

MATH 246 (Hamilton) Homework
MAT 1 supplement

The supplementary homework uses a parameters $\{n\}$ which are the third last NONZERO digit of your ID #, e.g. if your ID is 12345030 then $n = 4$. Print your name and this number at the top of every homework. These questions involve complex numbers $z = x + \mathbf{i}y$ which **MATLAB** treats as normal numbers.

Write down the number $a = N[n + 1/n]$ and define the polynomial

$$p(z) = z^4 + az^2 + 1 .$$

1. Use **MATLAB** to find the four roots ζ_k of

$$p(z) = 0$$

Check this by having **MATLAB** perform the calculation $p(\zeta_k)$.

2. For all complex constants c_k define the complex valued function

$$Z(t) = \sum_{k=1}^4 c_k e^{\zeta_k t}$$

Use **MATLAB** to differentiate the complex valued function $Z(t)$ wrt the real independent variable t . By **MATLAB** compute

$$\frac{d^4 Z}{d^4 t} + a \frac{d^2 Z}{d^2 t} + Z = 0$$

Check this by hand using Euler's formula

$$e^{x+iy} = e^x \{ \cos(y) + \mathbf{i} \sin(y) \}$$

3. Hence find the constants c_k so that $Z(t)$ solves the initial value problem

$$Z(0) = 0, Z'(0) = 1, Z''(0) = 0, Z'''(0) = -a$$

4. Finally, compute the ratio

$$\frac{Z''(1)}{Z(1)}$$