Discrete Structures **Homework 2** **Mar 19, 2012**

1. Express the following in terms of predicate logic (using nested quantifier and appropriately defined predicates).

   (a) If $|X| < |Y|$, then there can not be an onto function from X to Y.

   (b) If $|X| > |Y|$, then there can not be an one-to-one function from X to Y.

   (c) **Principle of mathematical induction** is an important proof technique which works as follows: Suppose we want to show that the predicate P is true for all positive integers n, we complete two steps.

   - **Basis step:** Show that P is true for 1.
   - **Inductive step:** Show that for every positive integer k, if P is true for k then P is true for $k+1$.

   (d) **The principle of Well Ordering** states that *"every nonempty set of positive integers has a minimum element"*.

   (e) **The Pigeon-hole Principle** states that if $n+1$ pigeons are placed in n pigeon-holes then some pigeon-hole must contain more than 1 pigeons.

2. Using definitions of the set operations show that if $X \subseteq Y$ and $X \subseteq Z$ then $X \subseteq Y \cap Z$.

3. In each of the following two sets, A and B, are given. Answer the following questions about each pair of them.

   (a) Is $A = B$?

   (b) Is $A \subset B$?

   (c) Is $B \subset A$?

   (d) Compare the cardinalities of A and B.

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\begin{array}{rll}
(i) & A = \mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) & B = \mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))) \\
(ii) & A = \mathcal{P}(X \cup Y) & B = \mathcal{P}(X) \cup \mathcal{P}Y) \\
(iii) & A = \mathcal{P}(X \cap Y) & B = \mathcal{P}(X) \cap \mathcal{P}(Y) \\
(iv) & A = \mathcal{P}(X \times Y) & B = \mathcal{P}(X) \times \mathcal{P}(Y)
\end{array}
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4. Let X be a set and let f_1 and f_2 be functions from A to $\mathbb{R}$. For $x \in X$ let $g(x) = f_1(x) + f_2(x)$ and $h(x) = f_1(x)f_2(x)$. Verify that g and h are functions.

5. In class we showed that $|\mathbb{Z}^+| = |\mathbb{N}|$ and that $|\mathbb{Z}^-| = |\mathbb{N}|$. Give a bijective function $f : \mathbb{N} \to \mathbb{Z}$ to show that $|\mathbb{N}| = |\mathbb{Z}|$ (i.e. cardinality of the set of natural numbers ($[\{0, 1, 2, 3, \ldots\}$) is the same as the cardinality of set of all integers).