I.4-1 A horizontal turbulent liquid jet of diameter D_1 and average velocity v_j strikes concentrically a vertical plate having an orifice at the center[30], thus exerting a force F on the plate, as shown in Fig. P1.4-1 (see p. 104). The jet leaving the orifice has a diameter D_2 and the same average velocity v_j. Determine the force needed at the steady state to hold the plate in place. Neglect the viscous and gravity forces.

ρ = density
n = normal vector to control area
dA = differential control area
A = overall control area
P = momentum

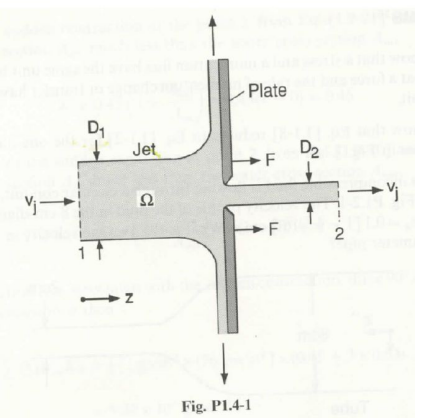

Fig. P1.4-1

Integral momentum-balance equation:

$$\frac{dP}{dt} = \left(\int\int_A \rho v(\boldsymbol{v}\cdot\boldsymbol{n})\, dA\right)_{in} - \left(\int\int_A \rho v(\boldsymbol{v}\cdot\boldsymbol{n})\, dA\right)_{out} + F_p + F_V + F_b$$

I'm letting my control volume be a hollow cylinder, where the outer diameter is D_1 and the hollow part is D_2

Since the control area is circular and perpendicular to direction of flow, and v_{out} is 0, and there is no F_V or F_b:
$0 = \left(\rho v_j^2 A\right)_{in} - 0 + F_p + 0 + 0$

$0 = \left(\rho v_j^2 A\right)_{in} + F_p$

Final:
$F_p = -\rho v_j^2 \pi\left(D_1^2 - D_2^2\right)$