

Classical effect of rotation on a 2D vector $\mathbf{r} = \langle x, y \rangle$ by rotation matrix $\phi_0 \mathbf{k}$, in the notation $\mathbf{r} \rightarrow \mathbf{r}' = (\phi_0 \mathbf{k})\mathbf{r}$, is,

$$\mathbf{r} \rightarrow \mathbf{r}' = (\phi_0 \mathbf{k})\mathbf{r} \quad [\text{I.1}]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} \cos \phi_0 & -\sin \phi_0 \\ \sin \phi_0 & \cos \phi_0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} p_x \\ p_y \end{bmatrix} \rightarrow \begin{bmatrix} \bar{p}_x \\ \bar{p}_y \end{bmatrix} = \begin{bmatrix} \cos \phi_0 & -\sin \phi_0 \\ \sin \phi_0 & \cos \phi_0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad [\text{I.2}]$$

That's classical. Now we need operators for rotations in quantum mechanics. We have two possible rotation operators: passive and active.

Passive: rotates coordinate system

active: rotates the state itself

Both passive and active lead to the same physical interpretation, but there's something distinct about them if we're paying attention to either the kets or the operators we're changing.

Helpful analogy: you may be wondering what it means to “rotate” a ket, especially if that ket isn't in an eigenstate of position. Well, the best I can offer you is analogy,

$$[\text{translation operator}] = T(\mathbf{a}) \xleftarrow{\text{analogous}} R(\phi_0 \mathbf{k}) = [\text{rotation operator}] \quad [\text{I.3}]$$

If you want to rotate the ket itself, you have the transformation law at best “analogous” to [I.1] that is,

$$|\psi\rangle \rightarrow |\psi_R\rangle = U[R]|\psi\rangle \quad [\text{I.4}]$$