

Given the vector equation :

$$(x, y, z) = (x_1, y_1, z_1) + t (a, b, c)$$

Making this a parametric equation :

$$x = x_1 + t a$$

$$y = y_1 + t b$$

$$z = z_1 + t c$$

Solving for t using the first equation :

$$t = \frac{x - x_1}{a}$$

Putting that into the second and third equations gives :

$$y = y_1 + \frac{b (x - x_1)}{a}$$

$$z = z_1 + \frac{c (x - x_1)}{a}$$

Solving the bottom equation for x gives :

$$x = \frac{a (z - z_1)}{c} + x_1$$

Plugging this into the y = equation

$$y = y_1 + \frac{b (2x - x - x_1)}{a}$$

$$y = y_1 + \frac{b \left( 2x - \left( \frac{a (z - z_1)}{c} + x_1 \right) - x_1 \right)}{a}$$

Simplifying a little :

$$y = y_1 + \frac{b \left( 2x - \frac{a (z - z_1)}{c} - 2x_1 \right)}{a}$$

$$a y = a y_1 + b \left( 2x - \frac{a (z - z_1)}{c} - 2x_1 \right)$$

$$a (y - y_1) = b \left( 2 (x - x_1) - \frac{a (z - z_1)}{c} \right)$$

Fully expanding :

$$a y - a y_1 = b \left( (2x - 2x_1) - \frac{(a z - a z_1)}{c} \right)$$

$$\frac{a y - a y_1}{b} + \frac{a z - a z_1}{c} = 2 (x - x_1)$$

Adding the two fractions :

$$\frac{a (c y + b z - c y_1 + b z_1)}{b c} = 2 (x - x_1)$$

Dividing by a and subtracting :

$$\frac{(c y + b z - c y_1 + b z_1)}{b c} - \frac{2 (x - x_1)}{a} = 0$$

Multiplying by  $(a * b * c)$  gives

$$-2 b c x + a c y + a b z + 2 b c x_1 - a c y_1 + a b z_1 = 0$$

$$2 b c x - a c y - a b z = 2 b c x_1 - a c y_1 + a b z_1$$

The right side is a constant, and for lack of a better term can be called e

On the left side

Let :

$$2 * b * c \rightarrow f$$

$$a * c \rightarrow -g$$

$$a * b \rightarrow -h$$

then it can be written as :

$$f * x + g * y + h * z = e$$

a plane.