Let us consider the simplest case, namely, a simple gear pair as shown in Fig. 1 (I hope you do not mind a sketch).

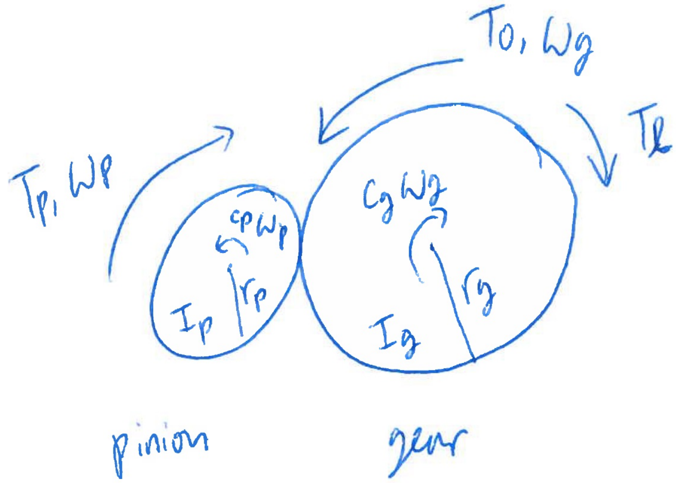

Figure 1: A simple gear pair

So, we have a pinion and a gear. I give an input torque T_p in the clockwise direction. Therefore, the pinion will rotate with ω_p angular velocity in clockwise and the gear ω_g in counter-clockwise. There is a load T_l against the gear motion. The bearing friction both in pinion and gear are considered by means of linearly-viscous damping coefficients c_p and c_g for pinion and gear, respectively. The friction between the gear mesh is neglected at this point. The moments of inertia of the pinion and the gear are I_p and I_g, respectively. Moreover, the radii of the pinion and the gear are r_p and r_g, respectively. My question is what the output torque T_o is because I want to find the efficiency of this gear pair.

I have tried four options for T_o and simulated them in MATLAB, but I have not found the correct results yet. Followings are the explanation of each option I tried for T_o.

1. The output torque T_o equals the load T_l

   If $T_o = T_l$, I would not get the correct efficiency η.

$$\eta = \frac{P_o}{P_i} = \frac{T_o \omega_g}{T_p \omega_p} = \frac{T_l \omega_g}{T_p \omega_p} \tag{1}$$

   where P_i and P_o are the input and output power, respectively.

   The gear pair constraint equation is

$$r_g \omega_g = r_p \omega_p \tag{2}$$

   and we can get

$$\frac{\omega_g}{\omega_p} = \frac{r_p}{r_g} \tag{3}$$

Substituting Eq. 3 to Eq. 1, η becomes

$$\eta = \frac{T_l r_p}{T_p r_g} \tag{4}$$

In Eq. 4, we can see that T_l/T_p has to be r_g/r_p in order to have $\eta = 1$. Thus, if T_l/T_p is not close to r_g/r_p, the efficiency could be very low, for example 0.2, which does not make sense because the efficiency of a gear pair is very high, around 0.95–0.98.

2. The output torque T_o equals the product of the gear moment of inertia I_g and angular acceleration α_g

   I was thinking that the output and input torques are

$$T_o = I_g \alpha_g \tag{5}$$

$$T_p = I_p \alpha_p \tag{6}$$

   But, after some symbolic analysis, I found that this is not true. Differentiating Eq. 2 with respect to time, I obtain the constraint equation in acceleration level as

$$r_g \alpha_g = r_p \alpha_p \tag{7}$$

   and

$$\frac{\alpha_g}{\alpha_p} = \frac{r_p}{r_g} \tag{8}$$

   Computing the efficiency

$$\eta = \frac{P_o}{P_i} = \frac{T_o \omega_g}{T_p \omega_p} = \frac{I_g \alpha_g \omega_g}{I_p \alpha_p \omega_p} \tag{9}$$

   Substituting Eqs. 3 and 8 to Eq. 9, η becomes

$$\eta = \frac{I_g r_p^2}{I_p r_g^2} \tag{10}$$

   Using Eq. 10, I can easily check the efficiency of the gear pair without simulation of dynamics model and find that the efficiency is incorrect.

3. The output torque T_o equals the product of constraint force F and radius of the gear r_g

   If I formulate the mathematical model by means of Newton-Euler, then I will get the constraint force F as shown in Fig. 2
   Then, I thought of the output torque as the $T_o = F r_g$ because it is the only positive torque in the gear. I did some simulation but the results are incorrect (the efficiency varies, not close to 1)

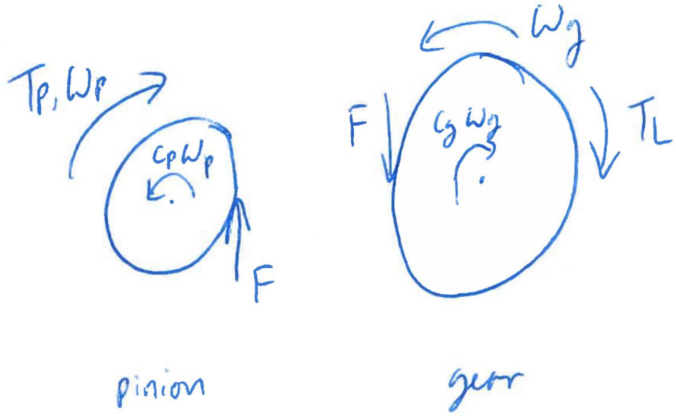

Figure 2: A free-body diagram of the gear pair

4. I used power conservation formula to find the output power

   Based on the power conservation, I obtain

$$P_i = P_o + P_l \tag{11}$$

   where P_l is the power loss.

   Using Eq. 11, I can find the output power P_o as follows

$$P_o = P_i - P_l = T_p\omega_p - c_p\omega_p^2 - c_g\omega_g^2 \tag{12}$$

   I then did some simulation but the results are incorrect (the efficiency varies, not close to 1)

I am still trying to figure out the problem. Please let me know if I made mistakes in my analysis. Thank you very much.