UNIVERSITY OF DUBLIN

XMA2E011

TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

SF Engineers
SF MSISS
SF MEMS

Trinity Term 2010

Module 2E011

Tuesday, April 27 LUCE LOWER 9.30 — 11.30

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ATTEMPT FIVE QUESTIONS

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. Consider the function

$$f(x,y,z) = \sqrt{y^2 - \sin(x+2z)}\,, \quad \text{and the point } P(0,2,0)\,.$$

(a) 12 marks. Find a unit vector in the direction in which f increases most rapidly at the point P.

(b) 3 marks. Find a unit vector in the direction in which f decreases most rapidly at the point P.

(c) 3 marks. Find the rate of change of f at the point P in these directions.

Show the details of your work.

Solution:

(a) f increases most rapidly in the direction of its gradient, so we compute

$$f_x(x,y,z) = \frac{-\cos(x+2z)}{2\sqrt{y^2-\sin(x+2z)}} \quad \Rightarrow \quad f_x(0,2,0) = -\frac{1}{4}\,,$$

$$f_y(x,y,z) = \frac{2y}{2\sqrt{y^2-\sin(x+2z)}} \quad \Rightarrow \quad f_y(0,2,0) = 1\,,$$

$$f_z(x,y,z) = \frac{-2\cos(x+2z)}{2\sqrt{y^2-\sin(x+2z)}} \quad \Rightarrow \quad f_x(0,2,0) = -\frac{1}{2}\,.$$

Thus, the gradient and its magnitude are equal to

$$\nabla f(0,2,0) = \left(-\frac{1}{4}\,,1\,,-\frac{1}{2}\right)\,, \quad ||\nabla f(0,2,0)|| = \sqrt{1^2 + \left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\sqrt{21}}{4}\,.$$

Therefore, the unit vector in the direction of the gradient is

$$\mathbf{u} = \frac{4}{\sqrt{21}}\left(-\frac{1}{4}\,,1\,,-\frac{1}{2}\right)\,.$$

(b) f decreases most rapidly in the direction opposite to its gradient, so the unit vector is

$$\mathbf{v} = -\frac{4}{\sqrt{21}}\left(-\frac{1}{4}\,,1\,,-\frac{1}{2}\right)\,.$$

(c) The rate of change of f at P in the direction of $\mathbf{u}$ is equal to

$$||\nabla f(0,2,0)|| = \frac{\sqrt{21}}{4} \approx 1.14564\,,$$

and the rate of change of f at P in the direction of $\mathbf{v}$ is equal to

$$-||\nabla f(0,2,0)|| = -\frac{\sqrt{21}}{4} \approx 1.14564\,.$$

2. Consider the surface

$$z = \ln \frac{\sqrt{2x^2+y^2}}{3}$$

   (a) 12 marks. Find an equation for the tangent plane to the surface at the point $P(2,-1,0)$.

   (b) 6 marks. Find parametric equations for the normal line to the surface at the point $P(2,-1,0)$.

Show the details of your work.

Solution:

(a) We first simplify

$$z = \ln \frac{\sqrt{2x^2+y^2}}{3} = \frac{1}{2}\ln(2x^2+y^2) - \ln 3\,.$$

Then, we compute the partial derivatives at $P(2,-1,0)$

$$\frac{\partial}{\partial x}\frac{1}{2}\ln(2x^2+y^2)|_{x=2,y=-1} = \frac{2x}{x^2+y^2}|_{x=2,y=-1} = \frac{4}{9}\,.$$

$$\frac{\partial}{\partial y}\frac{1}{2}\ln(2x^2+y^2)|_{x=2,y=-1} = \frac{y}{x^2+y^2}|_{x=2,y=-1} = -\frac{1}{9}\,.$$

The tangent plane equation is given by

$$z = 0 + \frac{4}{9}(x-2) - \frac{1}{9}(y+1) = \frac{4}{9}x - \frac{1}{9}y - 1\,.$$

(b) The normal line to the surface (and the tangent plane) is given by

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + t\big(-\frac{4}{9}\mathbf{i} + \frac{1}{9}\mathbf{j} + \mathbf{k}\big)\,.$$

3. Consider the portion of the cylinder $y^2 + z^2 = 8$ that is above the rectangle

$$R = \{(x, y) : -1 \le x \le 1\,,\ -2 \le y \le 2\}.$$

(a) 2 marks. Sketch the projection of the portion onto the xy-plane.

(b) 16 marks. Use double integration to find the area of the portion.

Show the details of your work.

Solution:

a) The projection is shown below

(b) The area is given by the formula

$$S = \iint_R \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA = \int_{-1}^{1}\int_{-2}^{2} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dy\, dx\,,$$

where

$$z = \sqrt{8 - y^2}$$

Computing the derivatives we get

$$\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + \frac{y^2}{8 - y^2}} = \frac{2\sqrt{2}}{\sqrt{8 - y^2}}\,.$$

Thus, we get

$$S = \int_{-1}^{1}\int_{-2}^{2} \frac{2\sqrt{2}}{\sqrt{8 - y^2}}\, dy\, dx = 4\int_0^2 \frac{2\sqrt{2}}{\sqrt{8 - y^2}}\, dy\,,$$

To compute the integral we do the substitution

$$y = 2\sqrt{2}\sin t\,, \quad dy = 2\sqrt{2}\cos t\, dt\,, \quad \sqrt{8 - y^2} = 2\sqrt{2}\cos t\,, \quad 0 \le t \le \frac{\pi}{4}$$

and get

$$S = 4\int_0^{\pi/4} 2\sqrt{2}\, dt = 2\sqrt{2}\,\pi\,.$$

4. (a) 3 marks. Show that the integral below is independent of the path

$$\int_{(-1,4)}^{(1,0)} (3x - 2y + 4)\, dx - (2x + 5y + 3)\, dy\,.$$

(b) 12 marks. Find the potential function $\phi(x, y)$

(c) 3 marks. Use the Fundamental Theorem of Line Integrals to find the value of the integral.

Show the details of your work.

Solution:

(a) We have

$$f(x, y) = 3x - 2y + 4\,, \quad g(x, y) = -(2x + 5y + 3)\,.$$

Thus

$$\partial_y f(x, y) = -2\,, \quad \partial_x g(x, y) = -2\,,$$

and therefore the integral is independent of the path.

(b) To compute the integral we find the potential function $\phi(x, y)$

$$\frac{\partial \phi}{\partial x} = 3x - 2y + 4 \quad \Rightarrow \quad \phi(x, y) = \frac{3}{2}x^2 - 2x\, y + 4x + C(y)\,.$$

To find $C(y)$ we use that

$$\frac{\partial \phi}{\partial y} = -2x + \frac{dC(y)}{dy} = -(2x+5y+3) \quad \Rightarrow \quad \frac{dC(y)}{dy} = -5y-3 \quad \Rightarrow \quad C(y) = -\frac{5}{2}y^2 - 3y + C\,.$$

Thus, we get

$$\phi(x, y) = \frac{3}{2}x^2 - 2x\, y + 4x - \frac{5}{2}y^2 - 3y + C\,.$$

By using the formula, we obtain

$$\int_{(-1,4)}^{(1,0)} (3x - 2y + 4)\, dx - (2x + 5y + 3)\, dy = \int_{(-1,4)}^{(1,0)} \frac{\partial \phi}{\partial x}\, dx + \frac{\partial \phi}{\partial y}\, dy = \phi(x, y)\big|_{(-1,4)}^{(1,0)}$$
$$= \frac{3}{2} + 4 - (\frac{3}{2} + 8 - 4 - 40 - 12) = 52\,.$$

5. (a) 5 marks. Express rectangular coordinates in terms of spherical coordinates

   (b) Use triple integral and spherical coordinates to

      i. 6 marks. Compute the volume of a ball of radius R.

      ii. 7 marks. Find the mass of the solid enclosed between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ if the density is

$$\delta(x, y, z) = \frac{e^{-(x^2+y^2+z^2)}}{\sqrt{x^2 + y^2 + z^2}}.$$

Show the details of your work.

Solution :

(a) We have

$$x = r\cos\theta\sin\phi\,, \quad y = r\sin\theta\sin\phi\,, \quad z = r\cos\phi$$

(b) i. We use the spherical coordinates to get

$$V = \iiint_V dV = \int_0^{2\pi}\left(\int_0^{\pi}\left[\int_0^R r^2 dr\right]\sin\phi\, d\phi\right) d\theta = \frac{4}{3}\pi R^3\,.$$

(b) ii. We use the spherical coordinates to get

$$\begin{aligned} M &= \iiint_V \delta(x, y, z) dV = \int_0^{2\pi}\left(\int_0^{\pi}\left[\int_2^3 \frac{e^{-r^2}}{r} r^2 dr\right]\sin\phi\, d\phi\right) d\theta \\ &= 4\pi \int_2^3 e^{-r^2} r dr = 2\pi(e^{-4} - e^{-9}) \approx 0.114305\,. \end{aligned} \tag{1}$$

6. (a) 14 marks. Solve the following initial value problem by the Laplace transform

$$y'' + 4\,y = 4\,u(t-\pi) - 4\,\delta(t-3\pi)\,, \qquad y(0) = 1\,, \quad y'(0) = 0\,.$$

   (b) 4 marks. Sketch the input function and the solution.

Show the details of your work.

Solution :

(a) We denote $Y(s) = \mathcal{L}(y)$, and then using the formulae

$$\mathcal{L}(y'') = s^2 Y(s) - sy(0) - y'(0)\,,\ \mathcal{L}(u(t-a)) = \frac{e^{-as}}{s}\,,\ \mathcal{L}(\delta(t-a)) = e^{-as}\,,$$

we get the algebraic equation

$$(s^2+4)Y(s) = 4\frac{e^{-\pi s}}{s} - 4e^{-3\pi s} + s\,.$$

Solving the equation for Y, we get

$$Y(s) = \frac{4e^{-\pi s}}{s(s^2+4)} - \frac{4e^{-3\pi s}}{s^2+4} + \frac{s}{s^2+4}\,.$$

Then we represent

$$\frac{4}{s(s^2+4)} = \frac{1}{s} - \frac{s}{s^2+4}\,.$$

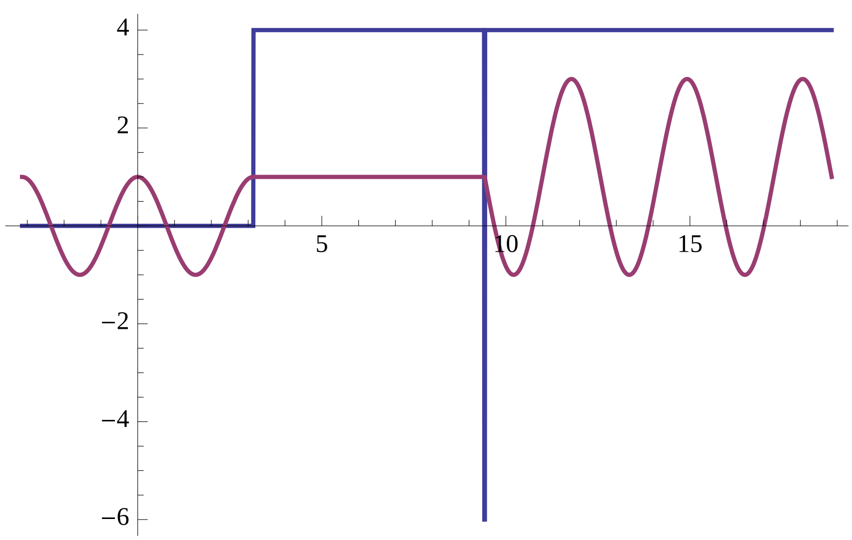
Finally we use the formulas of the inverse Laplace transform

$$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1\,, \quad \mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) = \cos 2t\,, \quad \mathcal{L}^{-1}\left(e^{-as}F(s)\right) = f(t-a)u(t-a)\,,$$

to get

$$\begin{aligned} y(t) &= u(t-\pi) - \cos 2(t-\pi)u(t-\pi) - 2\sin 2(t-3\pi)u(t-3\pi) + \cos 2t \\ &= \begin{cases} \cos 2t & \text{if} \quad 0 < t < \pi \\ 1 & \text{if} \quad \pi < t < 3\pi \\ 1 - 2\sin 2t & \text{if} \quad t > 3\pi \end{cases} \,. \end{aligned} \tag{2}$$

(b) The plot of the input function and the solution is shown below.



Useful Formulae

1. Let $\mathbf{r}(t)$ be a vector function with values in $\mathbf{R}^3$: $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$.

   (a) Its derivative is $\frac{d\mathbf{r}}{dt} = \left(\frac{df}{dt}, \frac{dg}{dt}, \frac{dh}{dt}\right)$.

   (b) The magnitude of this vector is $||\frac{d\mathbf{r}}{dt}|| = \sqrt{\left(\frac{df}{dt}\right)^2 + \left(\frac{dg}{dt}\right)^2 + \left(\frac{dh}{dt}\right)^2}$.

   (c) The unit tangent vector is $\mathbf{T} = \frac{\frac{d\mathbf{r}}{dt}}{||\frac{d\mathbf{r}}{dt}||}$.

   (d) The vector equation of the line tangent to the graph of $\mathbf{r}(t)$ at the point $P = (x_0, y_0, z_0)$ corresponding to $t = t_0$ on the curve is $\mathbf{R}(t) = \mathbf{r}_0 + (t - t_0)\,\mathbf{v}_0$, where $\mathbf{r}_0 = \mathbf{r}(t_0)$ and $\mathbf{v}_0 = \frac{d\mathbf{r}}{dt}(t_0)$.

   (e) The arc length of the graph of $\mathbf{r}(t)$ between t_1 and t_2 is $L = \int_1^2 ||\frac{d\mathbf{r}}{dt}||\,dt$.

   (f) The arc length parameter s having $\mathbf{r}(t_0)$ as its reference point is $s = \int_{t_0}^{t} ||\frac{d\mathbf{r}}{du}||\,du$.

2. Let σ be a surface in $\mathbf{R}^3$: $z = f(x, y)$

   (a) The slope k_x of the surface in the x-direction at the point (x_0, y_0) is $k_x = \frac{\partial z}{\partial x}(x_0, y_0)$.

   (b) The slope k_y of the surface in the y-direction at the point (x_0, y_0) is $k_y = \frac{\partial z}{\partial y}(x_0, y_0)$.

   (c) The equation for the tangent plane to the surface at the point $P = (x_0, y_0, z_0)$ is $z = z_0 + k_x(x - x_0) + k_y(y - y_0)$.

   (d) Parametric equations for the normal line to the surface at $P = (x_0, y_0, z_0)$ are $\mathbf{r}(t) = \mathbf{r}_0 + t(-k_x\mathbf{i} - k_y\mathbf{j} + \mathbf{k})$, $\mathbf{r}_0 = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k}$.

   (e) The volume under the surface and over a region R in the xy-plane is $V = \iint_R f(x, y)\,dA$.

   (f) The area of the portion of the surface that is above a region R in the xy-plane is $S = \iint_\sigma dS = \iint_R \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$.

   (g) The mass of the lamina with the density $\delta(x, y, z)$ that is the portion of the surface that is above a region R in the xy-plane is $M = \iint_\sigma \delta(x, y, z)\,dS = \iint_R \delta(x, y, z)\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$.

3. The local linear approximation of the function $z = f(x, y)$ at the point (x_0, y_0) is

   $L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$.

4. Let $f(x, y, z)$ be a function of three variables

   (a) The gradient of f is $\nabla f = (f_x, f_y, f_z)$.

   (b) f increases most rapidly in the direction of its gradient, and the rate of change of f in this direction is equal to $||\nabla f||$.

   (c) If f is smooth then its critical points satisfy $f_x = f_y = f_z = 0$.

5. Let R be a region in the xy-plane bounded by the curves $y = g(x)$, $y = h(x)$, $x = a$, $x = b$, and $g \le h$ for $a \le x \le b$. Then the double integral over the region is

   $\iint_R f(x, y)\, dA = \int_a^b \left[\int_{g(x)}^{h(x)} f(x, y) dy\right] dx$.

6. Let R be a region in the xy-plane bounded by the curves (in polar coordinates)

   $r = r_1(\theta)$, $r = r_2(\theta)$, $\theta = \alpha$, $\theta = \beta$ and $r_1 \le r_2$ for $\alpha \le \theta \le \beta$. Then the double integral over the region is

   $\iint_R f(r, \theta)\, dA = \iint_R f(r, \theta)\, r dr\, d\theta = \int_\alpha^\beta \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r, \theta) r dr\right] d\theta$.

7. Let R be a plain lamina with density $\delta(x, y)$.

   (a) Its mass is equal to $M = \iint_R \delta(x, y)\, dA$.

   (b) The x-coordinate of its centre of gravity is equal to $x_{cg} = \frac{1}{M} \iint_R x\, \delta(x, y)\, dA$.

   (c) The y-coordinate of its centre of gravity is equal to $y_{cg} = \frac{1}{M} \iint_R y\, \delta(x, y)\, dA$.

8. Let G be a simple solid whose projection onto the xy-plane is a region R. G is bounded by a surface $z = g(x, y)$ from below and by a surface $z = h(x, y)$ from above.

   (a) The triple integral over the solid is $\iiint_G f(x, y, z)\, dV = \iint_R \left[\int_{g(x,y)}^{h(x,y)} f(x, y, z)\, dz\right] dA$.

   (b) The volume of the solid is $V = \iiint_G dV = \iint_R [h(x, y) - g(x, y)]\, dA$.

9. Let G be a solid enclosed between the two surfaces (in spherical coordinates)

   $r = g(\theta\, , \phi)$, $r = h(\theta\, , \phi)$.

(a) The triple integral over the solid is
$\iiint_G f(r,\theta,\phi)\,dV = \int_0^{2\pi}\left(\int_0^{\pi}\left[\int_{g(\theta,\phi)}^{h(\theta,\phi)} f(r,\theta,\phi)\,r^2dr\right]\sin\phi\,d\phi\right)d\theta\,.$

(b) The volume of the solid is $V = \iiint_G dV = \int_0^{2\pi}\left(\int_0^{\pi}\left[\int_{g(\theta,\phi)}^{h(\theta,\phi)} r^2dr\right]\sin\phi\,d\phi\right)d\theta\,.$

(c) The mass of the solid with the density $\delta(r,\theta,\phi)$ is $M = \iiint_G \delta(r,\theta,\phi)\,dV\,.$

10. Let a region R_{xy} in the xy-plane be mapped to a region R_{uv} in the uv-plane under the change of variables $u = u(x,y)\,,\ v = v(x,y)$.

(a) The magnitude of the Jacobian of the change is $\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left|\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right|\,.$

(b) The integral over R_{xy} is $\iint_{R_{xy}} f(x,y)\,dA_{xy} = \iint_{R_{uv}} f\left(x(u,v),y(u,v)\right)\left|\frac{\partial(u,v)}{\partial(x,y)}\right|^{-1} dA_{uv}\,.$

11. The area of the surface that extends upward from the curve $x = x(t)\,,\ y = y(t)\,,\ a \le t \le b$ in the xy-plane to the surface $z = f(x,y)$ is given by the following line integral
$A = \int_C z\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt\,.$

12. Consider a line integral $\int_C f(x,y)\,dx + g(x,y)\,dy$, and let $P = (x_P,y_P)$ and $Q = (x_Q,y_Q)$ be the endpoints of the curve C.

(a) The line integral is independent of the path if $\partial_y f(x,y) = \partial_x g(x,y)\,.$

(b) Then there is a potential function $\phi(x,y)$ satisfying $\frac{\partial\phi}{\partial x} = f(x,y)\,,\ \frac{\partial\phi}{\partial y} = g(x,y)\,,$

(c) and the Fundamental Theorem of Line Integrals says that
$\int_C f(x,y)\,dx + g(x,y)\,dy = \int_P^Q \frac{\partial\phi}{\partial x}\,dx + \frac{\partial\phi}{\partial y}\,dy = \phi(x,y)\big|_P^Q = \phi(x_Q,y_Q) - \phi(x_P,y_P)\,.$

13. Let a closed curve C be oriented counterclockwise, and be the boundary of a simply connected region R in the xy-plane. By Green's Theorem we have
$\oint_C f(x,y)\,dx + g(x,y)\,dy = \iint_R\left(\frac{\partial g(x,y)}{\partial x} - \frac{\partial f(x,y)}{\partial y}\right)dA$

14. Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ be a vector field.

(a) If σ is the surface $z = f(x,y)$, oriented by upward unit normals $\mathbf{n}$, and R is the projection of σ onto the xy-plane then
flux $= \iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R\left(-M\frac{\partial f}{\partial x} - N\frac{\partial f}{\partial y} + P\right)dA\,.$

(b) If σ is the surface $z = f(x,y)$, oriented by downward unit normals $\mathbf{n}$, and R is the projection of σ onto the xy-plane then

flux $= \iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \left(M\frac{\partial f}{\partial x} + N\frac{\partial f}{\partial y} - P\right) dA\,.$

(c) According to the Divergence Theorem the flux of $\mathbf{F}$ across a closed surface σ with outward orientation is

flux $= \iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_V \operatorname{div}\mathbf{F}\,dV\,, \quad \operatorname{div}\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}\,.$

(d) If σ is an oriented smooth surface that is bounded by a simple, closed, smooth boundary curve C with positive orientation then, according to Stokes' Theorem

$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_\sigma (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS\,,\ \operatorname{curl}\mathbf{F} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}\,.$

15. The Laplace transform of a function $f(t)$ is the function $F(s)$ defined by

$F(s) = \mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)dt\,, \quad f(t) = \mathcal{L}^{-1}(F(s))\,.$

$Function$	$Transform$	$Function$	$Transform$
e^{at}	$\frac{1}{s-a}$	$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$e^{at}\sinh\omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$	$e^{at}\cosh\omega t$	$\frac{s-a}{(s-a)^2-\omega^2}$
$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$u(t-a)$	$\frac{e^{-as}}{s}$	$\delta(t-a)$	e^{-as}

16. Let $F(s) = \mathcal{L}(f(t))$, then $\mathcal{L}(f(t-a)u(t-a)) = e^{-as}F(s)$;

$\mathcal{L}\left(e^{at}f(t)\right) = F(s-a)\,;\ \mathcal{L}\left(tf(t)\right) = -\frac{dF(s)}{ds}\,;\ \mathcal{L}\left(f(kt)\right) = \frac{1}{k}F\left(\frac{s}{k}\right)\,.$

17. Let $Y(s) = \mathcal{L}(y)$, then $\mathcal{L}(y') = sY(s) - y(0)\,,\ \mathcal{L}(y'') = s^2Y(s) - sy(0) - y'(0)\,.$

18. Convolution. Let $f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$. Then $\mathcal{L}(f(t)*g(t)) = F(s)G(s)$