Modelling of a Bouncing Ball

18th September 2007

1 Objectives

The purpose of this practical is to familiarise yourselves with Simulink as a modelling and simulation environment by designing and simulating a model of the bouncing ball which you will be given. Once the model of the bouncing ball has been completed, you will be required to physically implement the system using op-amps.

2 Requirements

You are required to develop an accurate 1D model of the bouncing ball that you are supplied with for the purposes of simulation. The steps that you should follow are roughly set out below.

2.1 Development of the Model

You are required to derive a mass-spring-damper model of the bouncing ball in a single dimension. In reality the spring and damper are contained within the material of the bouncing ball that deforms to store energy during a collision. Once you have the differential equations associated with this model you will be able to determine which parameters will have to be obtained experimentally[1]. A diagrammatic representation of the model together with any free body diagrams and resultant differential equations should be presented in your report.

2.2 Obtaining Experimental Data

Devise and conduct a suitable experiment to determine the unknown parameters of the model you just created. Note that the values you obtain will be dependent on the floor used, make sure you keep this constant. The final values of the parameters together with all your measurements must be included in your report. Remember to justify your experimental method as well as the number of measurements which you have obtained.

2.3 Simulation of the Bouncing Ball

Now that you have a differential equation modelling your bouncing ball, simulate the system in Simulink by creating a block diagram of the model. When implementing the system in Simulink, take care to use blocks that can be easily implemented using op-amps. Include in your report the block diagram of the system together with plots of the state variables and the force exerted on the ball by the floor.

[1] HINT: The resultant model should be piece-wise linear and continuous.

2.4 Op-amp Implementation

Implement a practical simulation of your bouncing ball model using op-amps[2]. The optimality of your solution (ie. the number of op-amps used) will be considered when awarding marks. You may choose to either simulate your circuit first, or proceed directly to building the final circuit. If you created a simulation, include it in your report as it may help if you do not complete the physical implementation in time. The practical implementation will have to be demonstrated before you will be awarded marks for the operation of your circuit. The schematic of the completed op-amp must be included in your report.

The following items should be taken into consideration when designing your circuit.

Keep the gain of your op-amp circuits within practical limits The op-amps that you will be using are not ideal, nor is your power supply or your bread-board. Noise is present on the inputs and outputs of all your op-amps, therefore, if you implement an op-amp with a large gain (generally in excess of few 10's of thousands, depending on the op-amp), these noise artifacts are amplified and will negatively affect your results. To compensate for excessively high gains, you may include them into the gains of your integrators.

Integrators and Summers have gains too Not only inverting and non-inverting amplifiers have gains. Almost all op-amp systems have some form of gain. Use this to your advantage when building your circuit. It will reduce your part count.

Beware the inverting amplifier Most op-amp configurations are inverting. At first this may seem a little irritating, just keep track of the signs of the various signals and ensure that the loops are sign preserved.

Use practical resistor values The resistors available in the lab are part of the E12 series. Take this into consideration.

Stay within valid voltage levels The outputs of the op-amps are typically limited from -15V to +15V. Do not design a circuit where the required output voltage of the op-amp exceeds these values. It will not work.

The parameters you are trying to simulate are not voltages The state values that you are simulating with your op-amps are not voltages, therefore you should derive suitable conversion ratios to allow the simulation to remain within bounds (eg 1m = 1V or 1m/s = 100V). Use the results from your Simulink model to derive these relationships.

3 Outcomes

The final outcome of this practical will be a report highlighting the work done with your results. Additional marks will be awarded based on the quality and professionalism of your report. As a minimum your report should contain an aim, the results, a discussion or interpretation of the results and a conclusion. Things that will be considered when marking your report are,

- Significant figures
- Units
- Clear and legible figures and writing
- Correct labelling of axes of graphs

[2] HINT: Since the model is non-linear, you will require a non-linear circuit element.

- Grammar and spelling
- References

3.1 Questions

The following questions must be answered in your report.

1. Define the *Coefficient of Restitution* (CoR) and calculate it for your model.
2. What is the E12 series, what values are available in it and what other E-series are there?

3.2 Mark Allocation

The mark allocation for this practical is as follows.

Model and Experimental Results	30%
Simulink Results	20%
Op-Amp Results	20%
Op-Amp Optimality	20%
Report Structure	10%
Bonus Work	20%

The op-amp optimality will be evaluated in a competitive framework among your peers. If your op-amp simulation does not work, you will receive 0% for this section. Thereafter, marks will be awarded based on the relative optimality of your solution in relation to the average of the remaining working solutions. Thus, the mark that you obtain for this section will be based purely on your performance relative to your peers. This method also discourages copying as if everyone has the same number of op-amps in their solutions, everyone will receive 10% for this section.

The bonus work section allows you the opportunity to obtain more than 100% for your practical. Very little to no assistance will be rendered for this work as it tests your ability to tackle more sophisticated problems on your own.

4 Bonus Work

Derive and simulate, in Simulink, a higher order model of a bouncing ball that follows the trajectory given below in Figure 1[3]. The actual parameters required for this model will be very difficult to determine experimentally and so you will be allowed to use approximate values that produce reasonable results. The results only have to match the supplied figure in general shape, not an exact match. Include in your report a plot of all 6 state variables vs time, the constants and initial conditions used in your model as well as a plot to mimic the result shown in Figure 1 below.

[3]HINT: You will require at least 6 states to achieve this result.

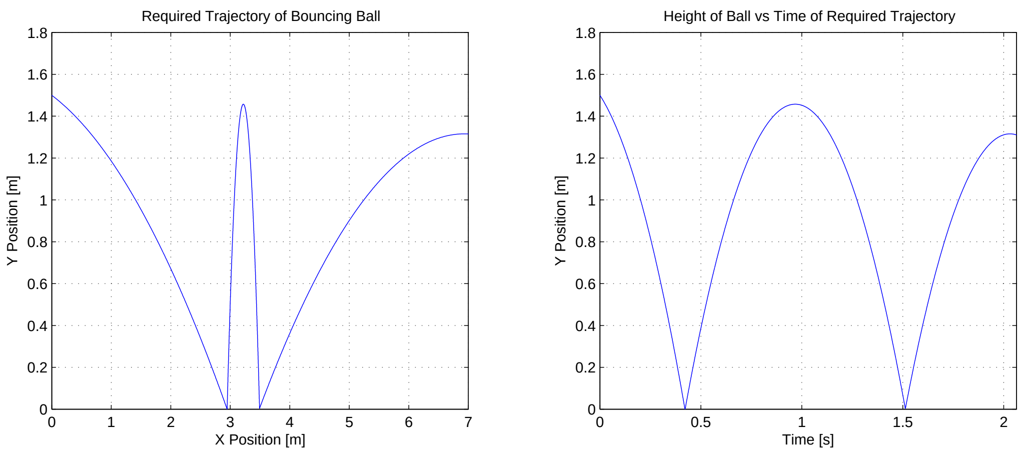

Figure 1: Required Trajectory of Bouncing Ball for Bonus Work Section.