Group Problem Solving I: Vectors and Integration

Name: ____________________

Partners: ____________________, ____________________

1 D kinematic equations:

$v(t) = \frac{dx}{dt}$, $a(t) = \frac{dv}{dt}$, $\Delta s = s(t_f) - s(t_i) = \int_{t_i}^{t_f} v(t)dt$, $\Delta v = \int_{t_i}^{t_f} a(t)dt$

At constant acceleration: $\Delta s = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$ $a = \frac{\Delta v}{\Delta t}$

$\int_{t_i}^{t_f} Ae^{-bt}dt = \frac{A}{-b}e^{-bt}|_{t_i}^{t_f}$

1. A steel ball is fired from a ballistic launcher at different angles. The launched ball has been found to travel from the edge of a table to land 30.0 cm from the far end of the table when starting from the height of the table and launched at an angle of 30.0° above the horizontal. When launched at 45.0°, the ball easily clears the table to land on the floor.

(a) Ignoring air resistance, write an expression for the x and y components of the acceleration of the ball while it is in the air. Next write an expression for the full acceleration vector in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. Here and below, you may assume that $t = 0$ at the instant that the ball leaves the launcher.

$a_x =$ ____________ $a_y =$ ____________

$\vec{a}(t) =$

(b) On its maximum setting the speed of the ejected steel ball is 5.4 m/s. Using this value, write an expression for the x and y components of the velocity of the ball when the angle of launch is 30.0°. Next, write an expression for the full velocity vector as a function of time.

$v_{x,30^0} =$ ____________ $v_{y,30^o} =$ ____________

$\vec{v}_{30^o}(t) =$

and when the angle of launch is 45.°,

$v_{x,45^0} =$ ____________ $v_{y,45^o} =$ ____________

$\vec{v}_{45^o}(t) =$

(c) From your expression for $v_{y,30^o}$, solve for the time that the ball was in the air, when the ball was launched at a 30.0° angle. (*Hint: what velocity would the ball hit the table at when it returns to the same height?*)

t_{30° = ____________

(d) Letting the edge of the table from which the ball was launched have coordinates (x(0),y(0))=(0,0), write an expression for the x and y components of the object's position vector. Next, write an expression for the full position vector.

$r_{x,30^\circ} = x_{30^o} =$ ____________ $r_{y,30^o} = y_{30^o} =$ ____________

$\overrightarrow{r}_{30^\circ}(t) =$

and when launched at 45.0°,

$r_{x,45^\circ} = x_{45^\circ} =$ ____________ $r_{y,45^o} = y_{45^o} =$ ____________

$\overrightarrow{r}_{45^\circ}(t) =$

(e) Using your results from (d), estimate the length of the table. Note that the steel ball landed about 30.0 cm from the end of the table when fired at a 30.0^o angle.

(f) Assuming that the table is 1.0 m high, find how far from the edge (at its base) of the table the ball shot at 45.0° first hits the ground.

2. (Part of this question is from P. 68 # 80 (3rd ed.) or P. 71 # 80 (2nd ed.) of Knight, Physics for Scientists and Engineers). Careful measurements have been made of Olympic sprinters in the 100-meter dash. A quite realistic model is that the sprinter's velocity is given by $v_x = a(1 - e^{-bt})$ where t is in s, v_x is in m/s and the constants a and b are characteristic of the sprinter. Sprinter Carl Lewis' run at the 1987 World Championships is modeled with $a = 11.81$ m/s and $b = 0.6887$ s^{-1}.

(a) Find an expression in terms of a and b for the distance traveled by a sprinter at time t. (Take the distance traveled at $t = 0$ to be 0 m). Your expression should simplify to have three terms.

(b) Assuming that Usain Bolt followed this model during his world record breaking run in Berlin, use the below provided split times to estimate values of a and b for Mr. Bolt. Then, using your model, estimate the speed at which Mr. Bolt was traveling as he reached each measured distance.

The split times for Usain Bolt at the IAAF World Championships in Berlin were: $t_{20m} = 2.89$ s, $t_{40m} = 4.64$ s, $t_{60m} = 6.31$ s, $t_{80m} = 7.92$ s, and $t_{100m} = 9.58$ s. (source: Biomechanics Project - Berlin 2009, http://berlin.iaaf.org/news/kind=101/newsid=53084.html)

(*Hint*: it is a good idea to use the approximation that the terms proportional to $e^{-bt_{100}}$ and $e^{-bt_{80}}$ can be dropped from your displacement expressions at 100 m and 80 m [since the exponential terms are smallest at large times], then use the resulting two equations to solve for a and b for Mr. Bolt. Then with values of a and b, evaluate the full expression for the velocity.)