rect relativistic theory must provide valid expressions for these four forces. These expressions, if stated in covariant form, will automatically provide the transformation properties of the forces. In this approach, since we understand electromagnetic forces, it remains to find expressions for the other three fundamental forces in a covariant form in some frame and assume this is correct in all inertial frames. It is assumed the transformations involve no terms that vanish in the chosen frame; for example, there is no need to arbitrarily add terms proportional to $(v/c)^3$. This program has been carried out for two of the remaining three forces (weak nuclear and strong nuclear) and for weak gravitational forces. It fails completely for strong gravitational effects. It is beyond the scope of the present text to probe more deeply in to this question.

The second approach of determining the correct relativistic force is to simply define force as being the time rate of change of the momentum. Then we write

$$\frac{dp_i}{dt} = F_i \tag{7.76}$$

where the p_i in Eq. (7.76) is some relativistic generalization of the Newtonian momentum that reduces to mv_i in the limit of small β. The simplest generalization is the one given in Eq. (7.36). This second approach has thus far failed to produce any results other than those predicted by the first approach.

7.7 ■ RELATIVISTIC KINEMATICS OF COLLISIONS AND MANY-PARTICLE SYSTEMS

The formulations of the previous sections enable us to generalize relativistically the discussion of Section 3.11 on the transformation of collision phenomena between various systems. The subject is of considerable interest in experimental high-energy physics. While the forces between elementary particles are only imperfectly known, and are certainly far from classical, so long as the particles involved in a reaction are outside the region of mutual interaction their mean motion can be described by classical mechanics. Further, the main principle involved in the transformations—conservation of the four-vector of momentum—is valid in both classical and quantum mechanics. The actual collision or reaction is taken as occurring at a point—or inside a very small black box—and we look only at the behavior of the particles before and after.

Because of the importance to high-energy physics, this aspect of relativistic kinematics has become an elaborately developed field. It is impossible to give a comprehensive discussion here. All that we can do is provide some of the important tools, and cite a few simple examples that may illustrate the flavor of the techniques employed. Although many collision experiments involve colliding beams, we shall, for simplicity, confine our attentions to problems where one of the particles is at rest in the laboratory frame. The generalization to both particles moving in the laboratory frame is straightforward.

The notion of a point designated as the center of mass obviously presents difficulties in a Lorentz-invariant theory. But the center-of-mass system can be suitably generalized as the Lorentz frame of reference in which the total spatial linear momentum of all particles is zero. That such a Lorentz frame can always be found follows from the theorem that the total momentum 4-vector is timelike for a system of mass points.

One such frame is the center-of-momentum frame. This is a frame in which the components of the spatial momentum of the initial particles add to zero. Such a frame obviously exists. Let us define E and $\mathbf{p}$ in Eq. (7.36) to be

$$E = \sum_{i=1}^{n} E_i \qquad \text{and} \qquad \mathbf{p} = \sum_{i=1}^{n} \mathbf{p}_i \tag{7.77}$$

where the sum is over the particles involved. The left-hand side of Eq. (7.38) becomes

$$\sum_{r,s} m_r m_s c^2 - \sum_{r,s} m_r m_s \gamma_r \gamma_s (\mathbf{v}_r \cdot \mathbf{v}_s). \tag{7.78}$$

This clearly is positive (*hint*: separate the negative terms in which $r = s$), so it is possible to find a frame in which the three-momentum, $\boldsymbol{p}$, equals zero. The Lorentz system, in which the spatial components of the total momentum are zero, is termed the *center-of-momentum system*, or more loosely, and somewhat incorrectly, as the center-of-mass system, and will be designated by the abbreviation "C-O-M system."

As an example, let us consider a particle of mass m_1 and momentum p^1 in the x-direction, which suffers a head-on collision with a particle of mass m_2 at rest in an experimenter's frame (called the laboratory frame). The initial 4-momentum is

$$p^{\mu} = ([m_1\gamma + m_2]c, m_1\gamma \mathbf{v}^1, 0, 0). \tag{7.79}$$

The length squared of momentum has the magnitude

$$p^{\mu} p_{\mu} = (m_1^2 + m_2^2 + 2m_1\gamma m_2)c^2. \tag{7.79$'$}$$

When components are given, we shall follow the practice of denoting the primed frame by primes on the indices. The two particles are denoted by subscripts 1 and 2 respectively.

In the C-O-M system, the total momentum is

$$\left([m_1\gamma_1' + m_2\gamma_2']c, 0, 0, 0\right), \tag{7.80}$$

since by definition the space part of the momentum vanishes,

$$m_1\gamma_1'\boldsymbol{\beta}_1'c + m_2\gamma_2'\boldsymbol{\beta}_2'c - 0, \tag{7.81}$$