluminum plates of area 150 cm^2, separated by 5 mm, with a 150 cm^2 piece of Bakelite between the plates. The designed capacitance is

$$C = \kappa\varepsilon_0 A/d$$

$$= (4.9)(8.85 \times 10^{-12} \text{ F/m})(150 \times 10^{-4} \text{m}^2)/(5 \times 10^{-3} \text{ m}) = 1.30 \times 10^{-10} \text{ F}.$$

The charge on the plates is

$$Q = CV = (1.30 \times 10^{-10} \text{ F})(1200 \text{ V}) = \boxed{1.56 \times 10^{-7} \text{ C}}.$$

The energy stored is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(1.30 \times 10^{-10} \text{ F})(1200 \text{ V})^2 = \boxed{9.37 \times 10^{-5} \text{ J}}.$$

(b) Because a single potential is available, from $E = V/d$ we see that the maximum field will be produced by the minimum separation. The Bakelite is not needed to have this electric field, so the system is the same, but with no Bakelite. The electric field is

$$E_0 = (1200 \text{ V})/(5 \times 10^{-3} \text{ m}) = \boxed{2.4 \times 10^5 \text{ V/m}}.$$

69. (a) We find the equivalent capacitance for N capacitors in series from

$$1/C_{series} = \sum(1/C_i) = N/C_1, \text{ which gives } C_{series} = C_1/N.$$

The energy stored is

$$U_{series} = \tfrac{1}{2}C_{series}V^2 = \boxed{\tfrac{1}{2}C_1V^2/N}.$$

(b) We find the equivalent capacitance for N capacitors in parallel from

$$C_{parallel} = \sum C_i = NC_1.$$

The energy stored is

$$U_{parallel} = \tfrac{1}{2}C_{parallel}V^2 = \boxed{\tfrac{1}{2}NC_1V^2}.$$

(c) The equivalent capacitance does not change, so we have

$$U_{series} = \tfrac{1}{2}C_{series}V^2 = \tfrac{1}{2}Q^2/C_{series} = \boxed{\tfrac{1}{2}Q^2N/C_1};$$

$$U_{parallel} = \tfrac{1}{2}Q^2/C_{parallel} = \boxed{\tfrac{1}{2}Q^2/NC_1}.$$

70. (a) Because there is no electric field within the metal plate, the system is two capacitors in series. If we call the separation for one of them $d_1 = 1d/3$, we find the equivalent capacitance from

$1/C_{\text{metal}} = 1/C_1 + 1/C_1 = d_1/\varepsilon_0 A + d_1/\varepsilon_0 A$, which gives

$C_{\text{metal}} = \varepsilon_0 A/2d_1 = 3\varepsilon_0 A/2d$

$= 3(8.85 \times 10^{-12}\ \text{F/m})(0.28\ \text{m}^2)/2(1.5\times 10^{-2}\ \text{m}) = 2.5 \times 10^{-10}\ \text{F} = \boxed{0.25\ \text{nF}}$.

(b) Because there is no field within the metal, the surface charge density induced on the intermediate plate is

$\sigma_{\text{ind}} = Q/A = Q/(0.28\ \text{m}^2) = 3.57Q\ \text{C/m}^2$.

(c) The original energy is

$U_1 = \frac{1}{2}Q^2/C_0 = Q^2 d/2\varepsilon_0 A$.

The new energy is

$U_2 = \frac{1}{2}Q^2/C_{\text{metal}} = Q^2 d/3\varepsilon_0 A$.

The ratio is

$\boxed{U_2/U_1 = 2/3 \quad \text{(a decrease)}}$.

(d) When a dielectric is inserted, the system is three capacitors in series. We find the equivalent capacitance from

$1/C_{\text{dielectric}} = 1/C_1 + 1/C_2 + 1/C_3$

$= d_1/\varepsilon_0 A + d_1/\kappa\varepsilon_0 A + d_1/\varepsilon_0 A$

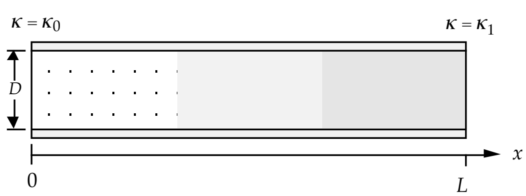
$= (2d_1 + d_1/\kappa)/\varepsilon_0 A = (2 + 1/\kappa)\, d_1/\varepsilon_0 A$, which gives

$C_{\text{dielectric}} = (3\varepsilon_0 A/d)[\kappa/(2\kappa + 1)]$.

The ratio is

$\boxed{C_{\text{dielectric}}/C_{\text{metal}} = 2\kappa/(2\kappa + 1)}$.

71.



We find the capacitance of the strip of the dielectric at x, with width dx, from

$dC = \kappa\varepsilon_0\, dA/D = \kappa\varepsilon_0 L\, dx/D$.

The strips that make up the capacitor are in parallel, so the equivalent capacitance is

$$C = \int dC = \int_0^L \frac{\kappa\varepsilon_0 L}{D}\, dx = \varepsilon_0 \int_0^L \left[\kappa_0 + \frac{(\kappa_1 - \kappa_0)x}{L}\right]\frac{L}{D}\, dx$$

$$= \frac{\varepsilon_0 L}{D}\left[\kappa_0 x + \frac{(\kappa_1 - \kappa_0)x^2}{2L}\right]\Bigg|_0^L = \frac{\varepsilon_0 L}{D}\left[\kappa_0 L + \frac{(\kappa_1 - \kappa_0)L^2}{2L}\right], \text{which reduces to}$$

$C = \boxed{\tfrac{1}{2}(\kappa_0 + \kappa_1)(\varepsilon_0 L^2/d)}$.

72. Using the estimates given, we have

$C = Q/V$

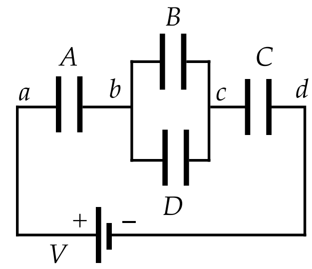
$\simeq (10^2\ \text{C})/(10^8\ \text{V}) \simeq \boxed{10^{-6}\ \text{F}}$.

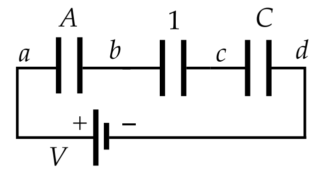
The energy stored in the capacitor is

$U = \frac{1}{2}CV^2 = \frac{1}{2}QV$

$\simeq \frac{1}{2}(10^2\ \text{C})(10^8\ \text{V}) \simeq \boxed{10^{10}\ \text{J}}$.

73. We find the equivalent capacitance of the circuit.
B and D are in parallel:
$C_1 = C_B + C_D = 4.3\ \mu\text{F} + 2.1\ \mu\text{F} = 6.4\ \mu\text{F}.$
We now have three capacitors in series:
$1/C_{\text{equ}} = 1/C_A + 1/C_1 + 1/C_C$
$= 1/(5.4\ \mu\text{F}) + 1/(6.4\ \mu\text{F}) + 1/(3.2\ \mu\text{F}),$
which gives $C_{\text{equ}} = 1.53\ \mu\text{F}$.
We find the charge on the equivalent capacitor, which is also the charge on each capacitor in series, from
$Q_{\text{equ}} = Q_A = Q_1 = Q_C = C_{\text{equ}} V_{ab}$
$= (1.53\ \mu\text{F})(3000\ \text{V}) = 4.6 \times 10^3\ \mu\text{C}.$
We find the potential differences from
$V_A = V_{ac} = Q_A/C_A = (4.6 \times 10^3\ \mu\text{C})/(5.4\ \mu\text{F}) = \boxed{8.5 \times 10^2\ \text{V}}$;
$V_B = V_D = V_{cd} = Q_1/C_1 = (4.6 \times 10^3\ \mu\text{C})/(6.4\ \mu\text{F}) = \boxed{7.2 \times 10^2\ \text{V}}$;
$V_C = V_{db} = Q_C/C_C = (4.6 \times 10^3\ \mu\text{C})/(3.2\ \mu\text{F}) = \boxed{1.43 \times 10^3\ \text{V}}$.

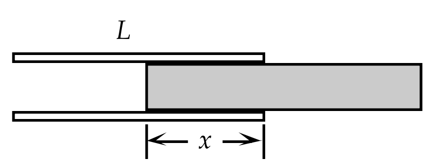




74. (a) We find the capacitance from
$C_0 = \varepsilon_0 A/d$
$= (8.85 \times 10^{-12}\ \text{F/m})(0.40\ \text{m}^2)/(3.0 \times 10^{-3}\ \text{m}) = \boxed{1.2 \times 10^{-9}\ \text{F}}$.
The maximum voltage is
$V_{\max} = E_{\max} d$, so the maximum charge is
$Q_{\max} = C_0 V_{\max} = C_0 E_{\max} d = (1.2 \times 10^{-9}\ \text{F})(2.7 \times 10^6\ \text{V/m})(3.0 \times 10^{-3}\ \text{m}) = \boxed{9.7 \times 10^{-6}\ \text{C}}$.
(b) $E_{\max} = Q_{\max}/Cd = Q_{\max}/\kappa C_0 d$
$= (9.7 \times 10^{-6}\ \text{C})/(6.0)(1.2 \times 10^{-9}\ \text{F})(3.0 \times 10^{-3}\ \text{m}) = \boxed{4.5 \times 10^5\ \text{V/m}}$.

75. The energy stored in the capacitor is
$U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2}(3.0 \times 10^{-6}\ \text{F})(1500\ \text{V})^2 = \boxed{3.4\ \text{J}}$.
Because the source is disconnected, the charge on the capacitor does not change, and we have
$C = \kappa C_0;\ V = V_0/\kappa.$
The energy stored after the dielectric is inserted is
$U = \frac{1}{2} C V^2 = \frac{1}{2}\kappa C_0 (V_0/\kappa)^2 = (1/\kappa)(\frac{1}{2} C_0 V_0^2) = (1/\kappa) U_0.$
We find the work required to insert the dielectric from
$W = \Delta U = (1/\kappa - 1) U_0$
$= (1/2.8 - 1)(3.4\ \text{J}) = \boxed{-2.2\ \text{J}}$.
The negative value means that the dielectric is drawn into the region between the plates.

76. We call the length of a plate L, so that $A = L^2$. We can treat the system as two capacitors in parallel:
$C = C_{\text{dielectric}} + C_{\text{air}}$
$= \kappa \varepsilon_0 L x/d + \varepsilon_0 L(L - x)/d$
$= (\varepsilon_0 L^2/d)[\kappa(x/L) + 1 - x/L)]$
$= \boxed{(\varepsilon_0 A/d)[1 + (\kappa - 1)x/A^{1/2}]}$.
The energy stored is
$U = \frac{1}{2} C V^2 = \boxed{(\varepsilon_0 A V^2/2d)[1 + (\kappa - 1)x/A^{1/2}]}$.

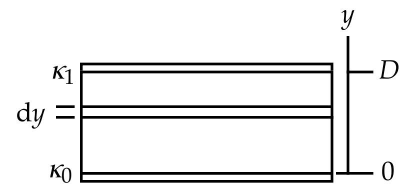


77. We take a strip of the dielectric perpendicular to the y-axis, with thickness Δy, as a capacitor. The capacitance of this strip is

$C_y = \kappa\varepsilon_0 L^2/\Delta y$.

All of the strips from $y = 0$ to $y = D$ are in series, so we find the total capacitance from

$1/C = \sum(1/C_i) = \sum(\Delta y/\kappa\varepsilon_0 A)$.



In the limit $\Delta y \to 0$, this sum becomes an integral:

$$\frac{1}{C} = \int_0^D \frac{dy}{\kappa\varepsilon_0 L^2} = \frac{1}{\varepsilon_0 L^2}\int_0^D \frac{dy}{\kappa_0 + \left[(\kappa_1 - \kappa_0)y/D\right]}$$

$$= \frac{D}{\varepsilon_0 L^2(\kappa_1 - \kappa_0)}\ln\left\{\kappa_0 + \left[(\kappa_1 - \kappa_0)y/D\right]\right\}\Bigg|_0^D = \frac{D}{\varepsilon_0 L^2(\kappa_1 - \kappa_0)}\ln\left[\frac{\kappa_0 + (\kappa_1 - \kappa_0)}{\kappa_0}\right]$$

$$= \frac{D}{\varepsilon_0 L^2(\kappa_1 - \kappa_0)}\ln\left(\frac{\kappa_1}{\kappa_0}\right).$$

The capacitance is

$C = \boxed{(\kappa_1 - \kappa_0)\varepsilon_0 L^2/[D\ln(\kappa_1/\kappa_0)]}$.

78. For two capacitors in series, the equivalent capacitance is

$C_{\text{equ, series}} = C_1C_2/(C_1 + C_2)$.

If we subtract this from one of the capacitances, we have

$C_i - C_{\text{equ, series}} = C_i^2/(C_1 + C_2) > 0$.

Because we can combine a series arrangement successively as pairs, the equivalent capacitance for a series combination is always less than any single capacitance. (Also see Problem 79.)

For capacitors in parallel, the equivalent capacitance is

$C_{\text{equ, parallel}} = \sum C_i = 2\mu\text{F} + 4\mu\text{F} + 9\mu\text{F} = \boxed{15\mu\text{F}}$.

so the equivalent capacitance is always greater than any single capacitance.

Thus, we arrange the three capacitors in series for the smallest equivalent capacitance:

$1/C_{\min} = 1/C_1 + 1/C_2 + 1/C_3 = 1/(2\mu\text{F}) + 1/(4\ \mu\text{F}) + 1/(9\ \mu\text{F})$, which gives $C_{\min} = \boxed{1.2\ \mu\text{F}}$.

79. We find the equivalent capacitance for a series arrangement from

$$\frac{1}{C_{\text{equ}}} = \sum_i\left(\frac{1}{C_i}\right).$$

If we multiply by the value of the jth capacitance, we get

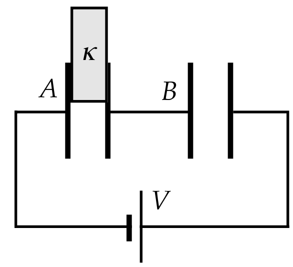
$$\frac{C_j}{C_{\text{equ}}} = \sum_i\left(\frac{C_j}{C_i}\right) = 1 + \sum_{i\neq j}\left(\frac{C_j}{C_i}\right).$$

Because the summation is positive, we have

$C_j/C_{\text{equ}} > 1$, for any value of j.

Thus the equivalent capacitance is less than any of the individual capacitances.

80. To distinguish the two capacitors, we label the one in which the dielectric is inserted A and the other one B. The two identical capacitors in series will be system 1, and the system with the dielectric will be system 2.



For system 1:

We find the equivalent capacitance from

$$C_1 = C_{A1}C_B/(C_{A1} + C_B) = CC/(C + C) = \tfrac{1}{2}C.$$

The charge on the equivalent capacitance is the charge on either capacitor:

$$Q_1 = Q_{A1} = Q_{B1} = C_1V = \tfrac{1}{2}CV.$$

The voltage drops are

$$V_{A1} = Q_{A1}/C_{A1} = (\tfrac{1}{2}CV)/C = \tfrac{1}{2}V;$$

$$V_{B1} = Q_{B1}/C_{B1} = (\tfrac{1}{2}CV)/C = \tfrac{1}{2}V.$$

The stored energy is

$$U_1 = \tfrac{1}{2}C_1V^2 = \tfrac{1}{4}CV^2.$$

For system 2:

We find the equivalent capacitance from

$$C_2 = C_{A2}C_B/(C_{A2} + C_B) = \kappa CC/(\kappa C + C) = [\kappa/(\kappa + 1)]C.$$

The charge on the equivalent capacitance is the charge on either capacitor:

$$Q_2 = Q_{A2} = Q_{B2} = C_2V = [\kappa/(\kappa + 1)]CV.$$

The voltage drops are

$$V_{A2} = Q_{A2}/C_{A2} = [\kappa/(\kappa + 1)]CV/\kappa C = [1/(\kappa + 1)]V;$$

$$V_{B2} = Q_{B2}/C_{B2} = [\kappa/(\kappa + 1)]CV/C = [\kappa/(\kappa + 1)]V.$$

The stored energy is

$$U_2 = \tfrac{1}{2}C_2V^2 = \tfrac{1}{2}[\kappa/(\kappa + 1)]CV^2.$$

The changes are

$$\Delta U = \tfrac{1}{2}[\kappa/(\kappa + 1) - \tfrac{1}{2}]\, CV^2 = \boxed{[(\kappa - 1)/2(\kappa + 1)]\tfrac{1}{2}CV^2};$$

$$\Delta Q_A = \Delta Q_B = [\kappa/(\kappa + 1) - \tfrac{1}{2}]\, CV = \boxed{[(\kappa - 1)/2(\kappa + 1)]CV};$$

$$\Delta V_A = [1/(\kappa + 1) - \tfrac{1}{2}]V = \boxed{[(1 - \kappa)/2(\kappa + 1)]V};$$

$$\Delta V_B = [\kappa/(\kappa + 1) - \tfrac{1}{2}]V = \boxed{[(\kappa - 1)/2(\kappa + 1)]V}.$$

Because $\kappa > 1$, the energy has increased. This energy is supplied by the source as the additional charge moves onto the plates.

CHAPTER 26 Currents in Materials

Answers to Understanding the Concepts Questions

1. The current is a measurement of the rate at which charge passes. Since charge is conserved, the rate at which electrons pass various points along a beam is the same no matter how the individual electrons in it may have been accelerated; otherwise you would say that some charge has been lost or gained along the way. In the part of the beam that has been sped up, the electrons have become more widely spaced. In this way the rate of passage remains the same.

2. Power dissipation occurs when electrical energy is converted into thermal energy. Microscopically, this is due to the collision between the drifting electrons with the lattice. Between successive collisions, an electron is accelerated by the electric field and builds up a certain drift velocity. As a result of the collision, the electron has lost all the "memory" of that drift velocity and has to start anew with zero average velocity. The electrons give up part of their kinetic energies through collisions with the lattice particles, which as a result oscillate faster on average, increasing the temperature. This is how electrical energy is turned into heat. The microscopic picture does agree with the observed linear relationship between voltage and current, as $\vec{J} = \sigma\vec{E}$, where is the conductivity [see Eq. (26-25)], which leads to $V = IR$.

3. The resistance is inversely proportional to the area, and for a fixed current the power dissipated, I^2R is therefore inversely proportional to the area. The thinner wire will get hotter.

4. Consider a certain segment of a wire, of length L, cross-sectional area A, and resistivity ρ. The power consumed is $P = I^2R = I^2(\rho L/A)$, and the thermal energy generated over a time interval t is then $Pt = I^2(\rho L/A)t$, which we equate to the energy needed to raise the temperature of the wire by ΔT: $Pt = I^2(\rho L/A)t = cm\Delta T = c(\rho' AL)\,\Delta T$, where ρ' is the density of the wire and c is its specific heat. Solve for ΔT: $\Delta T = I^2(\rho L/A)t/[c(\rho' AL)] = (I^2t/A^2)(\rho/c\rho')$. Thus
$$\Delta T_{\mathrm{Al}}/\Delta T_{\mathrm{Brass}} = (\rho_{\mathrm{Al}}/c_{\mathrm{Al}}\rho_{\mathrm{Al}}')/(\rho_{\mathrm{Brass}}/c_{\mathrm{Brass}}\rho_{\mathrm{Brass}}') = (\rho_{\mathrm{Al}}/\rho_{\mathrm{Brass}})(\rho_{\mathrm{Brass}}'/\rho_{\mathrm{Al}}')(c_{\mathrm{Brass}}/c_{\mathrm{Al}})$$
$$= (2.82/7)(8.9/2.7)(0.092/0.215) \approx 0.6 < 1,$$
so $\Delta T_{\mathrm{Al}} < \Delta T_{\mathrm{Brass}}$. The brass wire would get hotter.

5. It follows from Eq. (26-9) that for constant current and area the drift velocity only depends inversely on the density of free electrons. This is a characteristic of the material making up the conductor.

6. Imagine that the cylinder is made of N segments of equal length and equal cross-sectional area. Each segment has the same resistance and they are in series. When a certain current flows through the cylinder, each segment, with the same resistance, must have the same voltage drop (as $V= IR$). Thus the voltage difference applied across the entire cylinder is divided evenly over each identical segment, meaning that the voltage drops linearly over the length of the cylinder.

7. After the faucet is turned on, the water particles have to actually move from the faucet through the entire hose before they can flow out of the hose, and that accounts for the tine delay. In a wire, there are free electrons at every segment. The moment the power is switched on, an electric field is established, and every free electron instantly experiences the force of the electric field and start to drift. You don't have to wait for an electron to traverse from one end of the wire to the other end before a current is established. (In fact, the electrons drift incredibly slowly due to their frequent collisions with the lattice, at only several millimeters per hour for a typical current — so if you needed to wait for the electrons to move through the wire, you'd have a real long wait!)

8. In the free electron model, the temperature dependence of the resistivity [Eq. (26-25)] has to do with the time between collisions between electrons and the obstacle — fixed ions — that give rise to the drag force on the electrons. At very low temperatures the time between random collisions would increase; however, the accelerating field would still be present and there would still be a current. Thus the free electron model as it stands, with the accelerations due to the field as a small perturbation on the random motion, would have to be replaced with a "pinball" model, in which the only motion is due to the field and the electrons must "navigate," through multiple collisions, the forest of fixed ions. There would still be conduction even at $T = 0$ in this picture, whereas the free electron model would predict none.

9. In principle, yes it does. As T increases, not only doe the resistivity change, but also the length as well as the cross-sectional area of the wire increase. All these contribute to the temperature-dependency of electrical resistance. We do not, however, expect the dimensional change to be a major factor, since the fractional change in length and cross-sectional area is usually so small over a reasonable range of temperature variation.

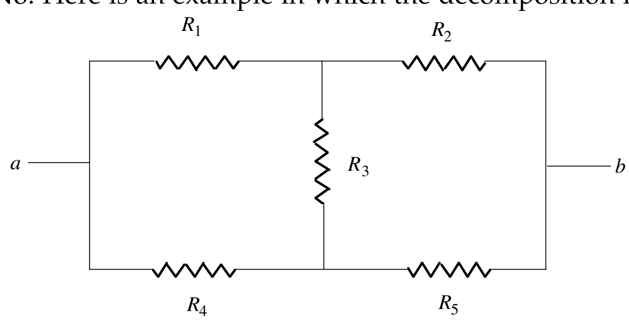
10. When a switch is thrown and charge flows, it is because there is an electric field in the wire. Free charge in the wire — mainly electrons — will move, but the wire itself remains neutral, because as many charges as leave a segment of wire from one end enter it from the other end.

11. The current in the wire is given by $I = JA = n_e evA$. As the electrons crowd to one side of the wire n_e increases, while the effective value of A decreases by the same factor. Thus I remains the same, as does the resistance of the wire.

12. There is no electric field inside a conductor when there is equilibrium. When charges are flowing due to a continuously applied potential we do not have equilibrium, and charges can flow inside the conductor.

13. According to Eq. (26-25), $\rho = m/n_e e^2\tau$. Here $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $n_e \approx 10^{29}$ /m^3 (see Example 26-3), and $\tau \approx 10^{-14}$ s. These data indeed yield $\rho \approx 10^{-8}\ \Omega \cdot \text{m}$.

14. No. Here is an example in which the decomposition is generally not possible:



15. When the power $P = VI$ becomes too large, there is too much power dissipated for the heat to be conducted away in time, and melting of the resistor material occurs. Since generally the potential V is held fixed, the current becomes too large if the resistance is too small.

16. The resistance R of the filament, along with the voltage V applied across it, determines the power consumption of the light bulb: $P = V^2/R$. Here $R = \rho L/A$, so $P = (V^2/\rho)(A/L)$. For a certain power rating, the ratio A/L must therefore be preserved. When choosing a certain diameter d (and, therefore, the cross-sectional area $A = \pi d^2/4$) of the filament, we must make sure that the corresponding length L of the filament is reasonable so that it can be coil up to fit inside the light bulb.

17. The resistance is proportional to the length of the resistor. This means that if two wires are tied together, the resistance is the sum of the resistances of the individual wires. This is consistent with the rule $R_{eq} = R_1 + R_2$ for two resistors in series.

18. The current in the wire is given by $I = JA = n_e evA$, which is proportional to the product of vA. Even though $v_2 < v_1$, we have $A_1 > A_2$. Moreover, $v_1 A_1 = v_2 A_2$, due to the continuity of flow of the charge carriers, analogous to the equation of continuity for the flow of incompressible fluids. Thus $I_1 = I_2$.

19. The total potential difference between A and B is fixed, i.e., $V_{AB} = V_{AC} + V_{CB}$ = constant. If the switch is closed then the overall resistance R_{eq} between A and B is reduced, and the current flowing through bulb 1, $I = V_{AB}/R_{eq}$, must increase. So bulb 1 will be brighter with the switch closed. As for bulb 2, when the switch is closed V_{AC}, the potential difference across bulb 1, increases (due to the increase in the current that flows through it), so V_{CB} must decrease. Thus bulb 2 becomes dimmer when the switch is closed.

20. As the diameter changes by a factor of 1/10 the cross-sectional area A changes by a factor of 1/100. Meanwhile, the volume of the wire remains fixed, so as the area changes by a factor of 1/100 the length L must change by a factor of 100. Overall, the resistance, which is proportional to L/A, must increase by a factor of $100/(1/100) = 10\,000$, or 10^4.

21. The resistors are connected in parallel. If one more resistor is added in parallel, the equivalent resistance is reduced, and the current, $I = V_{AB}/R_{eq}$, would increase.

22. For a network of resistors consisting of several parallel branches, the equivalent resistance is lower than that of the branch with the least resistance. So, to minimize R_{eq}, we can put the 1-Ω resistor in the lower branch and the other two in series in the upper branch. Then $R_{eq} < 1\ \Omega$. If we wish to maximize R_{eq}, then put the largest (4-Ω) resistor in the lower branch. You can easily verify these results with a straightforward calculation: $R_{eq} = [R_1^{-1} + (R_2 + R_3)^{-1}]^{-1} = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$, where R_1 is the resistance of the single resistor in the lower branch.

23. The term "high wattage" refers to the large amount of power dissipated in the bulb. More power is dissipated when the resistance is larger, and this is done in a light bulb by making the filament long and thin. A curled up filament allows for a longer filament in a small space.

Solutions to Problems

1. We find the average current density from

   $J_{av} = I/A = (0.46\ \text{A})/[\pi(2.2 \times 10^{-3}\ \text{m})^2/4] = \boxed{1.2 \times 10^5\ \text{A/m}^2}$.

   The charge that passes a fixed point is

   $q = It = (0.46\ \text{A})(1\ \text{s}) = \boxed{0.46\ \text{C}}$.

2. We find the density of carriers from

   $J_{av} = I/A = n_q q v_d$;

   $(1.2\ \text{A})/(4.2 \times 10^{-5}\text{m}^2) = n_q(1.6 \times 10^{-19}\ \text{C})(0.32 \times 10^{-5}\ \text{m/s})$, which gives $n_q = \boxed{5.6 \times 10^{28}\ \text{carriers/m}^3}$.

3. We find the drift speed from

   $J_{av} = I/A = n_q q v_d$;

   $(100\ \text{A})/(36 \times 10^{-6}\text{m}^2) = (5.7 \times 10^{28}\ \text{carriers/m}^3)(1.6 \times 10^{-19}\ \text{C})v_d$, which gives $v_d = 2.0 \times 10^{-4}\ \text{m/s}$.

   The time to travel the length of the cable is

   $t = L/v_d = (2\ \text{m})/(2.0 \times 10^{-4}\ \text{m/s}) = \boxed{1.0 \times 10^4\ \text{s}\ \ (2.8\ \text{h})}$.

4. We find the current from

   $I = JA$;

   $I_1 = (3 \times 10^5\ \text{A/m}^2)(0.02 \times 10^{-6}\text{m}^2) = 6 \times 10^{-3}\ \text{A} = \boxed{6.0\ \text{mA}}$.

   $I_2 = (13 \times 10^4\ \text{A/m}^2)(0.2 \times 10^{-6}\text{m}^2) = 2.6 \times 10^{-2}\ \text{A} = \boxed{26\ \text{mA}}$.

   $I_3 = (15 \times 10^4\ \text{A/m}^2)(2 \times 10^{-6}\text{m}^2) = \boxed{0.30\ \text{A}}$.

5. We find the number of electrons from the charge that passes the point:

   $N = Q/e = It/e = (0.092\ \text{A})(1\ \text{s})/(1.6 \times 10^{-19}\ \text{C}) = \boxed{5.8 \times 10^{17}\ \text{electrons}}$.

6. For the current density, we have

   $J = n_q q v_d$;

   $7.2 \times 10^2\ \text{A/m}^2 = (3.5 \times 10^{24}\ \text{carriers/m}^3)(1.6 \times 10^{-19}\ \text{C})v_d$, which gives $v_d = \boxed{1.3 \times 10^{-3}\text{m/s}}$.

7. For the current density, we have

   $J = I/A = n_q q v_d$;

   $(1.2\ \text{A})/[\pi(1.8 \times 10^{-3}\ \text{m})^2] = (8.5 \times 10^{28}\ \text{electrons/m}^3)(1.6 \times 10^{-19}\ \text{C/electron})v_d$, which gives

   $v_d = \boxed{8.7 \times 10^{-6}\text{m/s}}$.

   For the second wire, the only change is the area, so we have

   $v_{d2} = v_d R_1^2/R_2^2 = (8.7 \times 10^{-6}\ \text{m/s})(1.8\ \text{mm})^2/(1.2\ \text{mm})^2 = \boxed{2.0 \times 10^{-5}\text{m/s}}$.

8. For the current density, we have

   $J = I/A = n_q q v_d$;

   $(6.1 \times 10^{-3}\ \text{A})/(0.50 \times 10^{-4}\text{m}^2) = n_e(1.6 \times 10^{-19}\ \text{C})(3.5 \times 10^7\ \text{m/s})$, which gives

   $n_e = \boxed{2.2 \times 10^{13}\ \text{electrons/m}^3}$.

9. We find the number of electrons from the charge that passes the point:

   $N = Q/e = It/e = (200 \times 10^{-3}\ \text{A})(1\ \text{h})(3600\ \text{s/h})/(1.6 \times 10^{-19}\ \text{C}) = \boxed{4.5 \times 10^{21}\ \text{electrons}}$.

   We find the number of electrons in a 1-m length of the beam from the time to travel 1 m:

   $t = L/v$;

   $N = It/e = IL/ev = (200 \times 10^{-3}\ \text{A})(1\ \text{m})/(1.6 \times 10^{-19}\ \text{C})(3 \times 10^8\ \text{m/s}) = \boxed{4.2 \times 10^9\ \text{electrons}}$.

10. Because the current density is constant, we find the current from

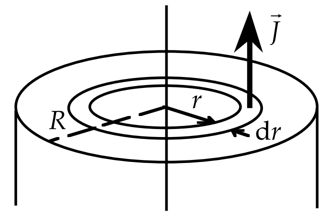
$$I_s = \iint \vec{J} \cdot d\vec{A} = \vec{J} \cdot \vec{A};$$

$$I_x = [(A\hat{i} + B\hat{j} + C\hat{k})\,\text{mA/cm}^2] \cdot (1\,\text{cm}^2)\,\hat{i} = \boxed{A\,\text{mA}}.$$

$$I_y = [(A\hat{i} + B\hat{j} + C\hat{k})\,\text{mA/cm}^2] \cdot (1\,\text{cm}^2)\,\hat{j} = \boxed{B\,\text{mA}}.$$

$$I_z = [(A\hat{i} + B\hat{j} + C\hat{k})\,\text{mA/cm}^2] \cdot (1\,\text{cm}^2)\,\hat{k} = \boxed{C\,\text{mA}}.$$

11. Because the current density is a function of the distance from the axis, we choose a circular ring for the differential area and integrate to find the current:



$$I = \iint_{area} \vec{J} \cdot d\vec{A} = \iint_{area} J\,dA = \int_0^R J_0\left(1 - \frac{r^2}{R^2}\right)2\pi r\,dr$$

$$= 2\pi J_0 \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= 2\pi J_0\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\Bigg|_0^R = \frac{\pi}{2} J_0 R^2.$$

12. We take the x-axis to the right. If v_e is the speed of the electrons, we have

$$\vec{J} = \vec{J}_{\text{electrons}} + \vec{J}_{\text{ions}} = n(-e)(-v_e)\,\hat{i} + ne(1.5 \times 10^{-3}\,v_e)\,\hat{i} = nev_e(1 + 1.5 \times 10^{-3})\,\hat{i} = (1.0015nev_e)\,\hat{i},$$

so the net current density is $\boxed{1.005nev_e \text{ to the right}}$.

13. Because a mol of NaCl contributes an Avogadro's number of positive ions and an equal number of negative ions, we find the density for each carrier from

$$n_+ = n_- = n = (0.1\,\text{mol/L})(6.02 \times 10^{23}\,\text{ions/mol})(10^3\,\text{L/m}^3) = 6.02 \times 10^{25}\,\text{ions/m}^3.$$

Because both types of carriers are present, we have

$$J = n_+q_+v_+ + n_-q_-v_- = nev_+ + n(-e)v_- = ne[v_+ - (-1.5v_+)] = 2.5nev_+;$$

$$40\,\text{A/m}^2 = (6.02 \times 10^{25}\,\text{ions/m}^3)(1.6 \times 10^{-19}\,\text{C/ion})(2.5v_+),$$ which gives

$$v_+ = \boxed{1.7 \times 10^{-6}\,\text{m/s}}, \quad v_- = \boxed{-2.5 \times 10^{-6}\,\text{m/s}}.$$

14. For the current density, we have

$$J = I/A = n_q q\,v_d;$$

$$(100\ \text{A})/\pi(2 \times 10^{-3}\,\text{m})^2 = (8.5 \times 10^{22}\,\text{electrons/cm}^3)(10^2\,\text{cm/m})^3(1.6 \times 10^{-19}\,\text{C/electron})v_d,$$

which gives $v_d = \boxed{5.9 \times 10^{-4}\,\text{m/s}}$.

If the diameter were doubled, A would increase by a factor of 4, so v_d would $\boxed{\text{decrease by a factor of 4}}$.

15. The time for the charged particle to circle the accelerator is $T = 2\pi R/v$. So the current is

$$I = q/T = qv/2\pi R = (1.6 \times 10^{-19}\,\text{C})(3 \times 10^8\,\text{m/s})/[2\pi(2.5 \times 10^3\,\text{m})] = \boxed{3.1 \times 10^{-15}\,\text{A}}.$$

16. Because each particle contributes the same current, we have

$$N = I_{\text{total}}/I = (42 \times 10^{-3}\,\text{A})/(3.1 \times 10^{-15}\,\text{A}) = \boxed{1.4 \times 10^{13}}.$$

$\boxed{17}$. We find the current from $I = Q/t = (10 \times 10^3\,\text{C})/(3.6 \times 10^3\,\text{s}) = \boxed{2.8\,\text{A}}$. The current density is

$$J = I/A = (2.8\,\text{A})/(50 \times 10^{-6}\,\text{m}^2) = \boxed{5.6 \times 10^4\,\text{A/m}^2}.$$

We find the free electron density from

$$n_e = \rho N_A/M = (2.7\,\text{g/cm}^3)(10^2\,\text{cm/m})^3(6.02 \times 10^{23}\,\text{atm/mol})/(27\,\text{g/mol}) = 6.0 \times 10^{28}\,\text{electrons/m}^3.$$

We find the drift speed from

$$v_d = J/n_e e = (5.6 \times 10^4\,\text{A/m}^2)/(6.0 \times 10^{28}\,\text{electrons/m}^3)(1.6 \times 10^{-19}\,\text{C/electron}) = \boxed{5.8 \times 10^{-6}\,\text{m/s}}.$$

18. We find the free-electron density from

$n_e = \rho N_A/M$

$= (19.3 \times 10^3\ \text{kg/m}^3)(6.02 \times 10^{23}\ \text{atm/mol})/[(197\ \text{g/mol})(10^{-3}\ \text{kg/g})]$

$= 5.90 \times 10^{28}\ \text{electrons/m}^3.$

We find the drift speed from

$v_d = J/n_e e = I/An_e e$

$= (0.3\ \text{A})/[\pi(0.5 \times 10^{-3}\ \text{m})^2(5.90 \times 10^{28}\ \text{electrons/m}^3)(1.60 \times 10^{-19}\ \text{C/electron})]$

$= \boxed{4.0 \times 10^{-5}\ \text{m/s}}.$

19. The total current must be the same on each side of the junction:

$I_{\text{total}} = 2I_1 = 3I_2;$

$2(3\ \text{A}) = 3I_2$, which gives $I_2 = 2$ A in each of the smaller wires.

We find the drift speed in the larger wires from

$v_{\text{in}} = J/n_e e = I_1/A_1 n_e e$

$= (3\ \text{A})/[\pi(0.1\ \text{cm})^2(7 \times 10^{22}\ \text{electrons/cm}^3)(1.60 \times 10^{-19}\ \text{C/electron})]$

$= 8.5 \times 10^{-3}\ \text{cm/s} = \boxed{8.5 \times 10^{-5}\ \text{m/s}}.$

We find the drift speed in the smaller wires from

$v_{\text{out}} = J/n_e e = I_2/A_2 n_e e$

$= (2\ \text{A})/[\pi(0.05\ \text{cm})^2(7 \times 10^{22}\ \text{electrons/cm}^3)(1.60 \times 10^{-19}\ \text{C/electron})]$

$= 2.3 \times 10^{-2}\ \text{cm/s} = \boxed{2.3 \times 10^{-4}\ \text{m/s}}.$

The combined area of the smaller wires is less than the combined area of the larger wires. Charge conservation is equivalent to mass conservation in water flow, so the smaller area requires a greater speed.

20. As a function of r, the drift speed is

$v = v_0(1 - r/R).$

This variable drift speed means the current density is a function of r. We find the total current by selecting a ring of radius r and thickness dr, then we add (integrate) the contributions from all of the rings:

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R n_q qv\, 2\pi r\, \mathrm{d}r = 2\pi n_q qv_0 \int_0^R \left(1 - \frac{r}{R}\right) r\, \mathrm{d}r$$

$$= 2\pi n_q qv_0 \left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^R = 2\pi n_q qv_0 \left(\frac{R^2}{2} - \frac{R^3}{3R}\right) = \frac{1}{3}\pi n_q qv_0 R^2.$$

For a constant drift speed of $\frac{1}{2}v_0$, the current is

$I' = n_q q(\frac{1}{2}v_0)\pi R^2 = \frac{1}{2}\pi n_q q\, v_0 R^2.$

The ratio of currents is

$I/I' = \frac{1}{3}/\frac{1}{2} = \boxed{\frac{2}{3}}.$

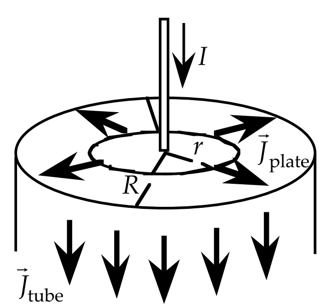
Even though the drift speed varies linearly, this ratio is not 1, since there is more area at the lower speeds.

21. With $d << R$, from symmetry, the current density in the tube is constant:

$J_{\text{tube}} = I/A = \boxed{I/2\pi Rd \text{ along the tube}}.$

The current density in the top plate is radial and depends on the distance from the axis r:

$J_{\text{plate}} = I/A = \boxed{I/2\pi rd \text{ radial}}.$



22. The variable charge density and drift speed means the current density is a function of r. We find the total current by selecting a ring of radius r and thickness dr, then add (integrate) the contributions from all of the rings:

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R nqv\, 2\pi r\, \mathrm{d}r = 2\pi q \int_0^R (n_0 - n'r)(v_0 - v'r^2) r\, \mathrm{d}r$$

$$= 2\pi q \int_0^R (n_0 v_0 r - n' v_0 r^2 - n_0 v' r^3 + n' v' r^4)\, \mathrm{d}r$$

$$= 2\pi q \left(n_0 v_0 \frac{r^2}{2} - n' v_0 \frac{r^3}{3} - n_0 v' \frac{r^4}{4} + n' v' \frac{r^5}{5} \right) \Bigg|_0^R \text{, which gives}$$

$$I = \boxed{\pi q R^2 (n_0 v_0 - 2n' v_0 R/3 - n_0 v' R^2/2 + 2n' v' R^3/5)}.$$

23. Because the material is the same, we have

$$R_2/R_1 = (L_2/A_2)/(L_1/A_1) = (L_2/L_1)(D_1/D_2)^2 = (\tfrac{1}{2})(2)^2 = 2;\ \boxed{R_2 = 2R_1}.$$

24. Because the length is the same, we have

$$R_{\text{gold}}/R_{\text{silver}} = (\rho_{\text{gold}}/A_{\text{gold}})/(\rho_{\text{silver}}/A_{\text{silver}}) = (\rho_{\text{gold}}/\rho_{\text{silver}})(D_{\text{silver}}/D_{\text{gold}})^2$$

$$1 = (1/1.5)(D_{\text{silver}}/D_{\text{gold}})^2 \text{, which gives } \boxed{D_{\text{silver}}/D_{\text{gold}} = 1.22}.$$

25. (a) We find the resistance from

$$R = \rho L/A = (2.82 \times 10^{-8}\ \Omega \cdot \text{m})(528\ \text{m})/(0.12 \times 10^{-4}\text{m}^2) = \boxed{1.2\ \Omega}.$$

(b) We form the ratio of resistances:

$$R_2/R_1 = (\rho_2/\rho_1)(L_2/L_1)(A_1/A_2);$$

$$1 = [(1.72 \times 10^{-8}\ \Omega \cdot \text{m})/(2.82 \times 10^{-8}\ \Omega \cdot \text{m})](1)[(0.12 \times 10^{-4}\text{m}^2)/\pi r_2^2] \text{, which gives}$$

$$r_2 = 1.5 \times 10^{-3}\ \text{m} = \boxed{0.15\ \text{cm}}.$$

26. (a) We find the resistance from

$$R = \rho L/A$$

$$= (1.72 \times 10^{-8}\ \Omega \cdot \text{m})(100\ \text{ft})(0.305\ \text{m/ft})/\{\tfrac{1}{4}\pi[(0.0403\ \text{in})(2.54 \times 10^{-2}\ \text{m/in})]^2\} = \boxed{0.637\ \Omega}.$$

(b) We find the length from

$$L_2 = R_2(L/R)$$

$$= (7.5\ \Omega)(100\ \text{ft})/(0.637\ \Omega) = \boxed{1.18 \times 10^3\ \text{ft}\ \ (359\text{m})}.$$

27. We find the resistance from

$$R = \rho L/A = (1.72 \times 10^{-8}\ \Omega \cdot \text{m})(10\ \text{m})/[\tfrac{1}{4}\pi(0.2588 \times 10^{-2}\ \text{m})^2] = \boxed{3.27 \times 10^{-2}\ \Omega}.$$

28. We find the resistance from

$$R = \rho L/A = (3.5 \times 10^{-5}\ \Omega \cdot \text{m})(20.0 \times 10^{-2}\ \text{m})/[\tfrac{1}{4}\pi(5.0 \times 10^{-3}\ \text{m})^2] = \boxed{0.36\ \Omega}.$$

We find the current from

$$I = V/R = (380\ \text{V})/(0.36\ \Omega) = \boxed{1.1 \times 10^3\ \text{A}}.$$

29. We find the resistance from

$$R_{20} = \rho_{20} L/A = (1.72 \times 10^{-8}\ \Omega \cdot \text{m})(2\ \text{m})/(36 \times 10^{-6}\ \text{m}) = \boxed{9.6 \times 10^{-4}\ \Omega}.$$

The increase in resistance is

$$\Delta R = R_{20}\alpha(T - 20°\text{C}) = (9.6 \times 10^{-4}\ \Omega)(0.0039/\text{C}°)(80°\text{C}) = \boxed{3.0 \times 10^{-4}\ \Omega}.$$

30. We find the radius from

$$R = \rho L/A = R = \rho L/\pi r^2;$$

$$10\ \Omega = (1.72 \times 10^{-8}\ \Omega \cdot \text{m})(175 \times 10^3\ \text{m})/\pi r^2 \text{, which gives } r = 9.8 \times 10^{-3}\ \text{m} = \boxed{9.8\ \text{mm}}.$$

31. The resistance of the wire of length L and cross-sectional area A is $R = \rho L/A$. But according to Ohm's law $R = V/I$, so $\rho L/A = V/I$, which we solve for L:
$$L = AV/\rho I = (0.20 \times 10^{-6}\,\text{m}^2)(120\text{ V})/[5.6 \times 10^{-8}\,)(15\text{ A})] = \boxed{29\text{ m}}.$$

32. The current density is given by
$$J_{\text{av}} = I/A = n_q q v_d;\ \text{or}\ \ v_d = J_{\text{av}}/n_q q.\ \text{Thus}$$
$$(v_d)_{\text{Cu}}/(v_d)_{\text{Al}} = (J_{\text{av}}/n_q q)_{\text{Cu}}/(J_{\text{av}}/n_q q)_{\text{Al}} = (n_q)_{\text{Al}}/(n_q)_{\text{Cu}} = (18 \times 10^{28}\,\text{m}^{-3})/(9 \times 10^{28}\,\text{m}^{-3}) = \boxed{2}.$$

33. The power consumed in a resistor is
$$P = IV = I^2R = V^2/R.$$
With a fixed potential difference, we have
$$(P_2 - P_1)/P_1 = (1/R_2 - 1/R_1)/(1/R_1) = (R_1 - R_2)/R_2.$$
If we assume that the temperature coefficient does not change with temperature, we get
$$(P_2 - P_1)/P_1 = \{R_{20}[1 + \alpha(T_1 - 20°\text{C})] - R_{20}[1 + \alpha(T_2 - 20°\text{C})]\}/R_{20}[1 + \alpha(T_2 - 20°\text{C})]$$
$$= \alpha(T_1 - T_2)/[1 + \alpha(T_2 - 20°\text{C})] = (0.0045/°\text{C})(-400°\text{C})/[1 + (0.0045/°\text{C})(1180°\text{C})]$$
$$= \boxed{-0.27}.$$
Because the resistance has increased, the power consumption has decreased.

34. We find the length of the equivalent single wire from
$$R = V/I = \rho L/A;$$
$$(1.5\text{ V})/(0.14\text{ A}) = (1.7 \times 10^{-8}\,\Omega\cdot\text{m})L/[\tfrac{1}{4}\pi(0.24 \times 10^{-3}\text{ m})^2],\ \text{which gives } L = 28.5\text{ m}.$$
The distance to the short is $d = L/2 = \boxed{14.3\text{ m}}$.

$\boxed{35}$. We find the current from
$$R = V/I = \rho L/A = \rho_0(1 + \alpha\,\Delta T)L/A.$$
Table 26–2 gives the resistivity at 20°C. At 25°C, we have
$$(50\text{ V})/I_{25} = (100 \times 10^{-8}\,\Omega\cdot\text{m})[1 + (4 \times 10^{-4}\,/\text{C}°)(5°\text{C})](0.50\text{ m})/[\tfrac{1}{4}\pi(0.5 \times 10^{-3}\text{ m})^2],$$
which gives $I_{25} = \boxed{19.6\text{ A}}$.
At 400°C, we have
$$(50\text{ V})/I_{400} = (100 \times 10^{-8}\,\Omega\cdot\text{m})[1 + (4 \times 10^{-4}\,/\text{C}°)(380°\text{C})](0.50\text{ m})/[\tfrac{1}{4}\pi(0.5 \times 10^{-3}\text{ m})^2],$$
which gives $I_{400} = \boxed{17.1\text{ A}}$.

36. If we ignore dimension changes, with I constant, we have
$$V_2/V_1 = R_2/R_1 = \rho_2/\rho_1 = \rho_{20}[1 + \alpha(T - 20°\text{C})]/\rho_{20} = 1 + \alpha(T - 20°\text{C});$$
$$(8.7\text{ mV})/(8.5\text{ mV}) = 1 + (0.0039/\text{C}°)(T - 20°\text{C}),\ \text{which gives } T = \boxed{26°\text{C}}.$$

37. From the expression for the resistance, $R = \rho L/A$, we form the ratio
$$R_{\text{Al}}/R_{\text{Cu}} = (\rho_{\text{Al}}/\rho_{\text{Cu}})(L_{\text{Al}}/L_{\text{Cu}})(A_{\text{Cu}}/A_{\text{Al}}) = (\rho_{\text{Al}}/\rho_{\text{Cu}})(L_{\text{Al}}/L_{\text{Cu}})(r_{\text{Cu}}/r_{\text{Al}})^2;$$
$$1 = [(2.8 \times 10^{-8}\,\Omega\cdot\text{m})/(1.7 \times 10^{-8}\,\Omega\cdot\text{m})](L/5L)(r_{\text{Cu}}/r_{\text{Al}})^2,\ \text{which gives } \boxed{r_{\text{Cu}}/r_{\text{Al}} = 1.74}.$$

38. We find the resistivity from
$$R = V/I = \rho L/A;$$
$$(12.0\text{ V})/(1.07\text{ A}) = \rho(100\text{ m})/(0.5\text{ mm}^2)(10^{-3}\text{ m/mm})^2,\ \text{which gives } \rho = 5.6 \times 10^{-8}\,\Omega\cdot\text{m}.$$
When we look at the values listed in Table 26–2, we see that the material is $\boxed{\text{tungsten}}$.

39. We find the length of the wire from
$$R = \rho L/A;$$
$$1.35\,\Omega = (1.72 \times 10^{-8}\,\Omega\cdot\text{m})L/[(1.5\text{ mm}^2)(10^{-3}\text{ m/mm})^2],\ \text{which gives } L = 118\text{ m}.$$
The number of turns around the spool is
$$N = L/\pi D = (118\text{ m})/[\pi(0.20\text{ m})] = \boxed{188\text{ turns}}.$$

40. We express the voltage in terms of the current and radius:
$V = IR = I\rho L/A = I\rho L/\pi r^2$.
If we use this for the two wires and take the ratio, we have
$V_1/V_2 = (I_1\rho L/I_2\rho L)(\pi r_2^2/\pi r_1^2) = (I_1/I_2)(r_2/r_1)^2$.
We see that r_2/r_1 will be minimum when V_2/V_1 has its maximum value of 1.5:
$1/1.8 = (1/2)(r_2/r_1)_{\text{min}}^2$, which gives $(r_2/r_1)_{\text{min}} = \boxed{1.05}$.

41. We find the resistance from
$R_{\text{Al}} = \rho_{\text{Al}} L/A_{\text{Al}}$
$= (2.8 \times 10^{-8}\ \Omega \cdot \text{m})(80\ \text{m})/\frac{1}{4}\pi(0.12 \times 10^{-2}\ \text{m})^2 = \boxed{2.0\ \Omega}$.
The mass of the wire is
$m_{\text{Al}} = \rho_{m,\text{Al}} A_{\text{Al}} L$
$= (2.7 \times 10^3\ \text{kg/m}^3)[\frac{1}{4}\pi(0.12 \times 10^{-2}\ \text{m})^2](80\ \text{m}) = \boxed{0.24\ \text{kg}}$.
We find the area of the copper wire from
$R_{\text{Cu}} = \rho_{\text{Cu}} L/A_{\text{Cu}}$;
$2.0\ \Omega = (1.7 \times 10^{-8}\ \Omega \cdot \text{m})(80\ \text{m})/A_{\text{Cu}}$, which gives $A_{\text{Cu}} = 6.8 \times 10^{-7}\ \text{m}^2$.
The mass of the copper wire is
$m_{\text{Cu}} = \rho_{m,\text{Cu}} A_{\text{Cu}} L = (8.9 \times 10^3\ \text{kg/m}^3)(6.8 \times 10^{-7}\ \text{m}^2)(80\ \text{m}) = \boxed{0.48\ \text{kg}}$.

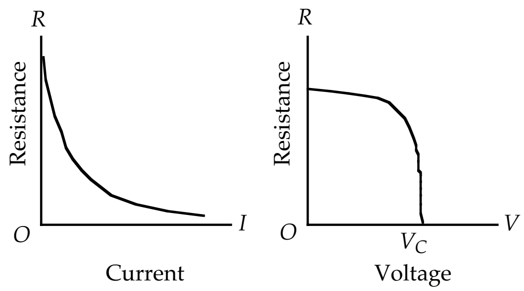
42. For the resistance, we have
$R = \rho L/A = \rho L/\pi r^2$;
$2\ \Omega = (1.72 \times 10^{-8}\ \Omega \cdot \text{m})L/\pi r^2$.
The mass of the wire is
$m = \rho_m AL$;
$1.5\ \text{kg} = (8.9 \times 10^3\ \text{kg/m}^3)\pi r^2 L$.
This gives us two equations with two unknowns, L and r. When we solve them, we get
$r = 6.2 \times 10^{-4}\ \text{m} = \boxed{0.62\ \text{mm}}$, and $L = \boxed{1.4 \times 10^2\ \text{m}}$.

43. For the resistance, we have
$R = \rho L/A$;
$5\ \Omega = (1.59 \times 10^{-8}\ \Omega \cdot \text{m})(10^3\ \text{m})/A$, which gives $A = 3.18 \times 10^{-6}\ \text{m}^2$.
We find the mass of the wire from
$m = \rho_m AL = (10.5 \times 10^3\ \text{kg/m}^3)(3.18 \times 10^{-6}\ \text{m}^2)(10^3\ \text{m}) = \boxed{33.4\ \text{kg}}$.

44. For the resistance, we have
$R = \rho L/A = \rho L/\pi(r_{\text{outside}}^2 - r_{\text{inside}}^2)$;
$3.5\ \Omega = (1.72 \times 10^{-8}\ \Omega \cdot \text{m})L/\pi[(2.75 \times 10^{-2}\ \text{m})^2 - (2.45 \times 10^{-2}\ \text{m})^2]$, which gives $L = \boxed{1.0 \times 10^5\ \text{m}}$.

45. For the resistance, we have
$R_1 = \rho L/A_1 = \rho L/\pi(r_{\text{outside}}^2 - r_{\text{inside}}^2)$
$= (1.72 \times 10^{-8}\ \Omega \cdot \text{m})(1\ \text{m})/\pi[(0.2 \times 10^{-2}\ \text{m})^2 - (0.1 \times 10^{-2}\ \text{m})^2] = \boxed{1.82 \times 10^{-3}\ \Omega}$.
For the solid wire, we have
$R_2 = \rho L/A_2 = \rho L/\pi r^2$.
If we divide the two equations, we get
$R_2/R_1 = 1 = r^2/(r_{\text{outside}}^2 - r_{\text{inside}}^2)$, which becomes
$r^2 = r_{\text{outside}}^2 - r_{\text{inside}}^2 = (0.2\ \text{cm})^2 - (0.1\ \text{cm})^2$, which gives $r = \boxed{0.17\ \text{cm}}$.
The ratio of masses is
$m_2/m_1 = \rho_m L A_2/\rho_m L A_1 = A_2/A_1 = r^2/(r_{\text{outside}}^2 - r_{\text{inside}}^2) = 1$.
The masses are the $\boxed{\text{same}}$.

46.

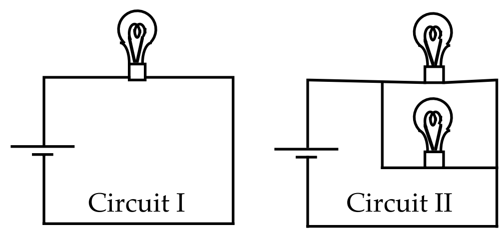


At V_C, $R \to 0$; V_C is the breakdown voltage.

47. When two identical lightbulbs are connected in series the current flowing through them is the same, and so is the power consumption. They should be identical in brightness. If a third lightbulb is connected in series with the second one, then the current flowing through the first lightbulb must split equally for the second and the third lightbulb, so the current in bulb 1 is 2 times as much as those in bulb 2 and 3. Since the power consumption is proportional to I^2, bulb 1 consumes 4 times as much power as bulbs 2 and 3, and is therefore 4 times as bright as the other two lightbulbs.

48. (a) Because the bulbs in Circuit II are in parallel, the potential difference across each bulb is the same as in Circuit I. Each bulb will have the same brightness, which will be the brightness of the bulb in Circuit I. (Note that more power will come from the battery.)

(b) If one bulb is removed from Circuit II, there will still be a closed circuit for the other bulb, which will be the same as Circuit I, so the brightness of the remaining bulb will not change. It does not matter which bulb is removed.



49. The voltage across the two-resistor combination is

$V = I(R + R_x)$, so $I = V/(R + R_x)$. Thus the voltage drop across the resistor X is

$V_x = IR_x = VR_x/(R + R_x)$; so

$8\text{ V} = VR_x/(10\ \Omega + R_x)$ and $12\text{ V} = VR_x/(5\ \Omega + R_x)$; from which we get $R_x = \boxed{5\ \Omega}$, $V = \boxed{24\text{ V}}$.

50. The effective resistance R of the three-resistor combination satisfies

$1/R = 1/R_1 + 1/R_2 + 1/R_3$.

With $R = 4\ \Omega$, $R_1 = 20\ \Omega$, and $R_2 = 12\ \Omega$, we find the value of the third resistor as

$R_3 = (1/R - 1/R_1 - 1/R_2)^{-1} = [1/(4\ \Omega) - 1/(12\ \Omega) - 1/(20\ \Omega)]^{-1} = \boxed{8.6\ \Omega}$.

$\boxed{51}$. The equivalent resistance of the five resistors is

$R_{\text{eq}} = \sum R_i = 5R_1 = 5(18\ \Omega) = \boxed{90\ \Omega.}$

The current in each resistor is the same:

$I = V/R_{\text{eq}} = (16\text{ V})/(90\ \Omega) = \boxed{0.18\text{ A}}$.

The total power dissipated is

$P = I^2 R_{\text{eq}} = (0.18\text{ A})^2(90\ \Omega) = \boxed{2.8\text{ W}}$.

52. The equivalent resistance of the two resistors is

$R_{\text{eq}} = \sum R_i = 2R_1 = 2(60\ \Omega) = 120\ \Omega$.

The current in each resistor is the same:

$I = V/R_{\text{eq}} = (120\text{ V})/(120\ \Omega) = 1.0\text{ A}$.

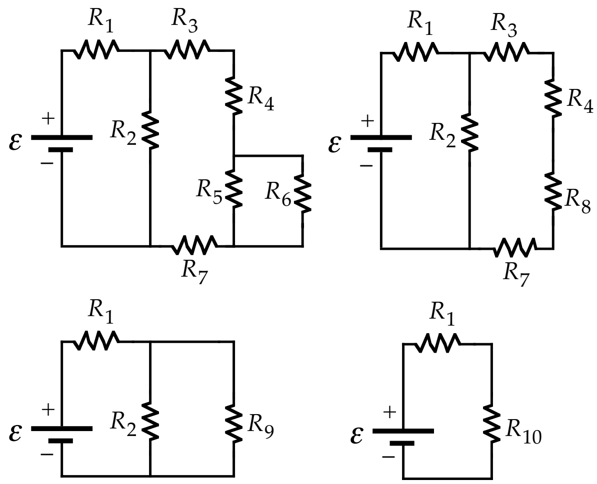
The total power dissipated is

$P = I^2 R_{\text{eq}} = (1.0\text{ A})^2(120\ \Omega) = \boxed{120\text{ W}}$.

53. We find the equivalent resistance of the two resistors from

$$1/R_{eq} = 1/R_1 + 1/R_2 = 1/R_1 + 1/(2R_1) = 3/(2R_1) = 3/[2(150\ \Omega)],$$ which gives $R_{eq} = \boxed{100\ \Omega}$.

54.



We combine R_5 and R_6, which are in parallel:

$$1/R_8 = 1/R_5 + 1/R_6 = 1/(5\ \Omega) + 1/(5\ \Omega),$$ which gives $R_8 = 2.5\ \Omega$.

We combine R_3, R_4, R_8, and R_7, which are in series:

$$R_9 = R_3 + R_4 + R_8 + R_7$$
$$= 2\ \Omega + 1.5\ \Omega + 2.5\ \Omega + 2\ \Omega = 8\ \Omega.$$

We combine R_2 and R_9, which are in parallel:

$$1/R_{10} = 1/R_2 + 1/R_9 = 1/(5\ \Omega) + 1/(8\ \Omega),$$

which gives $R_{10} = 3.1\ \Omega$.

We combine R_1 and R_{10}, which are in series:

$$R_{eq} = R_1 + R_{10} = 3\ \Omega + 3.1\ \Omega = \boxed{6.1\ \Omega}.$$

55. The current in the 24-Ω resistor is

$$I_1 = V_{AB}/(24\ \Omega) = (16\ \text{V})/\ (24\ \Omega) = \boxed{0.67\ \text{A}}.$$

Since the equivalent resistance of the upper branch of the circuit is

$R = 8\ \Omega + (12\ \Omega)(6\ \Omega)/(12\ \Omega + 6\ \ \Omega) = 12\ \Omega$, the current in the 8-Ω resistor is

$$I_2 = V_{AB}/(12\ \Omega) = (16\ \text{V})/\ (12\ \Omega) = \boxed{1.3\ \text{A}}.$$

This current is split between the two remaining resistors, with

$I_3 = [6\ \Omega/(6\ \Omega + 12\ \Omega)]\ I_2 = \frac{1}{3}(1.33\ \text{A}) = \boxed{0.44\ \text{A}}$ in the 12-Ω resistor and

$I_4 = I_2 - I_3 = 1.33\ \text{A} - 0.44\ \text{A} = \boxed{0.89\ \text{A}}$ in the 6-Ω resistor.

56. In Problem 55 we calculated the current flowing through each resistor. Then we may use $P = I^2R$ to find the power dissipated on each of them:

$P = I^2R = (0.667\ \text{A})^2(24\ \Omega) = \boxed{11\ \text{W}}$ (for the 24-Ω resistor);

$P = I^2R = (1.33\ \text{A})^2(8\ \Omega) = \boxed{14\ \text{W}}$ (for the 8-Ω resistor);

$P = I^2R = (0.889\ \text{A})^2(6\ \Omega) = \boxed{4.7\ \text{W}}$ (for the 6-Ω resistor);

$P = I^2R = (0.444\ \text{A})^2(12\ \Omega) = \boxed{2.4\ \text{W}}$ (for the 12-Ω resistor).

57. Call the lower branch with the single 12-Ω resistor branch 1 and the other one (which contains all the other resistors) branch 2. First, we find R_2, the equivalent resistance of branch 2. Combine the 2-Ω and 4-Ω resistors in parallel to obtain (2 Ω)(4 Ω)/(2 Ω + 4 Ω) = 1.33 Ω, which we add to the 8-Ω resistor and put the resultant 9.33-Ω resistor in parallel with the 24-Ω resistor: (9.33 Ω)(24 Ω)/(9.33 Ω + 24 Ω) = 6.72 Ω. Add this to the 6-Ω resistor to obtain R_2 = 12.72 Ω.

We now combine R_1 (= 12 Ω) and R_2 in parallel to obtain the equivalent resistance between A and B:

$R_{eq} = R_1 R_2/(R_1 + R_2) = (12.72\ \Omega)(12\ \Omega)/(12.72\ \Omega + 12\ \Omega) = 6.18\ \Omega.$

The voltage across AB, i.e., that across the 12-Ω resistor, is then

$V_{12\ \Omega} = I_{AB}R_{eq} = (20\ \text{A})(6.18\ \Omega) = \boxed{124\ \text{V}}.$

The current in the 12-Ω resistor is

$I_{12\ \Omega} = V_{AB}/(12\ \Omega) = (124\ \text{V})/(12\ \Omega) = \boxed{10.3\ \text{A}},$

And the current through the 6-Ω resistor is then

$I_{6\ \Omega} = 20\ \text{A} - 10.3\ \text{A} = \boxed{9.7\ \text{A}}$, which requires a voltage of

$V_{6\ \Omega} = I_{6\ \Omega}\,(6\ \Omega) = (9.7\ \text{A})(6\ \Omega) = \boxed{58\ \text{V}}.$

The voltage difference across the 24-Ω resistor is now

$V_{24\ \Omega} = 123.6\ \text{V} - 58.2\ \text{V} = \boxed{66\ \text{V}}$, which drives a current of

$I_{24\ \Omega} = (65.4\ \text{V})/(24\ \Omega) = \boxed{2.7\ \text{A}}$, leaving the current in the 8-Ω resistor as

$I_{8\ \Omega} = 9.7\ \text{A} - 2.7\ \text{A} = \boxed{7.0\ \text{A}}$, which requires a voltage of

$V_{8\ \Omega} = I_{8\ \Omega}\,(8\ \Omega) = (7.0\ \text{A})(8\ \Omega) = \boxed{56\ \text{V}}.$

The voltage applied on the two remaining resistors (4-Ω and 2-Ω) is then

$V_{4\ \Omega} = V_{2\ \Omega} = 123.5\ \text{V} - 58.2\ \text{V} - 56.0\ \text{V} = \boxed{9.3\ \text{V}}$, which drives a current of

$I_{4\ \Omega} = (9.3\ \text{V})/(4\ \Omega) = \boxed{2.3\ \text{A}}$ in the 4-Ω resistor and

$I_{2\ \Omega} = (9.3\ \text{V})/(2\ \Omega) = \boxed{4.7\ \text{A}}$ in the 2-Ω resistor.

58. The two parallel branches has an equivalent resistance of $R(R + x)/[R + (R + x)]$, so for the equivalent resistance of the entire load we have

$R_{eq} = 3R + R(R + x)/(2R + x) = x;$

$x^2 - 2Rx - 7R^2 = 0;$

$x = \boxed{(1+ \sqrt{8})R \approx 3.83\ R}.$

59. (a) We combine R_2 and R_3, which are in parallel:

$1/R_5 = 1/R_2 + 1/R_3 = 1/(75\ \Omega) + 1/(60\ \Omega),$

which gives $R_5 = 33.3\ \Omega$.

We combine R_1, R_5, and R_4, which are in series:

$R_{eq} = R_1 + R_5 + R_4 = 33\ \Omega + 33.3\ \Omega + 25\ \Omega,$

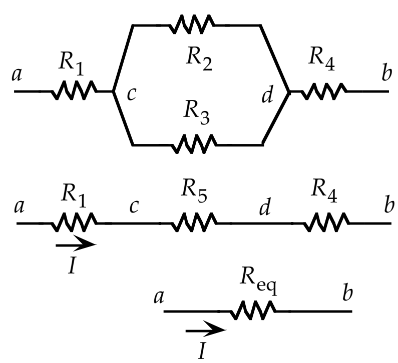
which gives $R_{eq} = \boxed{91.3\ \Omega}.$

(b) We find the current from

$I = V_{ab}/R_{eq} = (12\ \text{V})/(91.3\ \Omega) = 0.131\ \text{A}.$

The potential difference across the 75-Ω resistor is V_{cd}, which we find from

$V_{cd} = IR_5 = (0.131\ \text{A})(33.3\ \Omega) = \boxed{4.38\ \text{V}}.$



(c) From part (b), the current through the 33-Ω resistor is $I = \boxed{0.131\ \text{A}}.$

60. We find the drift speed from

$v_d = eE\tau/m$

$= (1.6 \times 10^{-19}\ \text{C})(2.0 \times 10^{-3}\ \text{V/m})(2.4 \times 10^{-14}\ \text{s})/(9.1 \times 10^{-31}\ \text{kg}) = \boxed{8.4 \times 10^{-6}\ \text{m/s}}.$

61. We estimate the mean free path as the average distance traveled between collisions:

$\lambda = v_{av}\tau = (2.7 \times 10^6\ \text{m/s})(2.4 \times 10^{-14}\ \text{s}) = \boxed{6.5 \times 10^{-8}\ \text{m}}.$

62. From Eq. 19–46 for the mean free path, we have
$\lambda = 1/(n\sigma\sqrt{2})$;
$3.7 \times 10^{-8}\text{ m} = 1/[(8.5 \times 10^{28}/\text{m}^3)\sigma\sqrt{2}]$, which gives $\sigma = \boxed{2.2 \times 10^{-22}\text{ m}^2}$.

63. In terms of the average time between collisions, the resistivity is
$\rho = m/ne^2\tau$.
The average time between collisions depends on the drift speed:
$\tau = mv_d/eE$.
When we combine these, we have
$\rho = meE/ne^2mv_d = E/nev_d$.
For the given r-dependence of v_d, we have
$\rho = E/[nev_0(1 - r/R)] = \boxed{\rho_0/(1 - r/R)}$, where $\rho_0 = E/nev_0$.

64. From kinetic theory, we have the average kinetic energy of the electron:
$K = \frac{3}{2}kT$,
which we use as the energy necessary to cross the energy gap. For the given elements, we have
Si: $(1.1\text{ eV})(1.6 \times 10^{-19}\text{ J/eV}) = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})T_{\text{Si}}$, which gives $T_{\text{Si}} = \boxed{8.5 \times 10^3\text{ K}}$.
Ge: $(0.7\text{ eV})(1.6 \times 10^{-19}\text{ J/eV}) = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})T_{\text{Ge}}$, which gives $T_{\text{Ge}} = \boxed{5.4 \times 10^3\text{ K}}$.
C: $(6\text{ eV})(1.6 \times 10^{-19}\text{ J/eV}) = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})T_{\text{C}}$, which gives $T_{\text{C}} = \boxed{4.6 \times 10^4\text{ K}}$.

65. For an ohmic resistor, we have
$P = IV = V^2/R$, or $R = V^2/P = (12\text{ V})^2/(65\text{ W}) = \boxed{2.2\ \Omega}$.

66. For an ohmic resistor, we have
$P = IV = V^2/R$, or $V^2 = PR = (1.5\text{ W})(1000\ \Omega)$, which gives $V = \boxed{39\text{ V}}$.

67. Cost = (rate)Pt = $(10\ ¢/\text{kWh})(100\text{ W})(1\text{ h})(10^{-3}\text{ kW/W}) = \boxed{1\ ¢}$.

68. For an ohmic resistor, we have
$P = IV = I^2R$, or $I = (P/R)^{1/2}$.
For the various resistors, we have
$P = 1/8\text{ W}$: $I = [(1/8\text{ W})/(100\ \Omega)]^{1/2} = 0.035\text{ A} = \boxed{35\text{ mA}}$;
$P = 1/4\text{ W}$: $I = [(1/4\text{ W})/(100\ \Omega)]^{1/2} = 0.050\text{ A} = \boxed{50\text{ mA}}$;
$P = 1/2\text{ W}$: $I = [(1/2\text{ W})/(100\ \Omega)]^{1/2} = 0.071\text{ A} = \boxed{71\text{ mA}}$;
$P = 1\text{ W}$: $I = [(1\text{ W})/(100\ \Omega)]^{1/2} = \boxed{0.10\text{ A}}$;
$P = 2\text{ W}$: $I = [(2\text{ W})/(100\ \Omega)]^{1/2} = \boxed{0.14\text{ A}}$.

69. For an ohmic resistor, we have
$P = IV = I^2R$, or $I = (P/R)^{1/2}$.
(a) $I = [(5\text{ W})/(160\ \Omega)]^{1/2} = \boxed{0.18\text{ A}}$.
(b) $I = [(3\text{ W})/(2.5 \times 10^3\ \Omega)]^{1/2} = 3.5 \times 10^{-2}\text{ A} = \boxed{35\text{ mA}}$.

70. $P = IV = (100 \times 10^{-6}\text{ A})(8 \times 10^6\text{ V}) = \boxed{800\text{ W}}$.

71. For an ohmic resistor, we have
$P = IV = V^2/R$, or $V^2 = PR$,
so the maximum allowed operating voltage is
$V_{\max} = \boxed{(PR)^{1/2}}$.

72. Cost = (rate)IV = (7 ¢/kWh)(10 A)(120 V)(10^{-3} kW/W) = $\boxed{8.4\text{ ¢/h}}$.

73. We find the energy dissipated as heat from
$U = Pt = I^2Rt = (100\text{ A})^2(9 \times 10^{-4}\ \Omega)(20\text{ s}) = \boxed{1.9 \times 10^2\text{ J}}$.

74. For an ohmic resistor, we have
$P = IV = V^2/R$, or $V^2 = PR = P\rho L/A$;
$(110\text{ V})^2 = (1250\text{ W})(10^{-6}\ \Omega\cdot\text{m})L/(0.2 \times 10^{-6}\text{m}^2)$, which gives $L = \boxed{1.9\text{ m}}$.

75. For an ohmic resistor, we have
$P = IV = V^2/R$, or $V^2 = PR = P\rho L/A$;
$(12\text{ V})^2 = (0.8 \times 10^3\text{ W})(1.72 \times 10^{-8}\ \Omega\cdot\text{m})L/(8 \times 10^{-6}\text{m}^2)$, which gives $L = \boxed{84\text{ m}}$.

76. (a) For an ohmic resistor, we have
$P = IV$, so the maximum power is
$P_{\text{max}} = I_{\text{max}}V = (15\text{ A})(110\text{ V}) = 1.65 \times 10^3\text{ W} = \boxed{1.65\text{ kW}}$.
(b) We find the maximum number of light bulbs from
$N = P_{\text{max}}/P_{\text{bulb}} = (1.65 \times 10^3\text{ W})/(75\text{ W}) = 22 \rightarrow \boxed{22\text{ bulbs}}$.

77. The energy taken out of the battery is
$U = Pt = IVt = (50 \times 10^{-3}\text{ A})(6\text{ V})(18\text{ h})(3600\text{ s/h}) = \boxed{1.94 \times 10^4\text{ J (5.4 kWh)}}$.

78. Assuming a constant resistance, we have
$P = IV = V^2/R$, or
$P_2/P_1 = (V_2/V_1)^2(R_1/R_2) = (V_2/V_1)^2$;
$P_2/(500\text{ W}) = [(105\text{ V})/(115\text{ V})]^2$, which gives $P_2 = \boxed{417\text{ W}}$.

79. The power dissipated on a wire of surface area A and surface temperature T is
$P = \sigma T^4 A$, where $A = \pi dl$, with l its length and d its cross-sectional diameter.
The current I is related to the power P and the resistance R of the wire as
$P = I^2R$, so
$I = (P/R)^{1/2} = [\sigma T^4(\pi dl)/(\rho l/\pi d^2)]^{1/2} = \text{constant} \times (T^2 d^{3/2})$,
which suggest that I depends more strongly on T (to the 2nd power) than on d (to the $\frac{3}{2}$-th power). So it would be preferable to $\boxed{\text{change the temperature}}$.

80. Because the powers add and the resistors are identical, we have
$P = I^2R_{\text{eq}} = I^2(3R) = 3I^2R = \boxed{3P_0}$.

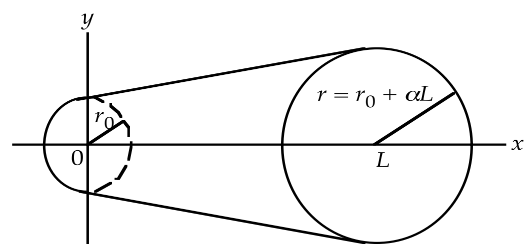
81. If we estimate that it takes 3 min to boil a 0.50-L pot, we have
$mc\,\Delta T = IVt$;
$(500\text{ cm}^3)(1\text{ g/cm}^3)(1\text{ cal/g}\cdot{}^\circ\text{C})(100^\circ\text{C})(4.185\text{ J/cal}) = (4\text{A})V(5\text{ min})(60\text{ s/min})$,
which gives $V \simeq \boxed{170\text{ V}}$.
We find the resistance from
$R = V/I \simeq 170\text{ V}/4\text{ A} \simeq \boxed{43\ \Omega}$.

82. We find the total energy used in the month from
U = \$25.33/(\$0.08/kWh) = 317 kWh.
The average current during the month was
$I = P/V = U/Vt$, so the charge that passed through the meter was
$Q = It = U/V$, and the number of electrons was
$N = Q/e = U/Ve = (317\text{ kWh})(10^3\text{ W/kW})(3600\text{ s/h})/(120\text{ V})(1.6 \times 10^{-19}\text{ C}) = \boxed{5.94 \times 10^{25}\text{ electrons}}$.

83. Because the volume is constant, we have
$A_2L_2 = A_1L_1$.
For a fixed voltage, the power dissipation is
$P = V^2/R = V^2A/\rho L$.
If we apply this to the two wires and divide the two expressions, we get
$$\begin{aligned} P_2/P_1 &= (V^2A_2/\rho L_2)(\rho L_1/V^2A_1) \\ &= (A_2/A_1)(L_1/L_2) = (L_1/L_2)^2 = (1/2)^2 = 1/4. \end{aligned}$$
$\boxed{\text{The power decreases by } 3/4}$.

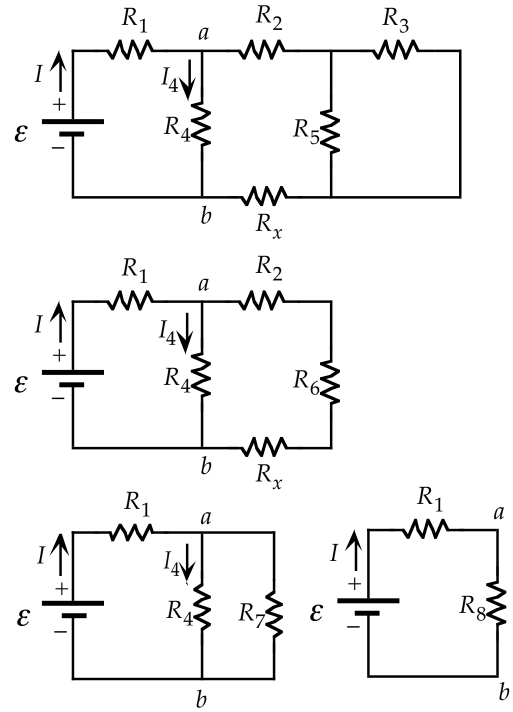
84. (a) We find the number of protons from
$$\begin{aligned} N &= Q/e = It/e \\ &= (5 \times 10^{-6}\text{ A})(1\text{ h})(3600\text{ s/h})/(1.6 \times 10^{-19}\text{ C}) = \boxed{1.1 \times 10^{17}\text{ protons}}. \end{aligned}$$
(b) Because each proton has an energy of 4 MeV, the total energy is
$$\begin{aligned} U_{\text{total}} &= NU \\ &= (1.1 \times 10^{17}\text{ protons})(4 \times 10^6\text{ eV/proton})(1.6 \times 10^{-19}\text{ J/eV}) = \boxed{7.2 \times 10^4\text{ J}}. \end{aligned}$$
(c) We find the power of the beam from
$$\begin{aligned} P &= U_{\text{total}}/t \\ &= (7.2 \times 10^4\text{ J})/(1\text{ h})(3600\text{ s/h}) = \boxed{20\text{ W}}. \end{aligned}$$

85. To find the resistance of the cylinder, we choose a vertical slice at a distance x from the origin, with radius $r = r_0 + \alpha x$ and thickness dx. We find the resistance by integrating over these slices:



$$R = \int \frac{\rho\,\mathrm{d}L}{A} = \int_0^L \frac{\rho\,\mathrm{d}x}{\pi r^2} = \frac{\rho}{\pi}\int_0^L \frac{\mathrm{d}x}{(r_0 + \alpha x)^2}$$
$$= -\frac{\rho}{\pi\alpha}\frac{1}{r_0 + \alpha x}\Big|_0^L = -\frac{\rho}{\pi\alpha}\left(\frac{1}{r_0 + \alpha L} - \frac{1}{r_0}\right) = \frac{\rho L}{\pi r_0 (r_0 + \alpha L)}.$$

86. (a) When the bulbs are connected in series, the equivalent resistance is
$R_{\text{series}} = \sum R_i = 10R_{\text{bulb}}$.
The power consumption is
$P = V_{ab}{}^2/R_{\text{eq}}$;
$50\text{ W} = (120\text{ V})^2/(10R_{\text{bulb}})$, which gives $R_{\text{bulb}} = \boxed{28.8\ \Omega}$.
(b) In part (a), the power consumption of each bulb is 5 W, which is the maximum power rating, so the voltage across each bulb, 12 V, must be the maximum allowed. With the limiting resistor connected in series with the parallel bulb combination, we find the equivalent resistance of the ten bulbs from
$1/R_{\text{parallel}} = \sum(1/R_i) = 10/R_{\text{bulb}} = 10/(28.8\ \Omega)$, which gives $R_{\text{parallel}} = 2.88\ \Omega$.
For the maximum consumption, the voltage across each bulb, and thus R_{parallel}, is 12 V, so we have
$V_{\text{parallel}} = I_{\text{total}}R_{\text{parallel}}$;
$12\text{ V} = I_{\text{total}}(2.88\ \Omega)$, which gives $I_{\text{total}} = 4.17$ A.
With the series resistor, we have
$V_{ab} = I_{\text{total}}(R_{\text{parallel}} + R_2)$;
$120\text{ V} = (4.17\text{ A})(2.88\ \Omega + R_2)$, which gives $R_2 = \boxed{25.9\ \Omega\ \text{ in series}}$.
The power loss in the added resistor is
$P_2 = I_{\text{total}}{}^2R = (4.17\text{ A})^2(25.9\ \Omega) = \boxed{4.5 \times 10^2\text{ W}}$.

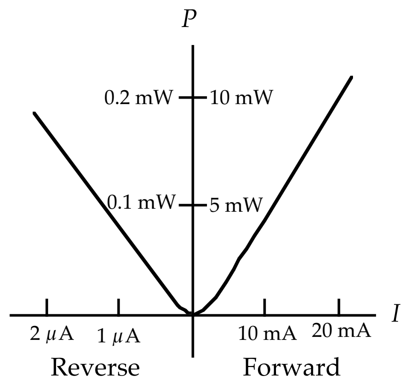


87. We can reduce the circuit to a single loop by successively combining parallel and series combinations.
We combine R_3 and R_5, which are in parallel:
$$1/R_6 = 1/R_3 + 1/R_5 = 1/(3\ \Omega) + 1/(6\ \Omega),$$
which gives $R_6 = 2\ \Omega$.
We combine R_2, R_6, and R_x, which are in series:
$$R_7 = R_2 + R_6 + R_x = 2\ \Omega + 2\ \Omega + R_x = 4\ \Omega + R_x.$$
We combine R_4 and R_7, which are in parallel:
$$1/R_8 = 1/R_4 + 1/R_7 = 1/(4\ \Omega) + 1/(4\ \Omega + R_x),$$
which gives $R_8 = (16\ \Omega + 4R_x)/(8\ \Omega + R_x)$.
We find the current in the single loop from
$$I = \mathcal{E}/(R_1 + R_8)$$
$$= (24\ \text{V})/[1\ \Omega + (16\ \Omega + 4R_x)/(8\ \Omega + R_x)]$$
$$= \boxed{[24(8\ \Omega + R_x)/(24\ \Omega + 5R_x)]\ \text{A}}.$$
We use the voltage across R_8 and across R_4:
$$V_{ab} = IR_8 = I_4R_4;$$
$$[24\ \text{A}\ (8\ \Omega + Rx)/(24\ \Omega + 5R_x)] \times [(16\ \Omega + 4R_x)/(8\ \Omega + R_x)] = I_4(4\ \Omega),$$
which gives
$$I_4 = [24\ (4\ \Omega + R_x)/(24\ \Omega + 5R_x)]\ \text{A}.$$
The power dissipated is
$$P_4 = I_4^2R_4$$
$$= [24(4\ \Omega + R_x)/(24\ \Omega + 5R_x)]^2(4\ \Omega)$$
$$= \boxed{[48(4\ \Omega + R_x)/(24\ \Omega + 5R_x)]^2\ \text{W}}.$$

88. The power dissipated in the bus bar is
$$P = I^2R = I^2\rho L/A = I^2\rho_0(1 + \alpha\,\Delta T)L/A;$$
$$0.2\ \text{W} = (100\ \text{A})^2(1.72 \times 10^{-8}\ \Omega\cdot\text{m})[1 + (0.0039\ /°\text{C})(300\ \text{K} - 20\ \text{K})](0.25\ \text{m})/A,$$
which gives $A = \boxed{4.5 \times 10^{-4}\ \text{m}^2} \simeq 1.5\ \text{cm} \times 3.0\ \text{cm}$.

89. The power delivered by the generator is
$$P = IV = (75\ \text{A})(12\ \text{V}) = 9.0 \times 10^2\ \text{W} = \boxed{0.9\ \text{kW}}.$$
For the generator to supply the energy required to raise the temperature of the water, we have
$$mc\,\Delta T = Pt_1;$$
$$(10^{-3}\,\text{m}^3)(10^2\ \text{cm/m})^3(1\ \text{g/cm}^3)(1\ \text{cal/g}\cdot°\text{C})(7.5°\text{C})(4.185\ \text{J/cal}) = (9.0 \times 10^2\ \text{W})t_1,$$
which gives $t_1 = \boxed{35\ \text{s}}$.
The mass of 0.5 L of water is $m = (0.1 \times 10^3\ \text{cm}^3)(1\ \text{g/cm}^3) = 500\ \text{g}$. For the water to boil away, we have
$$mc\,\Delta T + mL_v = Pt_2;$$
$$(500\ \text{g})(1\ \text{cal/g}\cdot°\text{C})(75°\text{C})(4.185\ \text{J/cal}) + (500\ \text{g})[(1\ \text{mol})/(18\ \text{g})](41 \times 10^3\ \text{J/mol}) = (9.0 \times 10^2\ \text{W})t_2,$$
which gives $t_2 = 1.4 \times 10^3\ \text{s} = \boxed{23\ \text{min}}$.

90.



We find the power from $P = IV$, estimating values from the plot. Note the change in scales on the two sides of the plot.
In the ideal diode, either the potential or the current is zero, so the power is zero.

91. (a) If the resistivity of the wire is ρ_0 at T_0, the resistance of the wire is
$R = \rho L/A = (\rho_0 L/A)(1 + \alpha\,\Delta T) = \boxed{(\rho_0 L/A)(1 + \alpha k t^2)}$.
For a constant potential, the current is
$I = V/R = VA/\rho_0 L(1 + \alpha k t^2) = I_0/(1 + \alpha k t^2)$.
(b) The power dissipated in the wire is
$P = IV = \boxed{VI_0/(1 + \alpha k t^2)}$.
(c) The rate at which the dissipated power changes is
$\mathrm{d}P/\mathrm{d}t = VI_0(-2\alpha k t)/(1 + \alpha k t^2)^2 = -2VI_0\alpha k t/(1 + \alpha k t)^2$.
Because $\mathrm{d}P/\mathrm{d}t < 0$, thermal equilibrium will be reached.

92. The resistance of the wire is $R = \rho L/A$, and the current is $I = JA = nev_dA$. The power dissipated, which becomes thermal energy, is
$P = I^2R = (nev_dA)^2\rho L/A = (nev_d)^2A\rho L$
$= [(8.5 \times 10^{28}\ \text{electrons/m}^3)(1.6 \times 10^{-19}\ \text{C})(1.2 \times 10^{-5}\text{m/s})]^2[\pi(0.5 \times 10^{-3}\ \text{m})^2](1.72 \times 10^{-8}\ \Omega\cdot\text{m})(3\ \text{m})$
$= 1.1 \times 10^{-3}\ \text{W}$.
For the wire to maintain its temperature, thermal energy must be removed at this rate, $\boxed{1.1 \times 10^{-3}\ \text{W}}$.

93. If a coil has length ℓ, the number of turns in the coil is
$N = \ell/2r$, so
$N_2/N_1 = r_1/r_2$.
Because a turn has a length πD, the length of a wire is
$L = N\pi D$, so
$L_2/L_1 = N_2D_2/N_1D_1 = r_1D_2/r_2D_1$.
The resistance of a wire is
$R = \rho L/A$, so we have
$R_2/R_1 = (\rho L_2/\pi r_2^{\,2})/(\rho L_1/\pi r_1^{\,2}) = (r_1D_2/r_2D_1)(r_1/r_2)^2 = (D_2/D_1)(r_1/r_2)^3$
$= [(8\ \text{cm})/(5\ \text{cm})][(0.6\ \text{mm})/(0.4\ \text{mm})]^3 = \boxed{5.4}$.

94. (a) From the conservation of charge, we know that the current must be constant along the wire. Because the area is also constant, we have

$$E = \rho J = \rho_0 J e^{-x/L} = (\rho_0 I / A) e^{-x/L} = \boxed{E_0 e^{-x/L}}.$$

(b) We take the reference level for V to be $V = 0$ at $x = L$, so $V = V_0$ at $x = 0$. We integrate the relation between the field and the potential, $E = -\,\mathrm{d}V/\mathrm{d}x$:

$$\int_{V_0}^{V} \mathrm{d}V = -\int_0^x E\,\mathrm{d}x' = -E_0\int_0^x e^{-x'/L}\,\mathrm{d}x';$$

$$V - V_0 = -E_0(-L)e^{-x'/L}\Big|_0^x = +E_0L\left(e^{-x/L} - 1\right) = -E_0L\left(1 - e^{-x/L}\right).$$

We can determine E_0 from our reference level:

$-V_0 = -E_0L(1 - e^{-1})$, which gives $E_0L = V_0/(1 - e^{-1})$.

The potential is

$$V = V_0 - V_0(1 - e^{-x/L})/(1 - e^{-1}) = \boxed{V_0(e^{-x/L} - e^{-1})/(1 - e^{-1})}.$$

(c) We choose a differential segment of the wire at x with length dx. We find the resistance by integration:

$$R = \int \frac{\rho\,\mathrm{d}L}{A} = \int_0^L \frac{\rho_0 e^{-x/L}\,\mathrm{d}x}{A} = -\frac{\rho_0 L}{A}e^{-x/L}\Big|_0^L$$

$$= -\frac{\rho_0 L}{A}\left(e^{-1} - 1\right) = \frac{\rho_0 L}{A}\left(1 - e^{-1}\right).$$

95. If we ignore the change in the dimensions of the wire, the resistance of the wire will be a function of the temperature:

$$R = R_0(1 + \alpha\,\Delta T) = R_0[1 + \alpha(T - T_0)].$$

Because all of the energy from Joule heating raises the temperature of the wire, we have

$P = I^2R = V^2/R = V^2/R_0[1 + \alpha(T - T_0)] = mc(\mathrm{d}T/\mathrm{d}t)$, which gives

$$\boxed{\mathrm{d}T/\mathrm{d}t = k/[1 + \alpha(T - T_0)],\quad \text{where } k = V^2/mcR_0}.$$

The solution of this equation will give the temperature as a function of time, $T(t)$.

We find the current from

$$I(t) = V/R = \boxed{V/R_0\{1 + \alpha[T(t) - T_0]\}}.$$

CHAPTER 27 Direct-Current Circuits

Answers to Understanding the Concepts Questions

1. Tap water is an excellent conductor, and if the appliance falls into the tub there is a danger that a large current will flow through the body of the person in the tub, causing bums and sometimes heart failure.

2. The voltmeter measures the terminal voltage V across the battery, and that depends on the current I drawn and the internal resistance r of the battery: $V = \mathcal{E} - Ir$. While $\mathcal{E}$ and r do not change appreciably, I can. (Even r can increase, if the battery gets warmer.)

3. The current that flows into the battery is the same as the current flowing out of the battery. Whether there is a potential drop Ir just before the current reaches the battery or whether the drop occurs just after the current leaves the battery is irrelevant, since either way there will be the same contribution to the loop rule, and that is all that counts.

4. It makes sense when the resistance of the wire is negligible in comparison with those of the resistors.

5. This is impossible, since by choosing the direction in which the emf is positive, one could create a situation in which one would be creating energy. Such "perpetual motion" machines violate energy conservation.

6. The battery could be all right, of course. Keep in mind, however, that it has to provide the right terminal voltage when hooked up to a working load, not just a voltmeter with presumably a very large resistance. As the resistance of the load decreases the current flowing in the battery increases, and the terminal voltage, $V = \mathcal{E} - Ir$, could drop appreciably. Here r is the internal resistance of the battery.

7. An unfair question. The circuit in a flash does not give the falling exponential characteristic of a pure RC circuit. Rather it uses solid-state devices to tailor the release of energy from a capacitor, typically of size 1000 μF, so that a current that is basically flat for a period of about 0.01 s results. If we work backwards and ask what value of R in a pure RC circuit would give a time constant $\tau = RC$ of 0.01 s with a capacitor or 1000 μF, we would find $R = \tau/C = (0.01\text{ s})/(10^{-3}\text{ F}) = 10\ \Omega$, a very reasonable value.

8. The time constant of an RC circuit is $\tau = RC$. To make the discharge time as short as possible we need to minimize τ, which means we want the lowest possible value of C out of the three capacitors. This can be done by connecting the three capacitors in series.

9. There is no disaster. With the new choice of positive direction for the current, I is related to Q as $I = -\,dQ/dt$. Thus from $IR - Q/C = 0$ we get $-R(dQ/dt) - Q/C = 0$, which again leads to the finite solution $Q = Q_0\, e^{-t/RC}$.

10. Let's construct a potential energy diagram with a fluid analogy in mind. The batteries "raise" the liquid, increasing its potential energy. A resistance corresponds to a drop of potential energy given by IR. When the current goes through resistors in series, it is as if it cascaded down several downward slopes. If current goes through two resistors in parallel, it splits up so "at the bottom" the two currents are reunited at the same potential. Consider now this diagram turned upside down. The batteries (with reversed polarities) now lower the potential, and IR must raise them. Since R is unchanged by the reversal, I must change sign.

11. The maximum possible emf you can get out of two emf sources is the sum of the two emf values, which is obtained when you connect the two sources in series. To ensure that each light bulb gets that maximum emf we can hook up the light bulbs in parallel and apply the combined emf across each of them simultaneously. This ensures maximum power consumption for each light bulb and therefore maximum brightness — assuming, of course, that the emf does not exceed the maximum value allowed by each light bulb (so none of them would burn out).

12. The effective net emf is the difference between the two emf values: $\mathcal{E}_{\text{eff}} = \mathcal{E}_1 - \mathcal{E}_2$, when they are connected + to + and – to –. The larger emf wins, of course, and the resulting current in the circuit is $I = \mathcal{E}_{\text{eff}}/R_{\text{eq}}$.

13. The effective emf that drives the current in the circuit is now $\mathcal{E}_{\text{eff}} = \mathcal{E}_1 + \mathcal{E}_2$, and the magnitude of the current increases.

14. Technically speaking this is certainly true. However, it is useless information. The value of the current itself depends on the rest of the circuit as well as on the value of the internal resistance, and so does the value of the "shifted" emf This is not a very useful way to think about a circuit. In contrast, the original emf is a constant which at least for an ideal battery does not vary with current.

15. In the circuit diagram depicted in Fig. 27-8(a), R_2 and R_3 are in series, as are R_5 and R_6, and the two branches are in parallel with R_4. The equivalent resistance of this three-branch combination is $R = [1/(R_2 + R_3) + 1/R_4 + 1/(R_5 + R_6)]^{-1}$. Put this in series with R_1 and we get a single-loop circuit. In general, such reduction is not possible for the circuit diagram depicted in Fig. 27-8(b), unless $R_2/R_3 = R_5/R_6$, in which case the voltage difference across R_4 is zero so that it can be removed from the circuit.

16. The teenagers provide a path for the current parallel to the wire. If the wire has no resistor along it, the resistance is low compared to that of the teenager, and most of the current flows through the wire. With the resistor, more of the current flows through the teenager, with more serious consequences. They are both dumb, but the one with the resistor is dumber.

17. The steady-state value of I_1 decreases as R_3 is increased.

18. When the lights are on, a current runs through the battery that powers the lights. Heat is generated as the current flows through the battery due to its internal resistance, and the battery warms up.

Solutions to Problems

1. Because the internal resistance of the battery is the only resistance in the single-loop circuit, we have
   $I = \mathcal{E}/r$;
   $80\ \text{A} = (12\ \text{V})/r$, which gives $r = \boxed{0.15\ \Omega}$.

2. The solar panel is a source of emf, so the power output is
   $P = V_{\text{term}}\, I$;
   $1200\ \text{W} = V_{\text{term}}(40\ \text{A})$, which gives $V_{\text{term}} = \boxed{30\ \text{V}}$.

3. For this single-loop circuit, we have
   $I = \mathcal{E}/(R + r)$;
   $1.99\ \text{A} = (3\ \text{V})/[(1.50\ \Omega + r)]$, which gives $r = \boxed{0.0075\ \Omega}$.

4. The energy contained in the battery is the total energy output:
   $U = IVt = (It)V = (30\ \text{A}\cdot\text{h})(3600\ \text{s/h})(30\ \text{V}) = \boxed{3.2 \times 10^6\ \text{J}}$.

5. For this single-loop circuit, we have
   $I = V_{\text{term}}/(R + r) = (5000\ \text{V})/(230\ \Omega + 20\ \Omega) = \boxed{20\ \text{A}}$.

6. For this single-loop circuit, we have
   $I = \mathcal{E}/(R + r)$;
   $170 \times 10^{-3}\ \text{A} = \mathcal{E}/(15\ \Omega + 0.06\ \Omega)$, which gives $\mathcal{E} = \boxed{2.56\ \text{V}}$.
   The terminal voltage of the battery is
   $V = \mathcal{E} - Ir = 2.56\ \text{V} - (170 \times 10^{-3}\ \text{A})(0.06\ \Omega) = \boxed{2.55\ \text{V}}$.

$\boxed{7}$. The terminal voltage of the battery is
   $V = \mathcal{E} - Ir$;
   $9.0\ \text{V} = 12\ \text{V} - (100\ \text{A})r$, which gives $r = \boxed{0.030\ \Omega}$.
   The power dissipated within the battery is
   $P = I^2 r = (100\ \text{A})^2(0.030\ \Omega) = \boxed{300\ \text{W}}$.

8. The terminal voltage of the battery is the voltage drop across the starter:
   $V_{\text{term}} = \mathcal{E} - Ir = IR$;
   $8\ \text{V} = 12\ \text{V} - Ir = I(0.11\ \Omega)$, which gives $I = 73\ \text{A}$, and $r = 0.05\Omega$.
   The rate at which heat is produced is the power dissipated within the battery. For 10 s we have
   $W = I^2 rt = (73\ \text{A})^2(0.05\ \Omega)(10\ \text{s})/(4.185\ \text{J/cal}) = 0.6 \times 10^3\ \text{cal} \simeq \boxed{0.6\ \text{kcal}}$.
   This would raise the temperature of a liter of electrolyte (water) by $\simeq 0.6°\text{C}$, which may decrease the internal resistance slightly. It is more important to raise the temperature of the oil.

9. The terminal voltage of the battery is
   $V_{\text{term}} = \mathcal{E} - Ir = \mathcal{E} - I(\alpha + \beta I)$.
   The power dissipated within the battery is
   $P = I^2 r = I^2(\alpha + \beta I)$.
   For $I = 1.0$ A, we have
   $V_1 = (12.0\ \text{V}) - (1.0\ \text{A})[0.15\ \Omega + (0.018\ \Omega/\text{A})(1.0\ \text{A})] = \boxed{11.8\ \text{V}}$.
   $P_1 = (1.0\ \text{A})^2[0.15\ \Omega + (0.018\ \Omega/\text{A})(1.0\ \text{A})] = \boxed{0.17\ \text{W}}$.
   For $I = 10.0$ A, we have
   $V_{10} = (12.0\ \text{V}) - (10.0\ \text{A})[0.15\ \Omega + (0.018\ \Omega/\text{A})(10.0\ \text{A})] = \boxed{8.7\ \text{V}}$.
   $P_{10} = (10.0\ \text{A})^2[0.15\ \Omega + (0.018\ \Omega/\text{A})(10.0\ \text{A})] = \boxed{33\ \text{W}}$.

10. The resistance of each bulb can be found from its rating: $R_1 = 2.5\text{ V}/0.5\text{ A} = 5.0\ \Omega$, and $R_2 = (110\text{ V})^2/10\text{ W} = 1.21\text{ k}\Omega$ (as $P_2 = V_2^2/R_2$). When connected in series the equivalent resistance of the two bulbs is $R = R_1 + R_2$, and when hooked up to a power supply of $\mathcal{E} = 110\text{ V}$ the current through each bulb is

$I = \mathcal{E}/(R_1 + R_2) = 110\text{ V}/(5.0\ \Omega + 1.21\text{ k}\Omega) = 0.0905\text{ A}$. The power consumed on each bulb is then

$P_1 = I^2 R_1 = (0.0905\text{ A})^2(5.0\ \Omega) = \boxed{0.041\text{ W}}$ and

$P_2 = I^2 R_2 = (0.0905\text{ A})^2(1.21\text{ k}\Omega) = \boxed{9.9\text{ W}}$.

11. If we go around the single loop in the direction shown, starting at point a, we have

$\sum \Delta V = -IR_2 + \mathcal{E} - IR_1 + \mathcal{E} = 0$, which gives

$I = 2\mathcal{E}/(R_1 + R_2)$.

If we go from a to b, we have

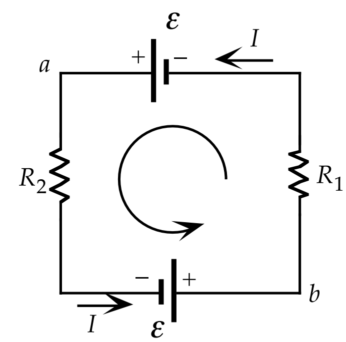
$V_a - IR_2 + \mathcal{E} = V_b = V_a$, or $\mathcal{E} = IR_2$.

When we combine this with the expression for the current, we get

$\mathcal{E} = 2\mathcal{E}R_2/(R_1 + R_2)$, which gives $\boxed{R_2 = R_1}$.

From the expression for the current, we see that

$I \to 0$ when $\boxed{R_2 \to \infty}$.



12. (a) Without the series resistor in the single-loop circuit, we have

$I = (\mathcal{E}_{\text{gen}} - N\mathcal{E}_{\text{batt}})/(r_{\text{gen}} + Nr_{\text{batt}})$

$= [(110\text{ V}) - 20(2.2\text{ V})]/[(0.50\ \Omega) + 20(0.06\ \Omega)] = 38.8\text{ A}$.

The terminal voltage of the generator is

$V_a - V_b = \mathcal{E}_{\text{gen}} - Ir_{\text{gen}} = (110\text{ V}) - (38.8\text{ A})(0.50\ \Omega) = \boxed{91\text{ V}}$.

(b) The terminal voltage of the bank of batteries is $V_d - V_c$.

Because the batteries are being charged, this must be greater than the total emf of the bank:

$V_d - V_c = N(\mathcal{E}_{\text{batt}} + Ir_{\text{batt}})$

$= 20[2.2\text{ V} + (38.8\text{ A})(0.06\ \Omega)] = \boxed{91\text{ V}}$.

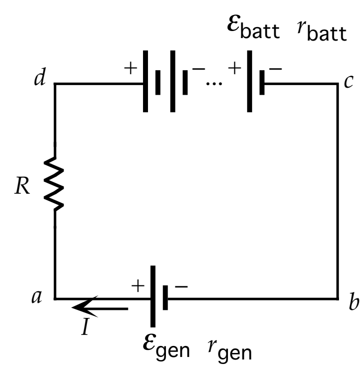
(c) With the series resistor in the single-loop circuit, we have

$I = (\mathcal{E}_{\text{gen}} - N\mathcal{E}_{\text{batt}})/(r_{\text{gen}} + Nr_{\text{batt}} + R)$;

$15\text{ A} = [110\text{ V} - 20(2.2\text{ V})]/[0.50\ \Omega + 20(0.06\ \Omega) + R]$, which gives $R = \boxed{2.7\ \Omega}$.

(d) The power dissipated in all the resistors is

$P = I^2(\sum R) = (15\text{ A})^2[0.50\ \Omega + 2.7\ \Omega + 20(0.06\ \Omega)] = \boxed{9.9 \times 10^2\text{ W}}$.



13. Because there are no resistors between points f and g, they must be at the same potential:

$V_{gf} = \boxed{0}$.

If we choose a path between two points, the potential difference is the sum of the potential differences of the segments of the path. Thus, we have

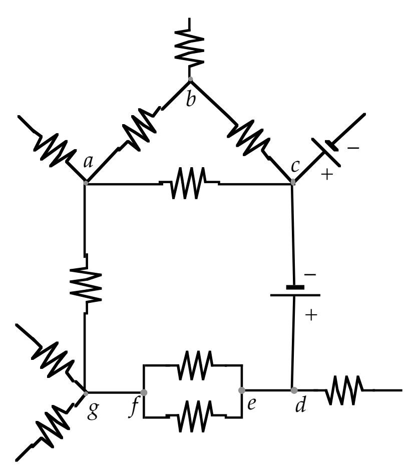
$V_{ag} = V_{af} = V_{ab} + V_{bc} + V_{cd} + V_{df}$

$= -V_{ba} - V_{cb} + V_{cd} + V_{df}$

$= -2\text{ V} - 3.5\text{ V} + 2\text{ V} - 0.5\text{ V} = \boxed{-4.0\text{ V}}$;

$V_{ca} = V_{cb} + V_{ba}$

$= 3.5\text{ V} + 2\text{ V} = \boxed{5.5\text{ V}}$.



14. (a) If we assume initially there is no internal resistance, we have

$I_0 = 2\mathcal{E}/R = 2(1.5\text{ V})/(10\ \Omega) = 0.30\text{ A}.$

The power delivered to the bulb is the power dissipated in the bulb:

$P_0 = I_0^2 R = (0.30\text{ A})^2(10\ \Omega) = \boxed{0.90\text{ W}}.$

(b) If the power delivered to the bulb, which is also the power dissipated in the bulb, decreases by one-third, we have

$P = I^2R;$

$\frac{2}{3}(0.90\text{ W}) = I^2(10\ \Omega)$, which gives $I = 0.25$ A.

For the single-loop circuit, we have

$I = 2\mathcal{E}/(R + 2r);$

$0.25\text{ A} = 2(1.5\text{ V})/[(10\ \Omega) + 2r]$, which gives $r = \boxed{1.0\ \Omega/\text{battery}}.$

15. For the conservation of current, we have

$\sum I_{\text{in}} = 0;$

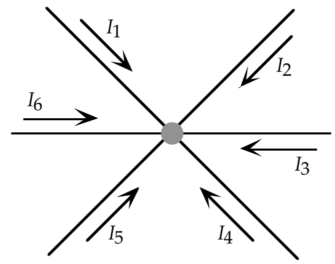
$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = 0;$

$(2\text{ A}) + (0.5\text{ A}) - (3\text{ A}) - 0.5I_6 - I_6 + I_6 = 0,$

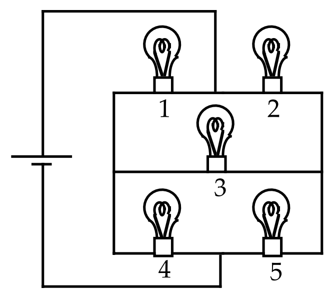
which gives $I_6 = -1.0$ A.

Thus we have

$I_4 = \boxed{+0.5\text{ A}},\ I_5 = \boxed{+1.0\text{ A}},\ I_6 = \boxed{-1.0\text{ A}}.$



16. From symmetry, the current will be the same in bulbs 1, 2, 4, and 5. Thus there will be no current through bulb 3, which will have $\boxed{\text{zero brightness}}$.



17. (a) With the switch open, the circuit consists of two branches in parallel, one with a resistance of $R_1 = (4\ \Omega + 12\ \Omega) = 16\ \Omega$, and the other with $R_2 = (8\ \Omega + 6\ \Omega) = 14\ \Omega$. The equivalent resistance is then

$R_{\text{eq}} = R_1R_2/(R_1 + R_2) = (16\ \Omega)(14\ \Omega)/(16\ \Omega + 14\ \Omega) = \boxed{7.5\ \Omega}.$

(b) We assume the current directions shown in the diagram.

We use conservation of current at points a and b:

$\sum I_{\text{in}} = 0;$

$I_1 - I_3 - I_4 = 0;\ \ I_2 + I_3 - I_5 = 0.$

We apply the loop rule for the two loops indicated in the diagram:

loop 1: $I_1(4\ \Omega) - I_2(8\ \Omega) + I_3(5\ \Omega) = 0;$

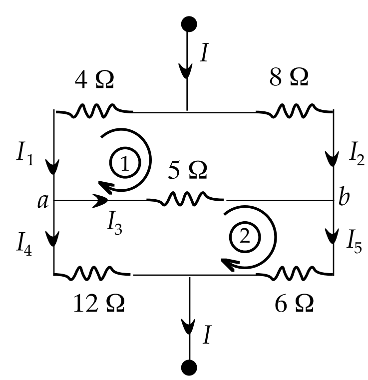
loop 2: $-I_3(5\ \Omega) - I_5(6\ \Omega) + I_4(12\ \Omega) = 0.$

Also, when an emf $\mathcal{E}$ is applied across the top and bottom of the circuit

$\mathcal{E} = I_1(4\ \Omega) + I_4(12\ \Omega) = I_2(8\ \Omega) + I_5(6\ \Omega).$

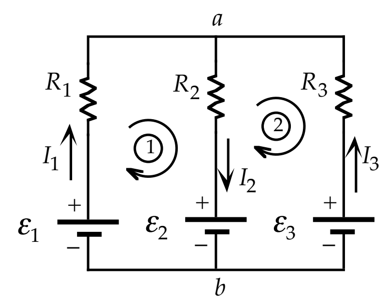
These equations yield $I_1 = 0.084\mathcal{E}/\Omega$ and $I_2 = 0.059\mathcal{E}/\Omega$. Thus

$R_{\text{eq}} = \mathcal{E}/(I_1 + I_2) = \mathcal{E}/(0.084\mathcal{E}/\Omega + 0.059\mathcal{E}/\Omega) = \boxed{7.0\ \Omega}.$



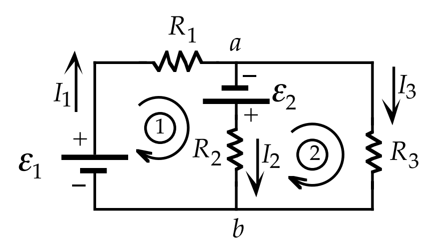
18. Let the current in the 8-Ω resistor be I_1, to the right, and that in the 10-Ω resistor be I_2, to the right. Then the current in the remaining two resistors is $I_1 + I_2$, to the left.
Apply the loop rule to the loop containing I_1 and I_2:
$$-I_1(8\ \Omega) - 6\ \text{V} + I_2(10\ \Omega) - 10\ \text{V} = 0.$$
Now apply the loop rule to the circumference of the circuit, starting clockwisely from point *a*:
$$-I_1(8\ \Omega) - 6\ \text{V} - (I_1 + I_2)(4\ \Omega) + 20\ \text{V} - (I_1 + I_2)(16\ \Omega) = 0.$$
Solve the two equations above to obtain
$I_1 = -0.409$ A, $I_2 = +1.27$ A. The current in the 16-Ω resistor is then
$$I_1 + I_2 = -0.409\ \text{A} + 1.27\ \text{A} = \boxed{+0.86\ \text{A, to the left}}.$$
The voltage difference between *a* and *b* is
$$V_b - V_a = 10\ \text{V} - (1.27\ \text{A})(10\ \Omega) = \boxed{-2.7\ \text{V}},\ \text{with } V_b < V_a.$$

$\boxed{19}$. We assume the current directions shown in the diagram.



We use conservation of current at point *a*:
$$\sum I_{\text{in}} = 0;$$
$$I_1 - I_2 + I_3 = 0.$$
We apply the loop rule for the two loops indicated in the diagram:
loop 1: $-I_2R_2 - \mathcal{E}_2 + \mathcal{E}_1 - I_1R_1 = 0;$
$-I_2(10\ \Omega) - 6\ \text{V} + 12\ \text{V} - I_1(5\ \Omega) = 0;$
loop 2: $+I_3R_3 - \mathcal{E}_3 + I_2R_2 + \mathcal{E}_2 = 0;$
$+I_3(12\ \Omega) - 9\ \text{V} + I_2(10\ \Omega) + 6\ \text{V} = 0.$
When we combine these equations, we get $I_1 = \boxed{+0.45\ \text{A}}$, $I_2 = \boxed{+0.38\ \text{A}}$, $I_3 = \boxed{-0.068\ \text{A}}$.

20. We assume the current directions shown in the diagram.



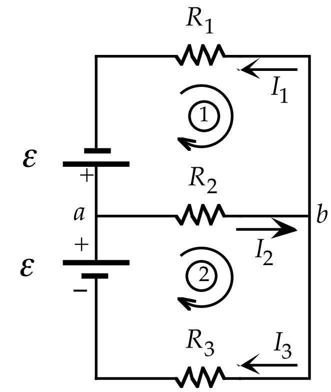
We use conservation of current at point *a*:
$$\sum I_{\text{in}} = 0;$$
$$I_1 - I_2 - I_3 = 0.$$
We apply the loop rule for the two loops indicated in the diagram:
loop 1: $\mathcal{E}_2 - I_2R_2 + \mathcal{E}_1 - I_1R_1 = 0;$
$5\ \text{V} - I_2(3\ \Omega) + 3\ \text{V} - I_1(2\ \Omega) = 0;$
loop 2: $-I_3R_3 + I_2R_2 - \mathcal{E}_2 = 0;$
$-I_3(4\ \Omega) + I_2(3\ \Omega) - 5\ \text{V} = 0.$
When we combine these equations, we get $I_1 = +1.577$ A, $I_2 = +1.616$ A, and
$I_3 = \boxed{-0.039\ \text{A}}$. The negative sign means that the current is up.

21. We combine the three resistors, which are in parallel:
$$1/R_{\text{eq}} = 1/R_1 + 1/R_2 + 1/R_3$$
$$= 1/(250\ \Omega) + 1/(420\ \Omega) + 1/(510\ \Omega),\ \text{which gives } R_{\text{eq}} = 120\ \Omega.$$
The potential difference across the equivalent resistance is
$$V_{ab} = IR_{\text{eq}} = (0.020\ \text{A})(120\ \Omega) = \boxed{2.4\ \text{V}}.$$
Because this is the potential difference across each resistor, we have
$$I_1 = V_{ab}/R_1 = (2.4\ \text{V})/(250\ \Omega) = 9.6 \times 10^{-3}\ \text{A} = \boxed{9.6\ \text{mA}};$$
$$I_2 = V_{ab}/R_2 = (2.4\ \text{V})/(420\ \Omega) = 5.7 \times 10^{-3}\ \text{A} = \boxed{5.7\ \text{mA}};$$
$$I_3 = V_{ab}/R_3 = (2.4\ \text{V})/(510\ \Omega) = 4.7 \times 10^{-3}\ \text{A} = \boxed{4.7\ \text{mA}}.$$
Note that we have conservation of current:
$$I = I_1 + I_2 + I_3 = (9.6 \times 10^{-3}\ \text{A}) + (5.7 \times 10^{-3}\ \text{A}) + (4.7 \times 10^{-3}\ \text{A}) = 0.020\ \text{A}.$$

22. Because no two resistors have the same current and no two resistors have the same potential difference across them, we cannot combine them in series or parallel.
We assume the current directions shown in the diagram.
We use conservation of current at point a:

$\sum I_{\text{in}} = 0;$

$I_1 - I_2 + I_3 = 0.$



We apply the loop rule for the two loops indicated in the diagram:

loop 1: $-\mathcal{E} + I_1R_1 + I_2R_2 = 0;$

loop 2: $-I_2R_2 - I_3R_3 + \mathcal{E} = 0.$

When we combine these equations, we get

$$\boxed{\begin{aligned} I_1 &= R_3\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3), \\ I_2 &= (R_1 + R_3)\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3), \\ I_3 &= R_1\mathcal{E}/(R_1R_2 + R_1R_3 + R_2R_3). \end{aligned}}$$

23. With identical batteries, the terminal voltage and the current through each battery are the same. When the batteries are connected in parallel, the terminal voltage is the voltage across the resistance, so we have

$\sum I_i = NI_i = I_a;$

$V_{ab} = \mathcal{E} - I_ir = I_aR.$

If we eliminate I_i from these equations, we get

$I_a = \boxed{\mathcal{E}/[R + (r/N)]}.$

When the batteries are connected in series, the current through each battery is the current through the resistance, so we have

$\sum V_i = NV_i = V_{cd};$ $(\mathcal{E} - I_br) = I_bR,$ which gives

$I_b = \boxed{\mathcal{E}/[r + (R/N)]}.$

In general, R will be much greater than r, so I_b will be greater than I_a.

24. We assume that the current going to point C is negligible. If I is the current through the resistors, which are in series, we have

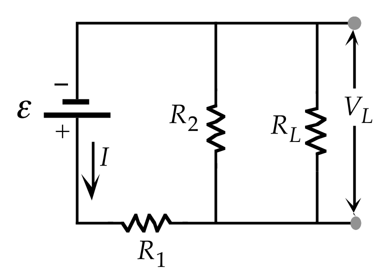
$V_{AB} = I(R_1 + R_2)$, and $V_{CD} = IR_2$.

If we eliminate I, we get

$V_{CD} = \boxed{[R_2/(R_1 + R_2)]V_{AB}}.$

25. We combine R_2 and R_L, which are in parallel:

$1/R = 1/R_2 + 1/R_L$, which gives $R = R_2R_L/(R_2 + R_L)$.



We now have a single-loop circuit, so the current is

$I = \mathcal{E}/(R_1 + R).$

The voltage across the load is the voltage across R:

$V_L = IR = \mathcal{E}R/(R_1 + R).$

When we use the expression for R, we get

$$\begin{aligned} V_L &= \mathcal{E}R_2R_L/[(R_1R_2 + R_1R_L + R_2R_L)] \\ &= (10\text{ V})(3.3\text{ k}\Omega)R_L/[(3.3\text{ k}\Omega)(3.3\text{ k}\Omega) + (3.3\text{ k}\Omega)R_L + (3.3\text{ k}\Omega)R_L] \\ &= 33R_L/(10.9 + 6.6R_L)\text{ V, with } R_L \text{ in k}\Omega. \end{aligned}$$

For the given loads, we have

$V_{20\text{ k}\Omega} = (33)(20\text{ k}\Omega)/[10.9 + 6.6(20\text{ k}\Omega)] = 4.62\text{ V},\quad \Delta V = \boxed{0.38\text{ V}};$

$V_{200\text{ k}\Omega} = (33)(200\text{ k}\Omega)/[10.9 + 6.6(200\text{ k}\Omega)] = 4.96\text{ V},\quad \Delta V = \boxed{0.04\text{ V}};$

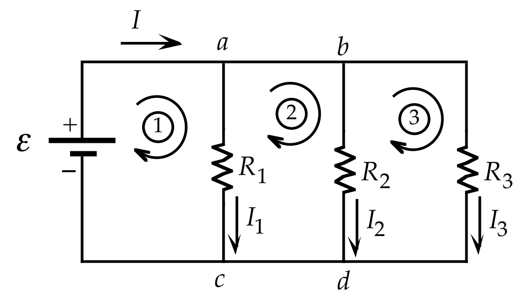
$V_{2\text{ M}\Omega} = (33)(2 \times 10^3\text{ k}\Omega)/[10.9 + 6.6(2 \times 10^3\text{ k}\Omega)] = 4.996\text{ V},\quad \Delta V = \boxed{0.004\text{ V}}.$

26. From the diagram, we have

$I_1 = V_{ac}/R_1 = \mathcal{E}/R;$

$55 \times 10^{-3}\ \text{A} = (2.8\ \text{V})/R,$

which gives $R = \boxed{51\ \Omega}.$



27. We can consider point a to be along the top and point b to be along the bottom, so the conservation of current gives

$\sum I_{\text{in}} = 0;$

junction a: $I_1 - I_2 - I_3 - I_4 = 0;$

junction b: $I_2 + I_3 + I_4 - I_1 = 0.$

Thus there is only $\boxed{\text{one independent junction}}$.

For the three loops indicated on the diagram, we have

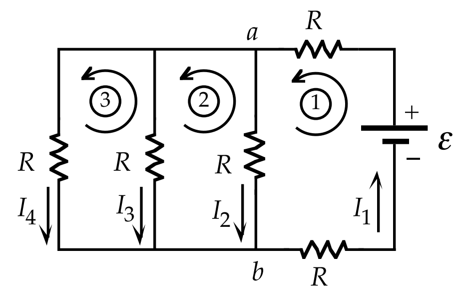
loop 1: $-I_1R - I_2R - I_1R + \mathcal{E} = 0;$

loop 2: $+I_2R - I_3R = 0;$

loop 3: $+I_3R - I_4R = 0.$

The solution of these four equations gives

$I_1 = \boxed{3\mathcal{E}/7R}, I_2 = I_3 = I_4 = \boxed{\mathcal{E}/7R}.$



28. For the conservation of current, we have

junction a: $I - I_1 - I_2 = 0;$

junction b: $I_1 - I_3 - I_4 = 0;$

junction d: $I_4 + I_5 - I = 0.$

For the three loops indicated on the diagram, we have

loop 1: $-I_1R - I_4R + \mathcal{E} = 0;$

loop 2: $-I_2R + I_3R + I_1R = 0;$

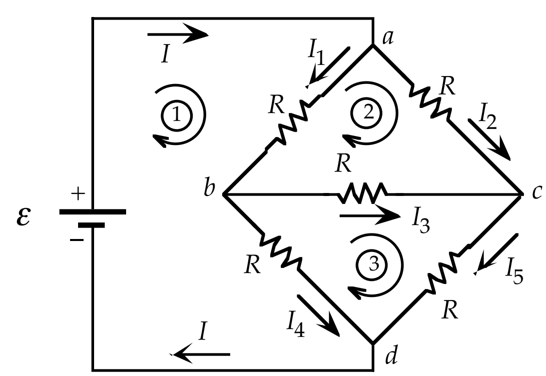
loop 3: $-I_3R - I_5R + I_4R = 0.$

When we solve these equations, we get

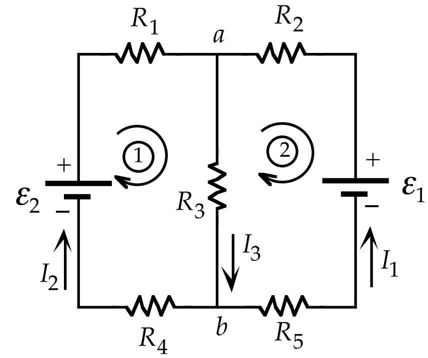
$I_3 = 0,\ I_1 = I_2 = I_4 = I_5 = \mathcal{E}/2R,\ I = \mathcal{E}/R.$

Because the current through the battery must be

$I = \mathcal{E}/R_{\text{eq}}$, we get $\boxed{R_{\text{eq}} = R}.$



From the symmetry of the resistance network, we know that there can be no current through the central resistor, which could be removed. Then we would have two pairs of series resistors in parallel, which gives $R_{\text{eq}} = R$.



$\boxed{29}$. (a) For the conservation of current at point a, we have

$\sum I_{\text{in}} = 0;$

$I_1 + I_2 - I_3 = 0.$

For the two loops indicated on the diagram, we have

loop 1: $\mathcal{E}_2 - I_2R_1 - I_3R_3 - I_2R_4 = 0;$

$+ 9\text{ V} - I_2(100\ \Omega) - I_3(50\ \Omega) - I_2(200\ \Omega) = 0;$

loop 2: $\mathcal{E}_1 - I_1R_2 - I_3R_3 - I_2R_5 = 0;$

$+ 6\text{ V} - I_1(150\ \Omega) - I_3(50\ \Omega) - I_1(250\ \Omega) = 0.$

When we solve these equations, we get

$I_1 = 0.0106\text{ A},\ I_2 = 0.0242\text{ A},\ I_3 = 0.0348\text{ A}.$

The power dissipated in the 50-Ω resistor is

$P_3 = I_3{}^2R_3 = (0.348\text{ A})^2(50\ \Omega) = 0.0605\text{ W} = \boxed{60.5\text{ mW}}.$

(b) If the terminals on the 6-V battery are reversed, we have the same equations, except for the sign of $\mathcal{E}_1$:

$I_1 + I_2 - I_3 = 0.$

loop 1: $\mathcal{E}_2 - I_2R_1 - I_3R_3 - I_2R_4 = 0;$

$+ 9\text{ V} - I_2(100\ \Omega) - I_3(50\ \Omega) - I_2(200\ \Omega) = 0;$

loop 2: $- \mathcal{E}_1 - I_1R_2 - I_3R_3 - I_2R_5 = 0;$

$- 6\text{ V} - I_1(150\ \Omega) - I_3(50\ \Omega) - I_1(250\ \Omega) = 0.$

When we solve these equations, we get

$I_1 = - 0.0165\text{ A},\ I_2 = 0.0281\text{ A},\ I_3 = 0.0116\text{ A}.$

The power dissipated in the 50-Ω resistor is

$P_3 = I_3{}^2R_3 = (0.0116\text{ A})^2(50\ \Omega) = 0.0068\text{ W} = \boxed{6.8\text{ mW}}.$

30. (a) The total resistance of the circuit is $R = 20(2\ \Omega) + 80\ \Omega = 120\ \Omega$, while the total emf is

$\mathcal{E} = 20(12\text{ V}) = 240\text{ V}$. The current is then

$I = \mathcal{E}/R = 240\text{ V}/120\ \Omega = \boxed{2.0\text{ A}}.$

(b) Now the total resistance of the circuit is

$R = (2\ \Omega)/20 + 20\ \Omega = 20.1\ \Omega$, while the total emf is

$\mathcal{E} = 12\text{ V}$. The current is then

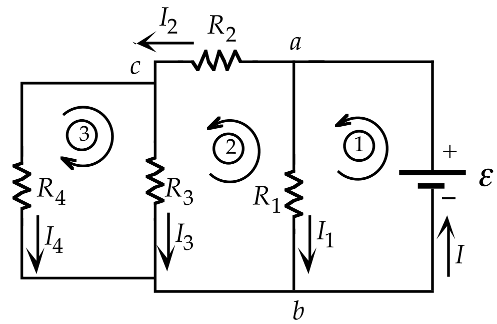
$I = \mathcal{E}/R = 12\text{ V}\ /20.1\ \Omega = \boxed{0.60\text{ A}}.$

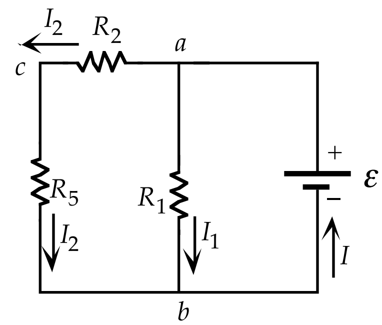
(c) Now the total resistance of the circuit is

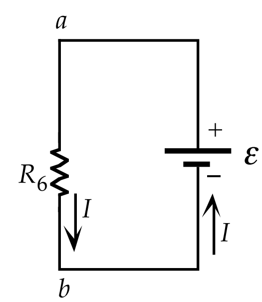
$R = 5(2\ \Omega)/4 + 40\ \Omega = 42.5\ \Omega$, while the total emf is

$\mathcal{E} = 5(12\text{ V}) = 60\text{ V}$. The current is then

$I = \mathcal{E}/R = 60\text{ V}\ /42.5\ \Omega = \boxed{1.4\text{ A}}.$







31. (a) We can reduce the circuit to a single loop by successively combining parallel and series combinations.

We combine R_3 and R_4, which are in parallel:

$$1/R_5 = 1/R_3 + 1/R_4$$
$$= 1/(100\ \Omega) + 1/(50\ \Omega),$$

which gives $R_5 = 33.3\ \Omega$.

We combine R_1 and $R_2 + R_5$, which are in parallel:

$$1/R_6 = 1/R_1 + 1/(R_2 + R_5)$$
$$= 1/(100\ \Omega) + 1/(20\ \Omega + 33.3\ \Omega),$$

which gives $R_6 = 34.8\ \Omega$.

Because $V_{ab} = 6$ V, we find I_2 from

$$I_2 = V_{ab}/(R_5 + R_2) = (6\text{ V})/(33.3\ \Omega + (0\ \Omega) = 0.113\text{ A}.$$

We can now find V_{cb} from

$$V_{cb} = I_2 R_5 = (0.113\text{ A})(33.3\ \Omega) = 3.75\text{ V}.$$

The current through the 50-Ω resistor is

$$I_4 = V_{cb}/R_4 = (3.75\text{ V})/(50\ \Omega) = \boxed{0.075\text{ A}}.$$

(b) For the conservation of current, we have

junction a: $I - I_1 - I_2 = 0$;

junction c: $I_2 - I_3 - I_4 = 0$.

For the three loops indicated on the diagram, we have

loop 1: $\mathcal{E} - I_1 R_1 = 0$;

$6\text{ V} - I_1(100\ \Omega) = 0$;

loop 2: $I_1 R_1 - I_2 R_2 - I_3 R_3 = 0$;

$I_1(100\ \Omega) - I_2(20\ \Omega) - I_3(100\ \Omega) = 0$;

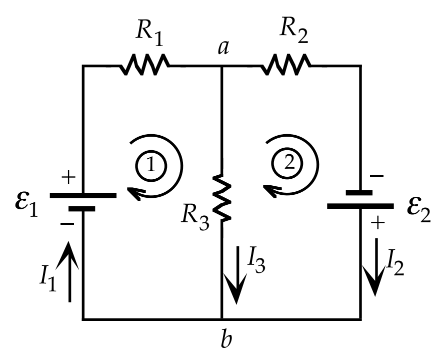
loop 3: $-I_3 R_3 + I_4 R_4 = 0$;

$-I_3(100\ \Omega) + I_4(50\ \Omega) = 0$.

When we solve these equations, we get

$I_1 = 0.060$ A, $I_2 = 0.113$ A, $I_3 = 0.038$ A, $I_4 = 0.075$ A.

Thus, the current through the 50-Ω resistor is $\boxed{0.075\text{ A}}$.



32. For the conservation of current at point a, we have

$\sum I_{\text{in}} = 0$;

$I_1 - I_2 - I_3 = 0$;

$I_1 - I_2 - 0.1\text{ A} = 0$.

For the two loops indicated on the diagram, we have

loop 1: $\mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0$;

$+3\text{ V} - I_1(5\ \Omega) - (0.1\text{ A})R_3 = 0$;

loop 2: $\mathcal{E}_2 + I_3 R_3 - I_2 R_2 = 0$;

$+6\text{ V} + (0.1\text{ A})R_3 - I_2(20\ \Omega) = 0$.

When we solve these equations, we get

$I_1 = 0.44$ A, $I_2 = 0.34$ A, and $R_3 = \boxed{8\ \Omega}$.

If $I_3 = -0.1$ A, the equations become

$I_1 = I_2 - 0.1$ A;

$+3\text{ V} - I_1(5\ \Omega) - (-0.1\text{ A})R_3 = 0$;

$+6\text{ V} + (-0.1\text{ A})R_3 - I_2(20\ \Omega) = 0$.

When we solve these equations, we get $I_1 = 0.28$ A, $I_2 = 0.38$ A, and $R_3 = -16\ \Omega$.

Because we cannot have a negative resistance, it is $\boxed{\text{not possible}}$ to have $I_3 = -0.1$ A.

33. For the conservation of current at point b, we have

$\sum I_{\text{in}} = 0;$

$I - I_1 - I_2 = 0;$

For the two loops indicated on the diagram, we have

loop 1: $\mathcal{E}_1 - I_1 r_1 - IR = 0;$

$+12\text{ V} - I_1(0.1\ \Omega) - I(5\ \Omega) = 0;$

loop 2: $\mathcal{E}_2 + I_2 r_2 - IR = 0;$

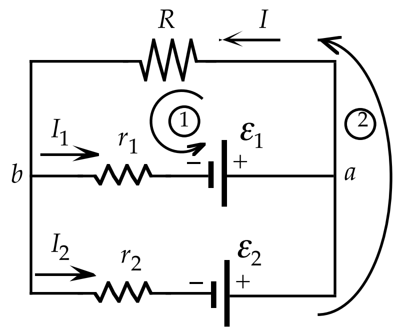
$+10\text{ V} - I_2(10\ \Omega) - I(5\ \Omega) = 0.$

When we solve these equations, we get

$I_1 = 2.52\text{ A}, I_2 = -0.17\text{ A, and } I = 2.35\text{ A}.$

The current through the load resistor is $\boxed{2.35\text{ A}}$.

$\mathcal{E}_1$ supplies $\boxed{2.52\text{ A}}$; $\mathcal{E}_2$ supplies $\boxed{\text{no current}}$, it is being charged by $\mathcal{E}_1$.



34. On the diagram, we show the potential difference applied between points A and B. Because all of the resistors are the same, symmetry means that the three currents leaving point A must be the same three currents entering point B. This means that there is no current in the resistor between points C and D, which can be removed without changing the currents. When we redraw the circuit, we see that we have three parallel branches between points A and B. The currents are

$I_1 = V_{AB}/R = (4\text{ V})/(1\ \Omega) = 4\text{ A};$

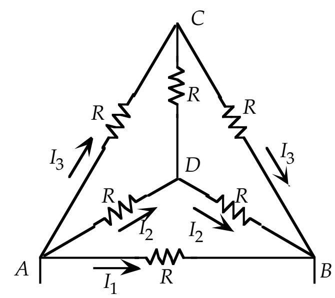
$I_2 = I_3 = V_{AB}/(R + R) = (4\text{ V})/(1\ \Omega + 1\ \Omega) = 2\text{ A}.$

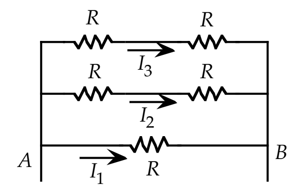
The power dissipated in each of the resistors is

$P_{AB} = I_1^2 R = (4\text{ A})^2(1\ \Omega) = \boxed{16\text{ W}};$

$P_{CD} = \boxed{0};$

$P_{\text{all others}} = I_2^2 R = (2\text{ A})^2(1\ \Omega) = \boxed{4\text{ W}}.$





35. In the original configuration, there are no series or parallel combinations; however, from the symmetry of the resistors, we know that the current that goes into point A must split equally to go through the cube. The three points on the other side of the three resistors must be at the same potential, so we can connect them with a wire without changing the currents. In the same way, the other three corners of the cube must be at equal potentials, so we can connect them with a wire. From the redrawn circuit, we see that we have three parallel combinations, two of which are the same:

$1/R_1 = 1/R_3 = (1/R) + (1/R) + (1/R),$

which gives $R_1 = R_3 = R/3;$

$1/R_2 = (1/R) + (1/R) + (1/R) + (1/R) + (1/R) + (1/R),$

which gives $R_2 = R/6;$

We combine these three in series to get

$R_{\text{eq}} = (R/3) + (R/6) + (R/3) = \boxed{5R/6}.$

The current in the equivalent resistor is

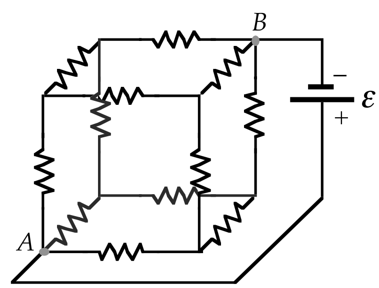
$I = V/(5R/6) = 6V/5R.$

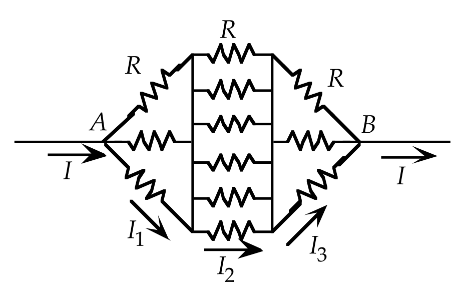
The current in a resistor connected to point A or point B is

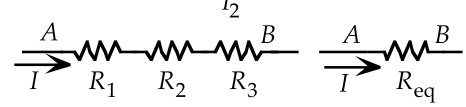
$I_1 = I_3 = I/3 = \boxed{2V/5R}.$

The current in each of the other resistors is

$I_2 = I/6 = \boxed{V/5R}.$





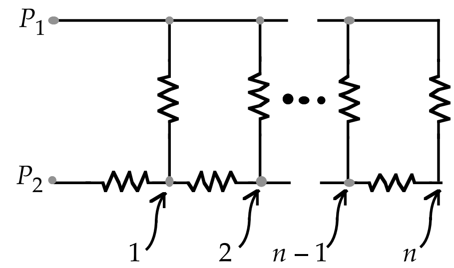


36. (a) For $n = 1$, we have two resistors in series:

$$R_1 = R + R = \boxed{2R}.$$

(b) For $n = 2$, we have a resistor in series with a parallel combination of a resistor and resistance R_1:

$$R_2 = R + R_1R/(R_1 + R)$$
$$= R + 2RR/(2R + R) = \boxed{5R/3}.$$



(c) For $n = 3$, we have a resistor in series with a parallel combination of a resistor and resistance R_2:

$$R_3 = R + R_2R/(R_2 + R)$$
$$= R + (5RR/3)/(5R/3 + R) = \boxed{13R/8}.$$

(d) For n rungs, we have a resistor in series with a parallel combination of a resistor and resistance R_{n-1}:

$$R_n = R + R_{n-1}R/(R_{n-1} + R).$$

In the limit of $n \to \infty$, we have

$R_n = R_{n-1} = R_{eq}$, which gives

$R_{eq} = R + R_{eq}R/(R_{eq} + R)$, which reduces to $R_{eq}^2 - RR_{eq} - R^2 = 0$.

The solutions to this quadratic equation are

$$R_{eq} = \tfrac{1}{2}(1 \pm \sqrt{5})R.$$

Because the resistance cannot be negative, we have $R_{eq} = \tfrac{1}{2}(1 + \sqrt{5})R = \boxed{1.618R}$.

37. The equivalent resistance of the circuit is $R = R_1R_V/(R_V + R_1) + R_2$, where R_V is the internal resistance of the voltmeter. The current in the circuit is then

$$I = \mathcal{E}/R = \mathcal{E}/[R_1R_V/(R_V + R_1) + R_2].$$

The voltage across R_2 is $V_2 = IR_2$, so the voltage across R_1 is

$$V_1 = \mathcal{E} - V_2 = \mathcal{E} - IR_2 = \mathcal{E} - \mathcal{E}R_2/[R_1R_V/(R_V + R_1) + R_2].$$

Plug in $\mathcal{E} = 6$ V, $R_1 = 1400\ \Omega$, $R_2 = 10$ kΩ, and $R_V = 200$ kΩ to obtain

$$V_1 = \boxed{0.732\text{ V}} \quad (\text{for } R_V = 200\text{ k}\Omega).$$

If R_V is changed to 10 MΩ, then from the formula above we get

$$V_1 = \boxed{0.737\text{ V}} \quad (\text{for } R_V = 10\text{ M}\Omega).$$

38. We have a single-loop circuit of the battery and the voltmeter. Because the terminal voltage of the battery is the voltage across the voltmeter, we have

$V_{terminal} = IR_V$;

$1.45\text{V} = I(60 \times 10^3\ \Omega)$, which gives $I = 2.4 \times 10^{-5}$ A.

For the battery, we have

$V_{terminal} = \mathcal{E} - Ir$;

$1.45\text{V} = 1.5\text{ V} - (2.4 \times 10^{-5}\text{ A})r$, which gives $r = \boxed{2.1\text{ k}\Omega}$.

39. Without the ammeter in the circuit, we have

$I = \mathcal{E}/R$.

With the ammeter in the circuit, we have

$I' = \mathcal{E}/(R_A + R)$.

When we combine these two equations, we get

$I'/I = R/(R + R_A) = 1/(1 + R_A/R) \simeq 1 - (R_A/R)$, if $R_A/R << 1$.

To get the desired accuracy, we want $I'/I \geq 0.999$, or $R_A/R \leq 0.001$.

The maximum allowable value of R_A is determined by the smallest value of R:

$R_A = 0.001(10\ \Omega) = \boxed{0.010\ \Omega}$.

40. The voltmeter is placed in parallel with the resistor, so the equivalent resistance is
   $1/R_{\text{eq}} = 1/R + 1/R_V$, which becomes
   $R_{\text{eq}}/R = R_V/(R + R_V) = 1/(1 + R/R_V) \simeq 1 - R/R_V$, if $R/R_V << 1$.
   To get the desired accuracy, we want $R_{\text{eq}}/R \geq 0.999$, or $R/R_V \leq 0.001$.
   The minimum allowable value of R_V is determined by the largest value of R:
   $R_V = R/0.001 = (5 \times 10^3\ \Omega)/0.001 = \boxed{5 \times 10^6\ \Omega}$.

41. We find the equivalent resistance for R and R_V, which are in parallel, from
   $1/R_{\text{eq}} = 1/R + 1/R_V$, which gives $R_{\text{eq}} = RR_V/(R + R_V)$.
   (a) $R_{\text{eq}} = (10\ \Omega)(10^5\ \Omega)/(10\ \Omega + 10^5\ \Omega) \simeq \boxed{10\ \Omega}$.
   (b) $R_{\text{eq}} = (10^5\ \Omega)(10^5\ \Omega)/(10^5\ \Omega + 10^5\ \Omega) = \boxed{5 \times 10^4\ \Omega}$.
   (c) $R_{\text{eq}} = (100 \times 10^6\ \Omega)(10^5\ \Omega)/(100 \times 10^6\ \Omega + 10^5\ \Omega) \simeq \boxed{10^5\ \Omega}$.
   The equivalent resistance has the value of the resistor when $R_V >> R$.

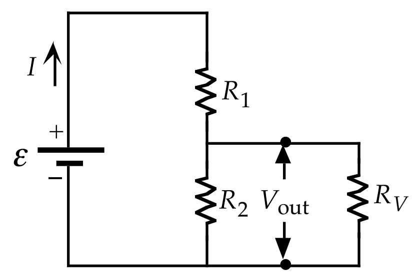
42. The resistance of the voltmeter is $R_V = R + R_A$. The maximum current through the voltmeter is the maximum current through the ammeter, so we have
   $V_{\max} = I_{\max}(R + R_A)$;
   $3\text{ V} = (5 \times 10^{-3}\text{ A})(R + 1.8 \times 10^{-4}\ \Omega)$, which gives
   $R = \boxed{0.6\text{ k}\Omega}$.

43. The equivalent resistance of the circuit is $R + r$, where we assumed that the internal resistance of the voltmeter is nearly infinity. The current in the circuit is then
   $I = \mathcal{E}/(R + r)$.
   The voltage across R, i.e., the reading of the voltmeter, is then
   $V = \mathcal{E} - Ir = \mathcal{E} - \mathcal{E}r/(R + r) = \mathcal{E}R/(R + r)$.
   For $R = 20\ \Omega$ we have $V = 23$ V, and for $R = 5\ \Omega$ we have $V = 16$ V; so
   $23\text{ V} = \mathcal{E}(20\ \Omega)/(20\ \Omega + r)$;
   $16\text{ V} = \mathcal{E}(5\ \Omega)/(5\ \Omega + r)$.
   Solve these two equations to obtain $\mathcal{E} = 26.9$ V and $r = 3.41\ \Omega$. Thus for $R = 50\ \Omega$
   $V = \mathcal{E}R/(R + r) = (26.9\text{ V})(50\ \Omega)/(50\ \Omega + 3.41\ \Omega) = \boxed{25\text{ V}}$.

44. To be able to accommodate 2×10^{-3} A, which is 10 times the current that causes the galvanometer to fully deflect, we need to add a resistor of resistance R in parallel to it to route 90% of the current, leaving only 10% of 2×10^{-3} A, or 2×10^{-4} A, to flow through the galvanometer. If the full-deflection voltage is V then
   $V = I_G R_G = I_R R$;
   $(2 \times 10^{-4}\text{ A})(20\ \Omega) = [(90\%)(2 \times 10^{-3}\text{ A})]R$; which yields
   $R = 2.2\ \Omega$.
   So one needs to $\boxed{\text{connect a 2.2-}\Omega\text{ resistor in parallel with the galvanometer}}$.
   To change the full-deflection potential to $V' = 0.2$ V, place a resistor of resistance R' in series with the galvanometer. Upon full deflection the current in both of them is $I_G = 2 \times 10^{-4}$ A, so
   $V' = I_G(R_G + R')$;
   $0.2\text{ V} = (2 \times 10^{-4}\text{ A})(20\ \Omega + R')$; which yields
   $R' = 0.98\text{ k}\Omega$.
   So one needs to $\boxed{\text{connect a 0.98-k}\Omega\text{ resistor in series with the galvanometer}}$.

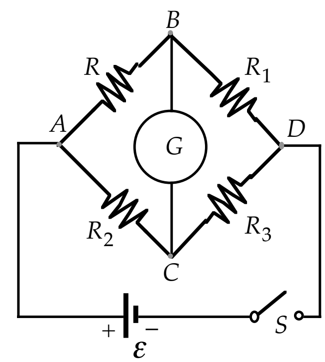
45. Because the shunt resistor is in parallel with the galvanometer, we have
$V_{\text{meter}} = I_G R_G = I_s R_s$, which gives $R_G/R_s = I_s/I_G$.
We use the junction at one side of the meter to find the total current through the meter:
$I = I_G + I_s = I_G(1 + I_s/I_G) = I_G(1 + R_G/R_s)$.

46. We combine R_2 and R_V, which are in parallel:
$1/R_3 = 1/R_2 + 1/R_V$, which gives $R_3 = R_2R_V/(R_2 + R_V)$.
For the resulting single-loop circuit, we have
$I = \mathcal{E}/(R_1 + R_3) = \mathcal{E}/[R_1 + R_2R_V/(R_2 + R_V)]$.
The output voltage is
$V_{\text{out}} = IR_3 = \{\mathcal{E}/[R_1 + R_2R_V/(R_2 + R_V)]\}[R_2R_V/(R_2 + R_V)]$
$= \mathcal{E}R_2R_V/(R_1R_2 + R_1R_V + R_2R_V)$
$= \mathcal{E}/(1 + R_1/R_2 + R_1/R_V)$.

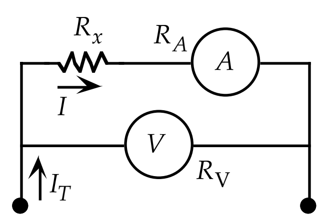


For the two voltmeters, we have
$R_V = 500\ \text{k}\Omega$:
$V_{\text{out}} = (1200\ \text{V})/[1 + (30\ \text{k}\Omega)/(50\ \text{k}\Omega) + (30\ \text{k}\Omega)/(500\ \text{k}\Omega)] = \boxed{723\ \text{V}}$.
$R_V = 100\ \text{M}\Omega$:
$V_{\text{out}} = (1200\ \text{V})/[1 + (30\ \text{k}\Omega)/(50\ \text{k}\Omega) + (30\ \text{k}\Omega)/(100 \times 10^3\ \text{k}\Omega)] = \boxed{750\ \text{V}}$.

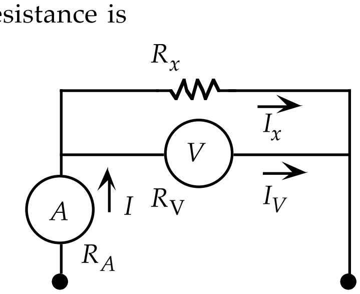
$\boxed{47}$. When there is no current through the galvanometer,
we have $V_{BC} = 0$, a current I_1 through R and R_1, and
a current I_2 through R_2 and R_3. Thus we have
$V_{AD} = I_1(R + R_1) = I_2(R_2 + R_3)$, and
$V_{AB} = V_{AC}$ or $I_1R = I_2R_2$.
When we divide these two equations, we get
$(R + R_1)/R = (R_2 + R_3)/R_2$, or
$1 + (R_1/R) = 1 + (R_3/R_2)$, which gives $R_1/R = R_3/R_2$.
The unknown resistance is
$\boxed{R = R_1R_2/R_3}$.



48. The voltage read by the voltmeter is also the voltage
across R_x and the ammeter:
$V = I(R_A + R_x)$, which gives
$R_x = \boxed{V/I - R_A}$.
We will have $R_x = V/I$ $\boxed{\text{when } R_A << V/I \text{ (when } R_A << R_x)}$.



49. Because the voltmeter and R_x are in parallel, their equivalent resistance is
$R_{\text{eq}} = R_VR_x/(R_V + R_x)$.
The voltage read by the voltmeter is also the voltage across R_{eq}:
$V = IR_{\text{eq}} = IR_VR_x/(R_V + R_x)$, which gives
$R_x = \boxed{(V/I)/(1 - V/IR_V)}$.
We will have $R_x = V/I$ when $V/IR_V << 1$, or
$\boxed{R_V >> V/I \text{ (when } R_V >> R_x)}$.



50. We find the resistance from

time constant = RC;

5×10^{-4} s = $R(16 \times 10^{-6}$ F), which gives $R = \boxed{31\ \Omega.}$

51. We find the capacitance from

time constant = RC;

2.0 s = $(10^5\ \Omega)C$, which gives $C = 2.0 \times 10^{-5}$ F = $\boxed{20\ \mu\text{F}}$.

52. We use the definitions of R and C to find the units of RC:

$RC = (V/I)(Q/V) = Q/I$ = coulomb/(coulomb/second) = second.

For the given data, we have

$R_1C_1 = (5 \times 10^6\ \Omega)(30 \times 10^{-6}\ \text{F}) = \boxed{150\ \text{s}}$;

$R_2C_2 = (8 \times 10^3\ \Omega)(3 \times 10^{-6}\ \text{F}) = 24 \times 10^{-3}\ \text{s} = \boxed{24\ \text{ms}}$;

$R_3C_3 = (20\ \Omega)(50 \times 10^{-12}\ \text{F}) = 1 \times 10^{-9}\ \text{s} = \boxed{1\ \text{ns}}$.

$\boxed{53}$. With the time constant as the flash time, we have

time constant = RC;

(1/500) s = $R(600 \times 10^{-6}$ F), which gives $R = \boxed{3.3\ \Omega}$.

54. From $Q = C\mathcal{E}(1 - e^{-t/RC})$, we obtain

$dQ/dt = C\mathcal{E}[-(-1/RC)e^{-t/RC}] = (\mathcal{E}/R)e^{-t/RC}$.

We substitute these equations into

$\mathcal{E} - R(dQ/dt) - Q/C$;

$\mathcal{E} - R(\mathcal{E}/R)e^{-t/RC} - (C\mathcal{E}/C)(1 - e^{-t/RC}) = \mathcal{E} - \mathcal{E}e^{-t/RC} - \mathcal{E} + \mathcal{E}e^{-t/RC} = 0$,

so Eq. (27–21) is satisfied.

55. From $Q = Q_0e^{-t/RC}$, we obtain

$dQ/dt = (-Q_0/RC)e^{-t/RC}$.

We substitute these equations into

$R(dQ/dt) + Q/C$;

$(-RQ_0/RC)e^{-t/RC)} + (Q_0/C)e^{-t/RC} = 0$, so Eq. (27–25) is satisfied.

56. (a) The time constant is

$RC = (3 \times 10^6\ \Omega)(350 \times 10^{-6}\ \text{F}) = 1050$ s.

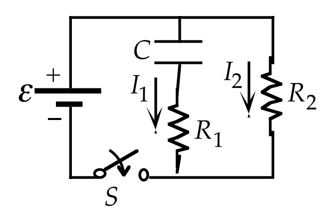
(b) For the charge on the capacitor, we have

$Q = Q_0(1 - e^{-t/RC}) = 0.90Q_0$, which gives $e^{-t/RC} = 0.10$.

For the charging current, we have

$I = (\mathcal{E}/R)e^{-t/RC} = [(50\ \text{V})/(3 \times 10^6\ \Omega)](0.10) = \boxed{1.7 \times 10^{-6}\ \text{A}}$.

57. Because there is no internal resistance in the battery, the potential difference across R_2 and across the capacitor branch is $\mathcal{E}$. The current in R_2 is constant:



$I_2 = \mathcal{E}/R_2$.

The charging current in the capacitor branch is

$I_1 = (\mathcal{E}/R_1)e^{-t/R_1C}$.

From the junction, the current in the battery is

$$\boxed{I_{\text{battery}} = I_1 + I_2 = (\mathcal{E}/R_1)e^{-t/R_1C} + (\mathcal{E}/R_2).}$$

58. The time constant of the circuit is

$RC = (350 \times 10^3\ \Omega)(20 \times 10^{-6}\ \text{F}) = 7.0\ \text{s}.$

The charging current in the resistor is

$I = (\mathcal{E}/R)e^{-t/RC}$, so the voltage across the resistor is

$V = IR = \mathcal{E}e^{-t/RC},$

$= (200\ \text{V})e^{-(4.0\ \text{s})/(7.0\ \text{s})} = \boxed{113\ \text{V}}.$

The charge on the capacitor is

$Q = C\mathcal{E}(1 - e^{-t/RC})$

$= [(20 \times 10^{-6}\ \text{F})(200\ \text{V})][1 - e^{-(4.0\ \text{s})/(7.0\ \text{s})}] = 1.7 \times 10^{-3}\ \text{C} = \boxed{1.7\ \text{mC}}.$

$\boxed{59}$. The possible capacitance values that we have are

$C_1 = C_2 = 5\ \mu\text{F};$

$C_{\text{parallel}} = C_1 + C_2 = 5\ \mu\text{F} + 5\ \mu\text{F} = 10\ \mu\text{F};$

$C_{\text{series}} = C_1C_2/(C_1 + C_2) = (5\ \mu\text{F})(5\ \mu\text{F})/[(5\ \mu\text{F}) + (5\ \mu\text{F})] = 2.5\ \mu\text{F}.$

We need to combine the resistors to produce one of the following resistance values:

$R_a = RC/C_1 = (1 \times 10^{-3}\ \text{s})/(5 \times 10^{-6}\ \text{F}) = 200\ \Omega;$

$R_b = RC/C_{\text{parallel}} = (1 \times 10^{-3}\ \text{s})/(10 \times 10^{-6}\ \text{F}) = 100\ \Omega;$

$R_c = RC/C_{\text{series}} = (1 \times 10^{-3}\ \text{s})/(2.5 \times 10^{-6}\ \text{F}) = 400\ \Omega;$

If we connect the 300-Ω resistors in parallel, we get

$R_3 = R_2R_2/(R_2 + R_2)$

$= (300\ \Omega)(300\ \Omega)/[(300\ \Omega) + (300\ \Omega)] = 150\ \Omega.$

We see that we can produce R_c by putting this combination in series with the 250-Ω resistor:

$R_c = R_1 + R_3 = 250\ \Omega + 150\ \Omega = 400\ \Omega.$

Thus we connect the 300-Ω resistors in parallel with each other and in series with the 250-Ω resistor and the two capacitors.

60. For a parallel-plate capacitor with separation d, we have

$C = \kappa\varepsilon_0 A/d.$

For the resistance of the dielectric, we have

$R = \rho d/A.$

The time constant is

$RC = (\rho d/A)(\kappa\varepsilon_0 A/d) = \kappa\varepsilon_0\rho$, which is independent of the area and separation.

61. We use the result of Problem 62 to find the time constant:

$RC = \kappa\varepsilon_0\rho$

$= (3.2)(8.85 \times 10^{-12}\ \text{F/m})(2 \times 10^{14}\ \Omega\cdot\text{m}) = 5.66 \times 10^3\ \text{s}.$

As the capacitor discharges, when 70% of the charge on the plates has leaked away, we have

$Q = Q_0e^{-t/RC} = 0.30Q_0$, which gives

$e^{-t/RC} = 0.30;\quad t/RC = 1.2.$

The time for 70% of the charge to leak away is

$t = 1.2RC = (1.2)(5.66 \times 10^3\ \text{s}) = \boxed{6.8 \times 10^3\ \text{s}\ \ (1.9\ \text{h})}.$

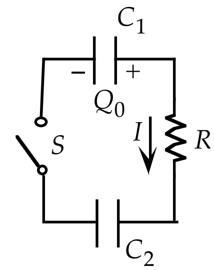
62. When the switch is closed, charge will move from C_1 to C_2; with a variable current I in the single loop or series circuit:

$I = -\,dQ_1/dt = dQ_2/dt$.

With the same current in the two capacitors, they are connected in series. We find the equivalent capacitance of the circuit from

$1/C = 1/C_1 + 1/C_2$, which gives

$C = C_2C_1/(C_1 + C_2)$.



The time constant of the circuit is RC.

Q_1, the charge on C_1, will decrease and Q_2, the charge on C_2, will increase until the potential difference will be the same across both capacitors, at which point the current becomes 0.

The charges will have reached their final value, which we find from

$V_1 = V_2$; or

$Q_{1f}/C_1 = Q_{2f}/C_2$.

Because there has been no loss of charge, we have $Q_0 = Q_{1f} + Q_{2f}$.

When we combine these two equations, we get

$Q_{1f} = Q_0C_1/(C_1 + C_2)$, and

$Q_{2f} = Q_0C_2/(C_1 + C_2)$.

Capacitor C_2 will charge just like a single capacitor from 0 to its final value Q_{2f}, so we have

$Q_2 = \boxed{[Q_0C_2/(C_1 + C_2)](1 - e^{-t/RC})}$.

At any time the total charge is conserved and equal to the initial charge Q_0. We find the charge on C_1 as a function of time from

$Q_1 = Q_0 - Q_2$, which gives

$Q_1 = \boxed{[Q_0C_1/(C_1 + C_2)] + [Q_0C_2/(C_1 + C_2)]e^{-t/RC}}$.

Note that at $t = 0$, this gives Q_0 and after a long time it gives Q_{1f}.

[It is also possible to solve the circuit equation, obtained by adding the potential drops around the loop: $Q_1/C_1 - IR - Q_2/C_2 = 0$. When the current is put in terms of the rate of change of the charge and the conservation of charge is used, a differential equation is obtained. The solution is the given equations.]

63. The power used by each appliance is $P_i = I_iV$, so the currents draw from the main supply are

$I_1 = P_1/V = (50\text{ W})/(120\text{ V}) = \boxed{0.417\text{ A}}$;

$I_2 = P_2/V = (60\text{ W})/(120\text{ V}) = \boxed{0.500\text{ A}}$;

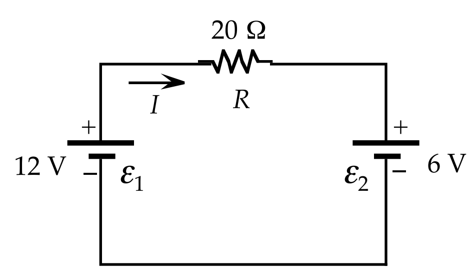
$I_3 = P_3/V = (20\text{ W})/(120\text{ V}) = \boxed{0.167\text{ A}}$.

64. (a) The current in the circuit will be clockwise.

For the single loop, we have

$I = (\mathcal{E}_1 - \mathcal{E}_2)/R$

$= (12\text{ V} - 6\text{ V})/(20\ \Omega) = \boxed{0.30\text{ A}}$.



(b) The rate at which energy is being stored in the smaller battery is

$P = I\mathcal{E}_2 = (0.30\text{ A})(6\text{ V}) = \boxed{1.8\text{ W}}$.

(c) The rate of energy dissipation in the resistor is

$P = I^2R = (0.30\text{ A})^2(20\ \Omega) = \boxed{1.8\text{ W}}$.

Note that the sum of the answers to parts (b) and (c) is the rate at which energy is being provided by the larger battery, 3.6 W.

65. Because the power dissipated in the effective resistance is the sum of the powers dissipated in the individual resistors, we have

$P_{eq} = \sum P_i = nP_i$;

30 W = n(5 W), which gives $n = 6$ resistors.

If we connect the six resistors in parallel, we have

$1/R_{eq} = \sum(1/R_i) = n/R_i$;

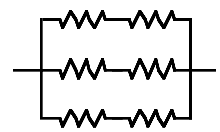
$1/(100\ \Omega) = 6/R_i$, which gives $R_i = 600\ \Omega$. We can connect six 600-Ω resistors in parallel.

If we connect the six resistors in series, we have

$R_{eq} = \sum R_i = nR_i$;

$100\ \Omega = 6R_i$, which gives $R_i = 16.7\ \Omega$. We can connect six 16.7-Ω resistors in series.

To have the same power dissipated in each resistor requires that the currents be the same, which means there must be symmetry in the arrangement of the resistors.

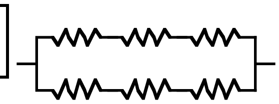


If we connect two series resistors in parallel with two more sets of two series resistors, we have

$1/R_{eq} = 1/2R_i + 1/2R_i + 1/2R_i = 3/2R_i$;

$1/(100\ \Omega) = 3/2R_i$, which gives $R_i = 150\ \Omega$.

We can connect two series 150-Ω resistors in parallel with two more sets of two series 150-Ω resistors.



If we connect three series resistors in parallel with a set of three series resistors, we have

$1/R_{eq} = 1/3R_i + 1/3R_i = 2/3R_i$;

$1/(100\ \Omega) = 2/3R_i$, which gives $R_i = 66.7\ \Omega$.

We can connect three series 66.7-Ω resistors in parallel with three series 66.7-Ω resistors.

66. The current for this single loop is

$I = \mathcal{E}/(R + r)$.

The power delivered to the external resistor is the power dissipated in the resistor:

$P_{ext} = I^2 R = \mathcal{E}^2 R/(r + R)^2$.

To find the value of R that maximizes the power, we set $dP_{ext}/dR = 0$:

$dP/dR = [\mathcal{E}^2/(r + R)^2] - [2\mathcal{E}^2 R/(r + R)^3] = \mathcal{E}^2(r - R)/(r + R)^3 = 0$, which gives $r = R$.

67. (a) The short-circuit current (when there is no load) provided by the battery is

$I = \mathcal{E}/r = 12.6\ \text{V}/0.05\ \Omega = \boxed{0.25\ \text{kA}}$.

(b) The voltage V across the battery terminal during recharging must overcome both $\mathcal{E}$ and the internal resistance of the battery:

$V = \mathcal{E} + Ir = 12.6\ \text{V} + (2.5\ \text{A})(0.05\ \Omega) = \boxed{12.7\ \text{V}}$.

(c) Energy stored = $\mathcal{E}It = (12.6\ \text{V})(2.5\ \text{A})(10\ \text{h})(3600\ \text{s/h}) = \boxed{1.1 \times 10^6\ \text{J}}$.

68. If the batteries are in parallel then the equivalent resistance of the circuit is $R + r/n$, and the emf is $\mathcal{E}$. The current is

$I_{parallel} = \mathcal{E}/(R + r/n) = n\mathcal{E}/(nR + r)$.

If the batteries are in series then the equivalent resistance of the circuit is $R + nr$, and the emf is $n\mathcal{E}$. The current is

$I_{series} = n\mathcal{E}/(R + nr)$.

If we compare $I_{parallel}$ with I_{series}, it is clear that $I_{parallel} > I_{series}$ if

$nR + r < R + nr$, or $R < r$.

Similarly, if $R > r$ then $I_{series} > I_{parallel}$.

Therefore, to maximize the current, put the batteries in series if $R > r$ and in parallel if $R < r$.

69. For this single-loop circuit, we have

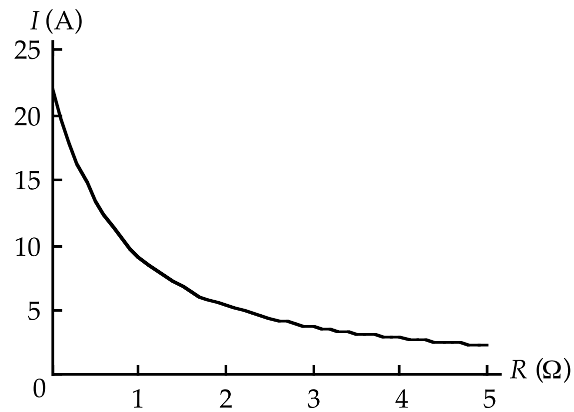
$I = \mathcal{E}/(R + r) = \mathcal{E}/[R + (\alpha + \beta I)]$, which is a quadratic equation for I: $\beta I^2 + (R + \alpha)I - \mathcal{E} = 0$.

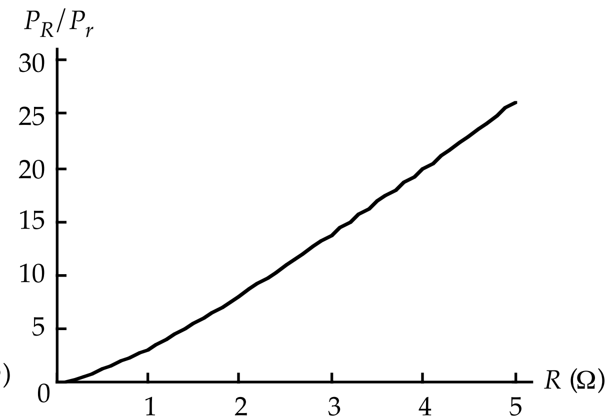
The positive solution is

$I = \boxed{\{-(R + \alpha) + [(R + \alpha)^2 + 4\beta\mathcal{E}]^{1/2}\}/2\beta}$.

The ratio of power delivered to the load to the power dissipated in the battery is

$P_R/P_r = I^2R/[I^2(\alpha + \beta I)] = R/(\alpha + \beta I)$.





70. Using $P = V^2/R$, we find the two resistances:

$R_1 = V^2/P_1$ and $R_2 = V^2/P_2$.

The equivalent resistance for the series connection is

$R_s = R_1 + R_2 = (V^2/P_1) + (V^2/P_2) = V^2(P_1 + P_2)/P_1P_2$.

The power generated is

$P_s = V^2/R_s = \boxed{P_1P_2/(P_1 + P_2)}$.

The equivalent resistance for the parallel connection is

$R_p = R_1R_2/(R_1 + R_2) = (V^4/P_1P_2)/(V^2/P_1 + V^2/P_2) = V^2/(P_1 + P_2)$.

The power generated is

$P_p = V^2/R_p = V^2/[V^2/(P_1 + P_2)] = \boxed{P_1 + P_2}$.

71. The two heating elements in the furnace are initially connected in parallel. The power dissipated in a resistor is $P = I^2R = V^2/R$. The resistances are

$R_1 = V^2/P_1 = (110\text{ V})^2/(1000\text{ W}) = 12.1\ \Omega$.

$R_2 = V^2/P_2 = (110\text{ V})^2/(2000\text{ W}) = 6.05\ \Omega$.

The power can be reduced by increasing the resistance, which means connecting them in series:

$P = V^2/(R_1 + R_2) = (110\text{ V})^2/(12.1\ \Omega + 6.05\ \Omega) = \boxed{667\text{ W}}$.

72. (a) For the series arrangement, we have

$I_s = (\mathcal{E} + \mathcal{E})/(R + 2r)$, which gives

$I_s = \boxed{\mathcal{E}/(\tfrac{1}{2}R + r)}$.

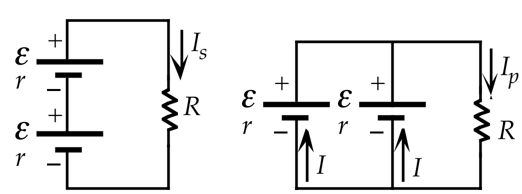
(b) For the parallel arrangement, we use symmetry to see that the current in each battery is the same. For the junction, we have

$I_p - I - I = 0$, or $I_p = 2I$.

For one loop, we have

$\mathcal{E} - I_pR - Ir = 0 = \mathcal{E} - I_pR - \tfrac{1}{2}I_pr$, which gives

$I_p = \boxed{\mathcal{E}/(R + \tfrac{1}{2}r)}$.



Series Parallel

For large R: $I_s \simeq 2\mathcal{E}/R$, $I_p \simeq \mathcal{E}/R$; so I_s is larger. For small R: $I_s \simeq \mathcal{E}/r$, $I_p \simeq 2\mathcal{E}/r$; so I_p is larger.

73. The devices are connected in parallel, so we have

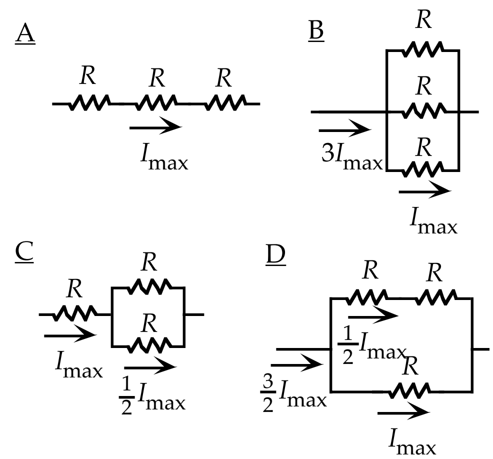
$$I = PV;$$
$$I_{\text{mixer}} = (800\text{ W})/(120\text{ V}) = \boxed{6.67\text{ A}}.$$
$$I_{\text{vacuum}} = (600\text{ W})/(120\text{ V}) = \boxed{5.00\text{ A}}.$$
$$I_{\text{chandelier}} = 10(60\text{ W})/(120\text{ V}) = \boxed{5.00\text{ A}}.$$

If all three devices are used at the same time, the fuse will blow. Each bulb draws 0.50 A. To find the number of bulbs that can be used without blowing the fuse, we have

$$I_{\max} = I_{\text{mixer}} + I_{\text{vacuum}} + NI_{\text{bulb}};$$

$15\text{ A} = 6.67\text{ A} + 5.00\text{ A} + N(0.50\text{ A})$, which gives $N = 6.7$ bulbs. Thus $\boxed{7\text{ bulbs}}$ will blow the fuse.

74. We find the maximum current through a resistor from



$$P_{1\max} = I_{\max}{}^2R;$$
$$2\text{ W} = I_{\max}{}^2(30\ \Omega),\text{ which gives } I_{\max} = 0.26\text{ A}.$$

When the three are connected in series, circuit A, the maximum current goes through each resistor, so we have

$$P_{A\max} = 3P_{1\max} = 3(2\text{ W}) = 6\text{ W}.$$

When the three are connected in parallel, circuit B, the maximum current goes through each resistor, so we have

$$P_{B\max} = 3P_{1\max} = 3(2\text{ W}) = \boxed{6\text{ W}}.$$

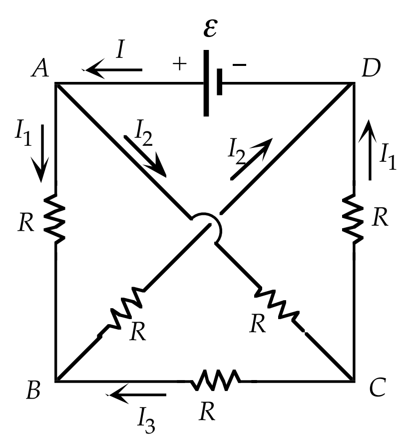
In circuit C, the resistor in series has the maximum current. From symmetry the current in the other two resistors will be $\frac{1}{2}I_{\max}$. The maximum power is

$$P_{C\max} = P_{1\max} + 2(\tfrac{1}{2}I_{\max})^2R$$
$$= P_{1\max} + \tfrac{1}{2}P_{1\max} = \tfrac{3}{2}(2\text{ W}) = \boxed{3\text{ W}}.$$

In circuit D, the branch with the single resistor has the maximum current. Because the total resistance in the other branch is $2R$, the current in the other branch will be $\frac{1}{2}I_{\max}$. The maximum power is

$$P_{D\max} = P_{1\max} + 2(\tfrac{1}{2}I_{\max})^2R$$
$$= P_{1\max} + \tfrac{1}{2}P_{1\max} = \tfrac{3}{2}(2\text{ W}) = \boxed{3\text{ W}}.$$

75. Normally, this circuit would have six currents, one for each branch. We have used the symmetry of the circuit to reduce the number of currents to four, as shown in the diagram.



For the junction equations, we have

junction A (or D): $I - I_1 - I_2 = 0$; (1)

junction B (or C): $I_1 + I_3 - I_2 = 0$. (2)

For the loop equations, we have

loop $ACDA$: $-I_2R - I_1R + \mathcal{E} = 0$; (3)

loop $BDCB$: $-I_2R + I_1R - I_3R = 0$; (4)

When we combine Eq. (2) and Eq. (4), we find

$\boxed{I_3 = 0}$ (as suggested by symmetry) and $I_1 = I_2$.

From Eq. (3), we get

$\boxed{I_1 = I_2 = \mathcal{E}/2R}$.

From Eq. (1), we get

$\boxed{I = 2I_1 = \mathcal{E}/R}$.

76. For a cylindrical wire, we have

$$R = \rho L/A,\ I = V/R = VA/\rho L,\ J = I/A = V/\rho L = E/\rho = V/\rho L, \text{ and } P = I^2R = I^2\rho L/A.$$

The two wires have the same length and area.

(a) When the wires are in series, the currents must be the same, so we have

$$J_{Al}/J_{Cu} = I_{Al}/I_{Cu} = \boxed{1}.$$

$$E_{Al}/E_{Cu} = J_{Al}\rho_{Al}/J_{Cu}\rho_{Cu} = \rho_{Al}/\rho_{Cu} = (2.82 \times 10^{-8}\ \Omega\cdot\text{m})/(1.72 \times 10^{-8}\ \Omega\cdot\text{m}) = \boxed{1.64}.$$

$$P_{Al}/P_{Cu} = (I^2\rho_{Al}L/A)/(I^2\rho_{Cu}L/A) = \rho_{Al}/\rho_{Cu} = \boxed{1.64}.$$

(b) When the wires are in parallel, the potential difference is the same, so we have

$$J_{Al}/J_{Cu} = I_{Al}/I_{Cu} = (VA/\rho_{Al}L)/(VA/\rho_{Cu}L) = \rho_{Cu}/\rho_{Al} = \boxed{0.61}.$$

$$E_{Al}/E_{Cu} = J_{Al}\rho_{Al}/J_{Cu}\rho_{Cu} = (J_{Al}/J_{Cu})(\rho_{Al}/\rho_{Cu}) = \boxed{1}.$$

$$P_{Al}/P_{Cu} = (I_{Al}{}^2\rho_{Al}L/A)/(I_{Cu}{}^2\rho_{Cu}L/A)$$

$$= (I_{Al}/I_{Cu})^2(\rho_{Al}/\rho_{Cu}) = (\rho_{Cu}/\rho_{Al})^2(\rho_{Al}/\rho_{Cu}) = \rho_{Cu}/\rho_{Al} = \boxed{0.61}.$$

(c) Because the currents and areas are the same, all wires have the same current density. For constant current, the electric field and the power loss is proportional to the resistivity. Thus silver, with the smallest resistivity, has the weakest field and the least power loss.

77. If there is no current through the ammeter, we find the current in the source loop from

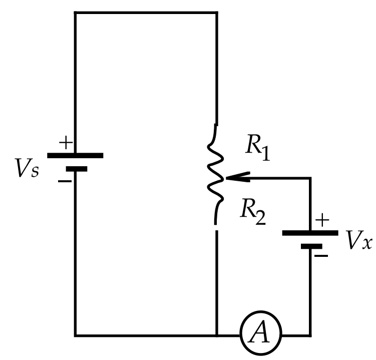
$$V_S - I_S(R_1 + R_2) = 0, \text{ which gives } I_S = V_S/(R_1 + R_2).$$

For the loop with the unknown, we have

$$V_x - I_SR_2 = 0.$$

When we use the expression for I_S, we get

$$V_x = V_SR_2/(R_1 + R_2).$$



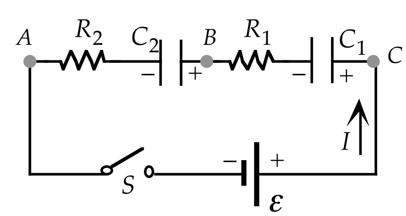
78. (a) Immediately after the switch is closed, there is no charge on the capacitors and thus no potential difference across them. For the loop we have

$$\mathcal{E} - I_iR_1 - I_iR_2 = 0;$$

$$9\text{ V} - I_i(300\ \Omega) - I_i(1000\ \Omega) = 0, \text{ which gives } I_i = 0.0069\text{ A}.$$

Between b and a we have

$$(V_B - V_A)_i = +I_iR_2 = (0.0069\text{ A})(1000\ \Omega) = \boxed{6.9\text{ V}}.$$



(b) After a long time, the current will be 0. The two capacitors are in series, with an equivalent capacitance:

$$1/C_{eq} = 1/C_1 + 1/C_2 = 1/(5\ \mu\text{F}) + 1/(2\ \mu\text{F}), \text{ which gives } C_{eq} = 1.43\ \mu\text{F}.$$

The final charge on either capacitor is

$$Q = C_{eq}\mathcal{E} = (1.43\ \mu\text{F})(9\text{ V}) = 12.9\ \mu\text{C}.$$

Between B and A we have

$$(V_B - V_A)_f = +Q/C_2 = (12.9\ \mu\text{C})/(1.43\ \mu\text{F}) = \boxed{6.4\text{ V}}.$$

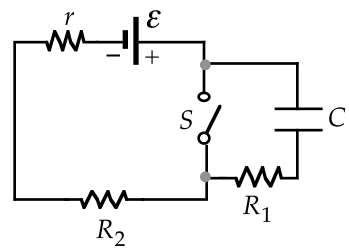
(c) The time constant of the circuit is

$$\text{time constant } = R_{eq}C_{eq} = (1\text{ k}\Omega + 0.3\text{ k}\Omega)(1.43\ \mu\text{F}) = 1.86\text{ ms}.$$

As a measure of how fast the circuit reaches a steady state, we use 10 time constants:

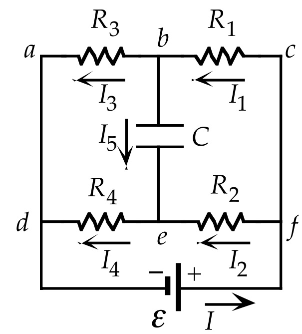
$$t = 10R_{eq}C_{eq} = 10(1.86\text{ ms}) = \boxed{19\text{ ms}}.$$

79. (a) With the switch open, we have a series circuit of the three resistors and the capacitor. For the time constant we have
time constant $= R_{eq}C = (r + R_1 + R_2)C$
$= [(0.04\ \Omega) + (0.1\ \Omega) + (2\ \Omega)](10\ \mu F) = \boxed{21.4\ \mu s}$.



(b) After a long time, there will be no current in the circuit. The battery emf will be across the capacitor:
$Q = C\mathcal{E} = (10\ \mu F)(240\ V) = 2400\ \mu C = \boxed{2.4\ mC}$.

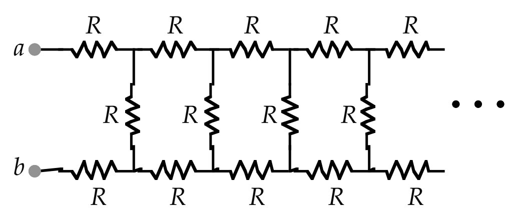
80. (a) After a long time there will be a steady state; there will be no current in the capacitor branch:
$I_5 = 0;\ I_1 = I_3$, and $I_2 = I_4$.



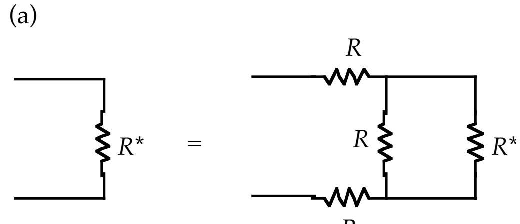
For the two resistor branches we have
$V_f - V_d = \mathcal{E} = I_2(R_2 + R_4)$;
$6\ V = I_2\ (180\ \Omega + 60\ \Omega)$, which gives $I_2 = 0.025\ A$;
$V_c - V_a = \mathcal{E} = I_1(R_1 + R_3)$;
$6\ V = I_1\ (35\ \Omega + 25\ \Omega)$, which gives $I_1 = 0.10\ A$.
Because $V_a = V_d$, we can find the relative potentials of b and e:
$V_b - V_e = (V_b - V_a) - (V_e - V_d) = I_1R_3 - I_2R_4$
$= (0.10\ A)(25\ \Omega) - (0.025\ A)(60\ \Omega) = +1.0\ V$.
The charge on the capacitor is
$Q = C(V_b - V_e) = (5\ \mu F)(1.0\ V) = \boxed{5\ \mu C}$.
Because $V_b > V_e$, $\boxed{\text{the top plate is positive}}$.
(b) The current through the 35-Ω resistor is $I_1 = \boxed{0.10\ A}$.

81. Because the emf has negligible resistance, the terminal voltage, which is the voltage across the capacitor, is the emf V_0. The resistance of the material in the capacitor is
$R = \rho L/A = L/\sigma A = d/\sigma\pi r^2$.
(a) We find the electric field between the capacitor plates from the potential gradient:
$E = \boxed{V_0/d}$.
(b) The current density depends on the electric field:
$J = \sigma E = \sigma V_0/d$.
The current is
$I = JA = (\sigma V_0/d)\pi r^2 = \boxed{\sigma\pi r^2 V_0/d} = V_0/R$.

82. When we add another rung, as shown in the diagram, we have the resistance R^* in parallel with a resistor, which has an equivalent resistance of
$R_{eq} = RR^*/(R + R^*)$.
This equivalent resistance is in series with two resistors.
Because the resistance does not change, we have
$R^* = 2R + [RR^*/(R + R^*)]$, which gives
$R^{*2} - 2RR^* - 2R^2 = 0$.
The solutions to this quadratic equation are
$R^* = (1 \pm \sqrt{3})R$.
Because the resistance cannot be negative, we have
$R^* = (1 + \sqrt{3})R = \boxed{2.732R}$.



(a)



(b)