On the Proper Understanding of the "Twin Paradox"

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ABSTRACT

In this paper, we will show that with proper understanding of the concept of the invariance of the "intervals" of events in Minkowski's spacetime, the problem of why (not the usual how) the "clock paradox" phenomenon happens can be easily understood without any confusion.

Keywords: Twin paradox, clock paradox, special relativity, invariance of intervals

1. INTRODUCTION

It has been over a century now that Einstein announced the theory of special relativity (SR) in 1905 [1]. Numerous experimental results exist that show both the accuracy and correctness of the SR theory. However, confusions still exist due to the improper understanding of some aspects of the theory. This is especially true concerning the so-called problem of the "twin paradox", or equivalently, the problem of the "clock paradox". Many textbooks certainly contribute to such confusions. Due to the nature of the topic, there are literally hundreds of references up to now, and it would be impractical to list them all here. Therefore, most of the references listed in this paper are meant to be examples only.

The "twin paradox" or the "clock paradox" problems can be briefly stated as follows: The times of two identical clocks are initially synchronized at the same point and time (same event), then we move one of the clocks away (usually at high speed to see substantial effects) and then move it back again. When we compare the elapse times of the two clocks when they meet again at the same point, it turns out that the elapsed time of the moving clock is slower than that of the stationary clock. Thus, the travelling twin is younger than the stationary twin. Nowadays, this phenomenon can be shown experimentally to be real to a very high degree of accuracies [2,3], using atomic clocks and decays of particles. We can also readily show using both the algebraic approaches (via Doppler shifts of light pulses) [4] and the geometric approaches (via Minkowski spacetime diagrams) [5] that how such phenomena arise. However, the explanation for why such phenomena happen is still lacking, giving rise to still wide spread confusions to the readers.

Some authors [6] suggest that Einstein's general relativity (GR) is required in order to understand the phenomenon. They insist that the moving clock must experiences numerous accelerations and decelerations in making the round trip, and the time difference can be accounted for by using only GR. This is not true. The effects of accelerations and decelerations will certainly increase the time difference from the value calculated by using SR alone. However, as will be shown later, we can configure situations whereby the moving clocks do not experience any acceleration or deceleration forces, at least in principle. So invoking GR does not really help in showing why such a phenomenon exists.

Many authors [7] in textbooks simply refuse to comment on the reasons that why the elapsed times of the two clocks are different. They just, more or less, stated that the phenomenon is real, and leave it at that [8]. At least one recent author [9] suggested very briefly that something weird must have happened during the acceleration and deceleration phases of the traveling clock.

A large number of authors follow Feynman's opinion [10] that the problem is non-symmetric, because the travelling clock feels the forces of accelerations and decelerations, so it "knows" that it is moving, and consequently slows down its rate of time accordingly. Although the problem is certainly non-symmetric, however, the non-symmetry is not due only to the accelerations and decelerations, but it is also a result of the events in the geometry of the spacetime itself. We will also argue that non-symmetry of frames of reference is not the cause of the phenomenon, since the phenomenon still exists during part of the arrangement where the two frames are strictly inertial and symmetric.

Many authors in the past, before we can actually conduct accurate experiments, simply refused to believe that the times indicated by the two clocks could be different [11]. Their opinions can now be totally ignored as they were proven experimentally to be wrong.

It is also interesting to note that Einstein has never directly answered the problem of the "clock paradox" [12]. He simply stated that the problem is due to the non-simultaneity of the events, and if the phenomenon is real then it is a very peculiar case. In addition, Einstein suggested that using SR alone, the problem could never be understood; in order to solve the problem completely, GR is needed [13]. This is true only if we want to take the effects of the accelerations and decelerations into account.

Reference [14] gives a comprehensive catalogue of the literature concerning the "clock paradox" problem up to the year 1970, and also describes most of the argument in support and in refute (which we now know that they are wrong) of the phenomenon. Interested readers should try to consult it.

It is the purpose of this paper to show that the "clock paradox" phenomenon is a natural consequent of one of Einstein's postulates that speed of light in empty space is constant or invariant with respect to an observer in any inertial frame of reference, resulting in the property of the invariance of the intervals (see section 2). The author hopes also that the following exposition will satisfy most readers on the question of why such phenomenon happens, and confusions, as far as this question is concerned, can be eliminated once and for all.

In order to keep the paper reasonably short, the author assumes that the readers are fairly knowledgeable with the SR theory, and know something about Minkowski's spacetime diagrams, and how to manipulate them. Especially how to read times and distances from such diagrams.

2. THE CONCEPT OF THE INTERVALS

In this paper, we will show that with proper understanding of the concept of the invariance of the "intervals" of events in Minkowski's spacetime, the problem of why (not the usual how) the "clock paradox" phenomenon happens can be easily understood without any confusion.

$$I = (X^2 + T^2)^{1/2} \tag{1}$$

Where X is the value of the spatial distance and $T = jct$, where j denotes a complex number, c is the speed of light in free space and t is the time. It is also evident that the value of T^2 is always negative. We can further see that the value of I can be real or complex depending upon the values of X^2 and T^2.

Figure 1 shows a typical spacetime diagram, where the two 45° dashed lines denote the worldlines of photons moving to the right (+c) and the left (-c) respectively, where c is the upper limit of the speed of any particle. The four hyperbolic lines represent the cases of $I=\pm$j, $I=\pm1$, are used to scale a unit time and a unit distance lengths along the time and the spatial axes in each appropriate quadrant respectively.

The most important property of the interval, as has been pointed out by Minkowski, is that it is invariant with respect to coordinate transformations, which can be easily proven algebraically [16]. Remember that in SR, we must use the set of Lorentz transformation equations to transform the

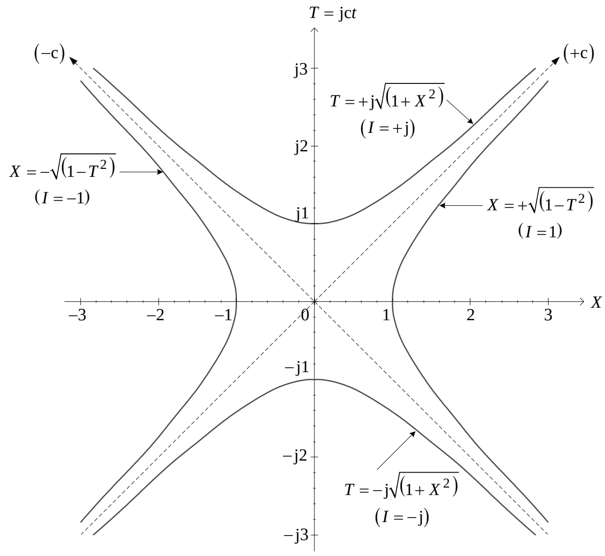

Figure 1 A typical Minkowski's spacetime diagram. The two diagonal dashed lines are the worldlines of photons travelling to the right (+c) and to the left (-c). The four hyperbolic graphs are used for unit lengths scaling of the axes in spacetime in each of its quadrant.

coordinates. As will be apparent later on, this property holds the key to a deeper understanding of the "clock paradox" problem. The author feels that this invariant property of the interval has not been emphasized enough in the literature or even in most textbooks. In fact, starting off with the invariance of intervals, most phenomena in SR can be derived extremely easily. For students new to SR, they should accept this property as one of the important laws of nature, in the same way that they are told to accept other laws of nature.

3. THE CASES OF TWO INERTIAL FRAMES OF REFERENCE

In order to gain a better insight into the "clock paradox" problem; we will, firstly, consider the cases of two inertial frames of reference. Strictly speaking, observers in two inertial frames can meet at only one common event, that is when the two observers are at the same spatial coordinates at the same time, and the two observers can use the opportunity to synchronize their clocks. As they move apart, the observer in each frame can measure the time of an event in the other frame. The resulting measurements will indicate to each observer that the clock of the moving observer is slow by exactly the same factor of $(1 - v^2/c^2)^{1/2}$, where v is the relative speed between the two frames and c is the speed of light in free space. So the natural question arises that whose clock is correct? Because of the symmetry of this effect, we usually referred to such an arrangement as the symmetrical case. This is the most important effect that renders confusions into the phenomenon of the "clock paradox". In most textbooks students are told not to worry about it, since the two observers in their inertial frames can never meet again. Therefore, there is no way for them to compare their respective times. In fact, there is a way for the two observers to compare their times by transmitting their measured times of the events to each other, say, by using encoded beams of light. In any case, such questions are meaningless, since the events in the two frames are different and distinct. Therefore, each observer is measuring the times of two separate situations, which should really not lead to any paradox. The two observers (assuming they are well educated) should also understand perfectly well that the clock in the frame which is moving relative to the observer's own frame appears to be slow because of the SR effect.

Next we will consider a more important case which should lead to better insight for the "clock paradox" problem, by still using two inertial frames of reference, but there is only a single **physical event** in one of the two frames, which can be shown graphically using spacetime diagrams as in figures 2a or 2b. Therefore, strictly speaking, the two inertial frames are now non-symmetrical.

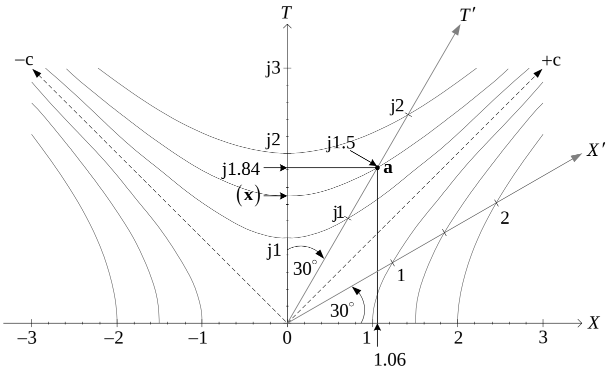

Figure 2a Shows two inertial frames of reference, where the frame T-X is stationary and the frame T'-X' is moving to the right with a speed of 0.5774c.

In figure 2a, without any loss of generality, we can let the frame *T-X* be a stationary frame, and frame *T'-X'* is moving to the right with a speed of, say, approximately 0.5774c. Therefore, the angle between the *T* and the *T'* axes, and the angle between the *X* and the *X'* axes are both 30º. We will further let **a** be a physical event with an interval j1.5 situated on the *T'* axis, which is the intersection of the *T'* axis with the hyperbolic line of constant interval equals to j1.5. Therefore, the time of the event as measured by an observer in the frame *T'-X'* is j1.5, while the spatial coordinate as measured by the same observer is zero. It is immediately seen that the interval of the event **a**, as measured by the observer in the frame *T'-X'*, is indeed j1.5. The time and the spatial coordinates of the event **a**, as measured by the observer in the frame *T-X*, are approximately j1.84 and 1.06 respectively. It should be noted from the spacetime diagram that these two values are measured by drawing the lines parallel to the *X* and the *T* axes, since in order to make the measurement, the observer in the frame *T-X* must imagine that he is situated at the event **a**'s coordinates. Do not confuse that the two parallel lines represent some sort of remote measuring signal beams! It is then quite straightforward to calculate that as far as the observer in the frame *T-X* is concerned, the interval value of the event **a** also equals to j1.5.

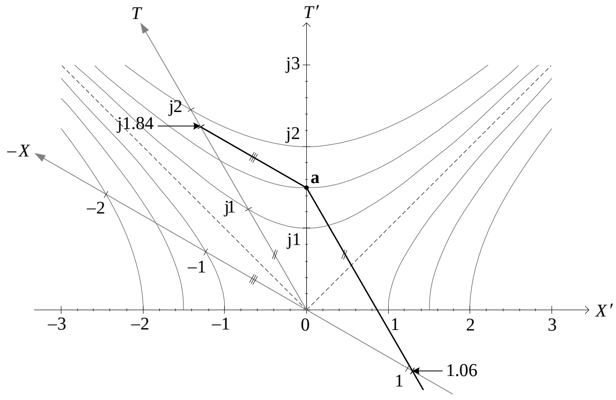

Figure 2b Shows the opposite case of figure 2a, where the frame T'-X' is now stationary and the frame T-X is moving to the left with a speed of 0.57

Figure 2b shows the case when we let the frame *T'-X'* be stationary and the frame *T-X* is then moving to the left with a speed of approximately 0.5774c instead. This is the reverse situation to the case of figure 2a. The event **a** must remain on the line of j1.5 interval. It is immediately seen that in the frame *T'-X'*, the time coordinate is j1.5 and the spatial coordinate is zero, while in the frame *T-X*, the time and the spatial coordinates remain approximately equal to j1.84 and 1.06 respectively.

One of the obvious results from figures 2a and 2b is that for any two inertial frames having coincident events at their origins, and suppose there is only one frame that possesses another physical event, say **a**, along the time axis. The time of the event **a** as measured by an observer in the event frame will be the slowest (or the smallest value). This follows immediately from the invariant property of the interval. Since measurement from the other frame will have non-zero time and spatial coordinates; therefore, in order to keep the same value of interval, the time coordinate must be faster (or larger) than the value as measured from the event frame, to take into account the non-zero spatial coordinate value. This can be readily seen from the definition of the interval, i.e. $I = (X^2 - |T|^2)^{1/2}$. We will make use of this conclusion to gain insight to the "clock paradox" problem later on.

It is important and informative to consider what would happen if we were to put another physical event, say **x** on the *T* axis of figure 2a also having a time value of j1.5, then we can readily show that an observer in the *T'-X'* frame will read the time of the event **x** to be j1.84, exactly symmetrical to the case of the event **a** as described previously. Such a consideration is the fundamental cause of all the confusions and controversies surrounding the "clock paradox" problems stated previously. However, before we go into a state of despair, let us consider the situation more carefully. Since the events **a** and **x** are different and distinct physical events, the fact that both observers measure different times for both events are perfectly allowable, and the resulting measured values follow the SR theory exactly. Both times will be real to each observer, as we will argue for in the following paragraphs. In fact, if the measured times of the two events in each observer's frame were to be the same value, then it is time that we should start despairing for the future of the SR theory!

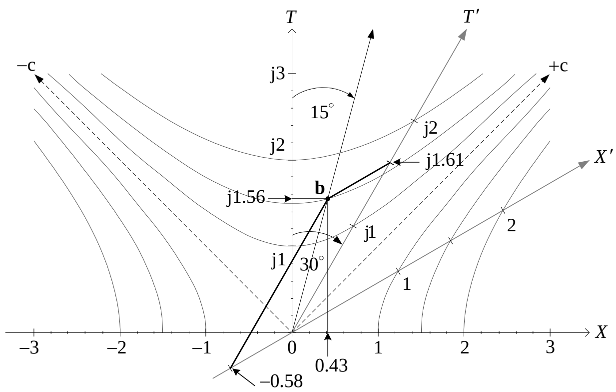

Figure 3a Shows a more general case of figure 2a.

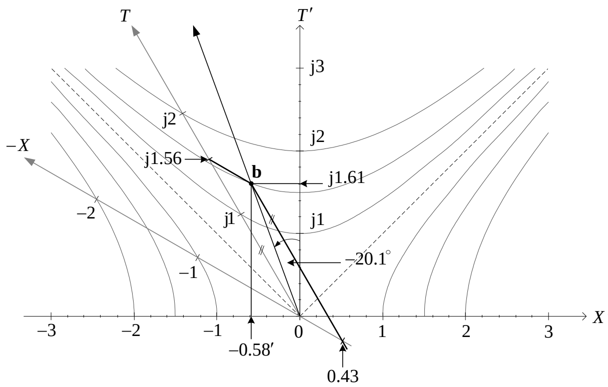

Figure 3b Shows a more general case figure 2b

Before making further observations, we will consider more general cases of figures 3a and 3b, where in the figure 3a, the frame *T-X* is stationary and the frame *T'-X'* is moving to the right with a speed of 0.5774c, which is the same as in figure 2a. We will arbitrarily choose a physical event **b** having j1.5 interval and is moving to the right with a speed of 0.2679c. Therefore, the angle of the worldline of the event **b** will be 15° with respect to the *T* axis. Using such a spacetime diagram, the measured values of the time and spatial coordinates of the event **b** by the observer in the frame *T-X* are j1.56 and 0.43 respectively, while the same respective values from the point of view of an observer in the frame *T'-X'* are j1.61 and −0.58. We can readily check that both sets of coordinates yield the same value of interval equals to j1.5.

Next we transform the frames, as in figure 2b, so that now the frame *T'-X'* is stationary and the frame *T-X* is moving to the left with a speed of 0.5774c. We will also transform the worldline of the event **b**, so that the relationships to the two frames are still the same. In order to do this we can use the "velocity addition" formula as first announced by Einstein, which can be written as:

$$w = (u + v) \,/\, (1 + uv \,/\, c^2) \qquad (2)$$

where u is the speed of the frame *T-X* with respect to the frame *T'-X'* after transformation, v is the speed of the event **b** with respect to the frame *T-X* before transformation and w is the speed of the event **b** with respect to the frame *T'-X'* after transformation. Equation (2) can be expressed in terms of various appropriate tangents of angles by dividing through with c. We can then easily substituting the appropriate tangent numbers to obtain the angle between the worldline of the event **b** with respect to the time axis of frame *T'-X'*, which is approximately −20.1°, where the minus sign indicates that the worldline will be on the left side of the *T'* axis, but the event itself is still on the line of constant j1.5 interval. From the graph of figure 3b we can obtain the time and the spatial coordinate values with respect to frame *T'-X'* to be j1.61 and −0.58 respectively. The respective values of the time and the spatial coordinates of the frame *T-X* are j1.56 and 0.43. These two sets of values are exactly the same as previously obtained from the figure 3a.

From the results of figures 2a, 2b, 3a and 3b, we can conclude that provided the relationships of the coordinates of a physical event with respect to the two frames of reference that possess coincidental origins are the same, the values of both the time and the spatial coordinates of the event, as measured by an observer in the same frame of reference, will be invariant whether we consider the frame to be stationary or moving. This make perfect sense, since each observer in his own frame feels that the frame is stationary, irrespective of the way it is shown in the spacetime diagrams. In fact, in the cases of figures 2a, 2b, 3a and 3b it is easy to transform the two frames by substituting appropriate values into the "velocity addition" formula (by either using phantom perpendicular frames or using hyperbolic geometry), so that neither of the two frames is now stationary, and the resulting measured values of the coordinates of the events **a** and **b** by the respective observers are still the same as in the corresponding cases of figures 2a, 2b, 3a and 3b! Therefore, the time and the spatial values of the event as measured by both observers in their own frames must be real to the respective observers, since they are always the same values, even if the values from the two frames of the same event are different by the natural results of non-simultaneity.

The two conclusions in this section are all we need to completely understand the phenomenon of the "clock paradox", as will be shown in section 5. In fact, all the effects of the "clock paradox" exist even when we consider only two inertial frames! Since the values as measured by the two observers can be remotely transmitted to each other. Thus, using a third frame of reference for the return trip simply increases the values of the elapsed times of both observers, and it also provides a convenient way to compare the times of the two clocks at the end of the trip.

4. THE "CLOCK PARADOX" WITHOUT GENERAL RELATIVITY

In this section we will demonstrate a method that can be used in the "clock paradox" problem without the need to consider the effects of various accelerations and decelerations, at least in principle. Such a method was first proposed by Lord Halsbury in 1957 [17], as a "triplet" or a "three clocks" problem, and can be briefly stated as follows:

Let **A**, **B** and **C** be three inertial frames of reference. Frame **A** is stationary with a **clock 1** situated initially at its origin. Frame **B** is moving away to the right of frame **A** with a **clock 2**, and frame **C** is moving to the left toward frame **A** with a **clock 3**, where both **clocks 2** and **3** are initially situated at the respective origins of their frames. At the event **a,** the origins of frame **A** and frame **B** coincide with each other, and the times of **clocks 1** and **clock 2** are synchronized. At the event **b**, the spatial origins of frame **B** and frame **C** coincide with each other, and the time of **clock 2** is transferred to **clock 3**. At the event **c**, the spatial origin of frame **C** coincides with the spatial origin of frame **A**, and the times of **clocks 1** and **clock 3** can then be compared.

It is somewhat surprising that Lord Halsbury's method is not more widely known in the literature. This is probably due to early objections that **clocks 1** and **clock 3** cannot be readily synchronized, since they are not together at the initial event **a**. However, as we have concluded in section 3, the times of **clock 1** and **clock 2** are real, so in transferring the time of **clock 2** to **clock 3**, and there is no need to synchronize **clock 1** and **clock 3**.

5. SPACETIME DIAGRAMS FOR THE "CLOCK PARADOX" PROBLEM

In this section, we will use the actual spacetime diagrams of the "clock paradox" to reinforce our argument in section 3. Figures 4a shows the simplest version of such a spacetime diagram, where we will assume the inertial frame *T*-*X* to be stationary with **clock 1** situated at its origin. Another inertial frame *T'*-*X'* is moving to the right at a speed of, say, 0.5774c. Therefore, the angle between the time axes *T'* and *T* is +30°. The **clock 2** is situated at the origin of the frame *T'*-*X'* which coincides with the

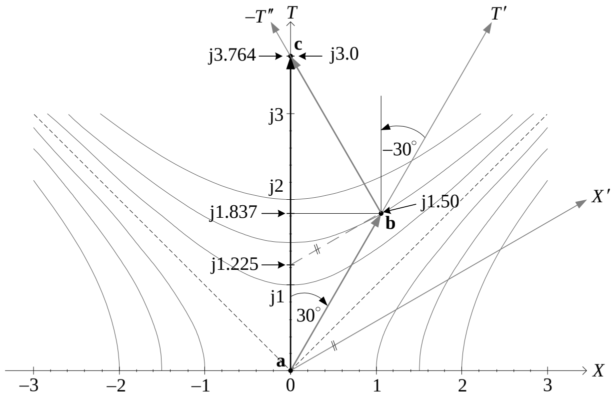

Figure 4a Shows the simplest spacetime diagram of the "clock paradox" problem, using the frame *T*-*X* (stationary frame) as the reference frame.

origin of the frame *T*-*X* at the event **a**. Thus the frame *T'*-*X'* represents the outward-bound portion of the trip of the **clock 2**. We will further assume that when the **clock 2** registers a time of j1.5 at the event **b**, it starts to turn back by moving to the left with the same speed, thus the **b** event is a physical event. Furthermore, **clock 2** is now in a different inertial frame, say, *T''*-*X''*, and the angle between the time axes *T* and *T''* is −30°. At the event **c** the **clock 1** will again be coincidental with the **clock 2**, and the observers in their respective frames can compare the elapsed times on their respective clocks. From this example, we can see that the times registered by the **clock 1** at the events **b** and **c** will be approximately j1.837 and j3.674 respectively, while at the same two events, the **clock 2** will register time values of j1.5 and j3.0 respectively. Therefore, the **clock 1** runs faster than the **clock 2** by approximately j0.674 unit time. In other words, the travelling twin is younger than the stationary twin.

We can understand this phenomena completely by noting, firstly, that during the outward bound trip, the spacetime relationships between the two clocks are continuously constant between the two events **a** and **b**, so the time of **clock 1** will be such that the continuously changing different values of intervals of the moving **clock 2** will be preserved by **clock 1** at every instant for the whole outward-bound trip. Thus, at the event **b** the interval as measured by the observers in the frames *T*-*X* and *T'*-*X'* will be equally j1.50. Therefore, the times as read by their observers of the two clocks are real, as we have concluded from section 3. We can apply exactly the same reasoning to the inward-bound portion of the trip. Thus, resulting in two different times of **clock 1** and **clock 2** when they meet again. We can certainly understand this results by using the two conclusions from section 3. Firstly, the **clock 1**'s time is faster than the time of **clock 2** because **clock 2**'s spatial coordinate at the event **b** is zero, whereas the spatial coordinate as measured by the observer of **clock 1** at the event **b** is non-zero and the time of **clock 1** has to be faster in order to preserve the interval value of the event **b**. The same argument also applies to the inward-bound portion of the trip. Secondly, because the times register by the two clocks will be invariant, irrespective of how we consider which frame to be stationary. Therefore, the times as measured by the respective observers will be real to the observers. At the end of the trip when the two observers meet again, they will see no real controversy, since they will understand that their respective times are different because **clock 1** and **clock 2** have been through different regions of spacetime. In fact, following our reasoning, it would be shocking if their times were to be the same!

It is interesting and illuminating for the observer of **clock 2** at the event **b** to measure the time on the T axis, which will be approximately j1.225. As pointed out previously, this is the root of the confusions, since it is different from the time as registered by **clock 1** at the same event, which is j1.837. In fact, the time j1.225 as measured by the outward-bound observer is physically meaningless (which will be shown later on), since there is no physical event at that time on the T axis. Even if we were to arbitrarily put an event there, the time j1.225 will then belong to an entirely different and separate problem. This is because the outward-bound observer's measurement must stop at the event **b**, and further measurement must be made by in the inward-bound frame. Now the time as measured by the outward-bound observer just before the event **b** is j1.225, but the time on the T axis as measured by the inward-bound observer just after the event **b** is j2.45; thus, there is an **apparent** jump of the time on the T axis just before and just after the event **b**, as measured by the travelling observer. This should not be surprising, since the travelling observer changes the standard base of time measurement just after the event **b** (which will be explained in more detail later on). Therefore, it is clear that the two sets of measurements during the outward-bound and the inward-bound trips are separate problems in themselves, as well as when compared to the original problem. However, it should be noted that after the event **b**, the time on the T axis as measured by the travelling observer will always be slower than the time on the T' axis, and at the end, at the event **c**, the respective times on the T and the T' axes will be j3.674 and j3.0 exactly as previously stated. We will consider the physical significance (or non-significance) of such apparent time jump a bit later.

Figures 4b and 4c illustrate the cases when we consider the outward-bound and the inward-bound frames to be stationary respectively. The readers can inspect the diagrams and see that they lead to exactly the same results of figure 4a. Thus, confirming that the times as registered by **clock 1** and **clock 2** are invariant to the way we consider any one frame to be stationary. In inspecting the figure 4b where the outward-bound portion of the trip is stationary, the angle between the inward-bound time axis (T'') and the T' axis has to be found again by using the "velocity addition" formula, resulting in an angle of approximately -41°, and the same procedure has to be used in figure 4c to obtain the angle of the outward-bound time axis to the T axis, when we consider the inward-bound frame to be stationary.

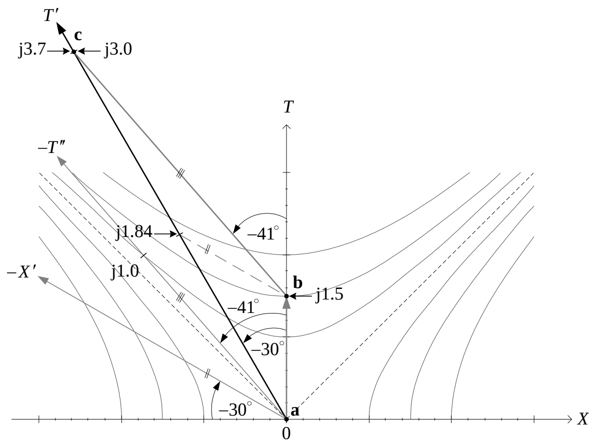

Figure 4b Shows the case of using the outward-bound frame as the reference.

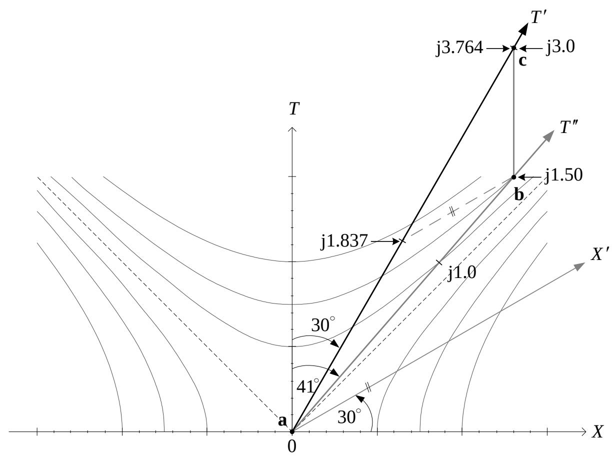

Figure 4c Shows the case of using the inward-bound frame as the reference.

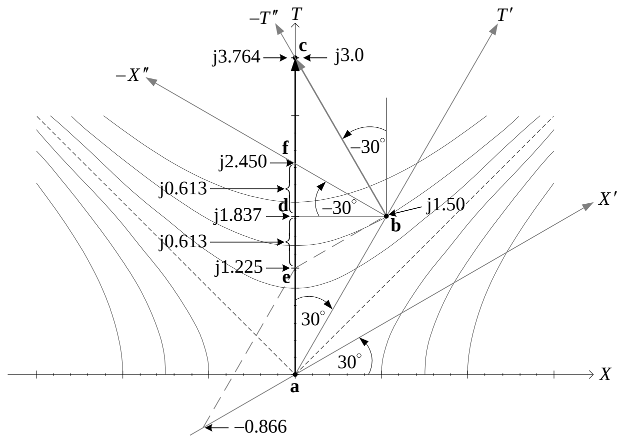

Figure 5 Shows a more detailed consideration of the figure 4a. Note that the equality of the times between the events **e-d** and **d-f** is a special case of equal inward and outward-bound speeds. It is not true in general.

Figure 5 shows a more detailed break down of figure 4a, where the three separate sets of reference frames *T-X*, *T'-X'* and *T"-X"* are clearly shown. Just before the event **b,** the outward-bound observer measured the time of the event **e** on the *T* axis as j1.225; and just after the event **b**, the inward-bound observer measured the time at the event **f,** also on the *T* axis, as j2.450. Thus, indicating an apparent net (instantaneous) time jump of j1.225! From the figure we can readily see that just before the event **b**, the travelling observer uses the line **e-b** (which is parallel to the *X'* axis) to measure the time of the event **e** on the *T* axis, but just after the event **b**, the travelling observer uses the line **f-b** (which is parallel to the *X"* axis) to measure the time of the event **f**. This is clearly the reason for the apparent time jump. It is also clearly seen, from the point of views of the observers in the outward-bound and the inward-bound portions of the trips, that these are separate problems. It should also be noted that the times of the events **e** and **f** are the last and the first times that the travelling observers can measure just before and after the event **b** respectively. There are no other last and first times measurement possible. From figure 5, the time of the event **e** is slower than the time of the event **d** by j0.613; similarly, the time of the event **f** is faster than the time of the event **d** by an equal amount, but this is only a special case of equal outward-bound and inward-bound speeds, and we should not infer any special physical significance into the equality of the time differences. In general, when the speeds are unequal, the time differences will also be unequal. In fact, from figure 5, we can readily see that the value of time at the event **f** as read by the inward-bound observer will always be greater that the time at the event **d**. Therefore, it really does not matter what the values of the times at the events **e** and **f** are, the inward-bound observer will always read the time of **clock 1** at the event **c** as j3.764, while his own time on **clock 2** will read j3.0, exactly as expected. This argument tends to support the author's assertion initially that the values of times at the arbitrarily introduced events **e** and **f** are physically meaningless!

From the aforementioned discussions, it should be cleared to the readers that there are indeed three separate problems, and they should be considered separately on their own to avoid confusions. However, the final combined results will be the correct one. The problem that contains real physical events is the important one, and in analyzing such a problem there will be no apparent non-physical effects. In any case, no matter how we choose to analyze the problem, it seems that the law of the invariance of interval reigns supreme in nature, preventing any possibility of an impossible situation.

6. CONCLUSIONS

In this paper, we have shown that by carefully considering the concept of the invariance of the interval for a pair of physical events in spacetime, we can eliminate all the confusions that surround the phenomenon of the "clock paradox", including, but not limited to, the believe that such an effect requires non-symmetrical frames of reference, or requires accelerations and decelerations, or requires general relativity to properly understand the problem, or requires an absolute frame of reference. The non-symmetry required is not in the frames of reference, but rather in the events that are attached to the frames.

There is certainly no new physics on offer in this paper. However, we believe that the way we looked at the physical significance of the invariance of the intervals of physical events in spacetime is new, and enable us to completely show why the "clock paradox" phenomenon happens. Most of the former publications seem to show how such phenomenon comes about, rather than why, and will only leave the readers to feel more uncomfortable and confused.

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