NOTES ON GRAPHING. 253, SMITH

1) Find points where the function is not defined, and thus find the domain of the function, (or the domain may be given). The domain will consist of some intervals, either with or without endpoints. Shade the domain on the x axis, and use open or closed brackets to indicate which endpoints are missing and which are included. Draw dotted vertical lines at the missing endpoints. These lines may turn out to be vertical asymptotes but do not have to be. (We will assume that our functions are all continuous where defined.)

2) Find all the first order critical points (places where the function itself is defined but the first derivative is either not defined or is zero), and plot them by a dot. If the first derivative is actually zero there, put a short horizontal line through the dot to remind yourself that the graph levels off there for an instant. (The first order critical points do not have to be either local max's or local min's, but they are the only possible points where a local max or local min can occur.)

3) Find the values of the function at all included endpoints of the domain and plot them, and find all one sided limits at missing endpoints of the domain; if ∞ or $-\infty$ is an endpoint, it is like a missing endpoint so find the limit. If the limit of f(x) as x approaches ∞ or $-\infty$, is a finite number c, then draw a dotted horizontal line at height c. This line will be a horizontal asymptote.

4) Find all second order critical points (points where the function is defined, but the second derivative is not defined or is zero; if the first derivative is not defined, the point is considered both a first order and a second order critical point), and plot them, and mark them with a star ($*$) instead of a dot. The second order critical points are the only possible flexes (points where concavity changes), but they do not have to be flexes.

5) If necessary, test the first order critical points to see whether they are local max, local min, or neither, and the second order critical points to see if they are flexes or not. For some points this information is obvious just by looking at the graph, but using derivatives helps make sure and guard against errors. Identify intervals of increasing and decreasing, either by looking at the critical points already plotted and using the limits already found if possible, or by using the first derivative if necessary.

6) Graph the function, using the previous information. It is sometimes helpful to plot the y intercept by setting x=0 and finding f(0) if f is defined there, and to plot the x intercepts by setting f(x) = 0 and solving for x if possible. This should be done when it is easy, but it is not always easy, and not always useful.

The most important information to find is the domain, the critical points, and the one sided limits at missing endpoints. This already gives you a good idea of the graph. After those in importance come the second order critical points. If there are no critical points, plot x and y intercepts, or just plug in and plot a few easy points to have somewhere to begin.