Example Problem 4–1

The shaft shown in Figure 4–7 is supported by two bearings and carries two V-belt sheaves. The tensions in the belts exert horizontal forces on the shaft, tending to bend it in the x–z plane. Sheave B exerts a clockwise torque on the shaft when viewed toward the origin of the coordinate system along the x-axis. Sheave C exerts an equal but opposite torque on the shaft. For the loading condition shown, determine the principal stresses and the maximum shear stress on element K on the front surface of the shaft (on the positive z-side) just to the right of sheave B. Follow the general procedure for analyzing combined stresses given in this section.

Solution

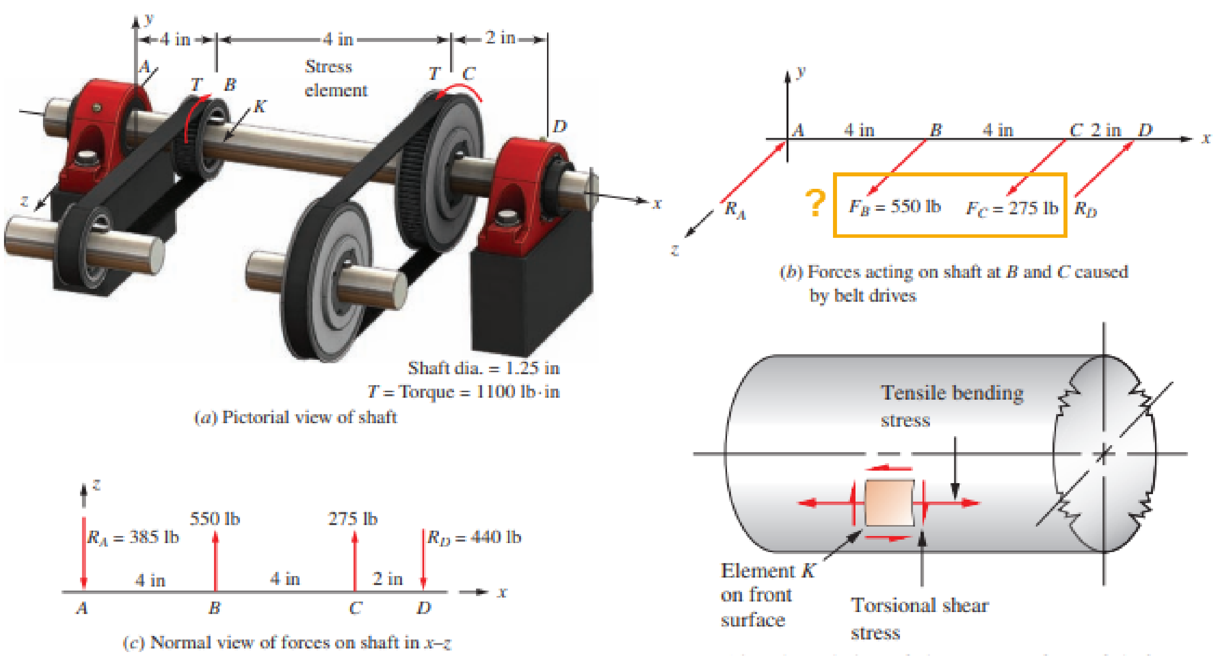
Objective: Compute the principal stresses and the maximum shear stresses on element K.

Given: Shaft and loading pattern shown in Figure 4–7. The forces at locations B and C are identified as 550 lb and 275 lb, respectively. These forces are determined by the tight side and slack side of the belt forces discussed in Section 12–3.

Analysis: Use the general procedure for analyzing combined stresses.

Results: Element K is subjected to bending that produces a tensile stress acting in the x-direction. Also, there is a torsional shear stress acting at K. Figure 4–8 shows the shearing force and bending moment diagrams for the shaft and indicates that the bending moment at K is 1540 lb · in. The bending stress is therefore

$$\sigma_x = M/S$$
$$S = \pi D^3/32 = [\pi(1.25\text{ in})^3]/32 = 0.192\text{ in}^3$$
$$\sigma_x = (1540\text{ lb}\cdot\text{in})/(0.192\text{ in}^3) = 8030\text{ psi}$$



(*a*) Pictorial view of shaft

(*b*) Forces acting on shaft at B and C caused by belt drives

(*c*) Normal view of forces on shaft in x–z plane with reactions at bearings

(*d*) Enlarged view of element K on front of shaft

where
the x-direction is parallel to the centerline,
the y-direction is perpendicular to it, and
the angle θ is the angle of inclination of the tight side of the chain with respect to the x-direction.

These two components of the force would cause bending in both the x-direction and the y-direction. Alternatively, the analysis could be carried out in the direction of the force, F_c, in which single plane bending occurs.

If the angle θ is small, little error will result from the assumption that the entire force, F_c, acts along the x-direction. *Unless stated otherwise, this book will use this assumption.*

V-Belt Sheaves

The general appearance of the V-belt drive system looks similar to the chain drive system. But there is one important difference: Both sides of the V-belt are in tension, as indicated in Figure 12–6. The tight side tension, F_1, is greater than the "slack side" tension, F_2, and thus there is a net driving force on the sheaves equal to

➪ Net Driving Force

$$F_N = F_1 - F_2 \qquad (12\text{–}7)$$

The magnitude of the net driving force can be computed from the torque transmitted:

➪ Net Driving Force

$$F_N = T/(D/2) \qquad (12\text{–}8)$$

But notice that the bending force on the shaft carrying the sheave is dependent on the *sum*, $F_1 + F_2 = F_B$. To be more precise, the components of F_1 and F_2 parallel to the line of centers of the two sprockets should be used. But unless the two sprockets are radically different in diameter, little error will result from $F_B = F_1 + F_2$.

To determine the bending force, F_B, a second equation involving the two forces F_1 and F_2 is needed. This is provided by assuming a ratio of the tight side tension to the slack side tension. For V-belt drives, the ratio is normally taken to be

$$F_1/F_2 = 5 \qquad (12\text{–}9)$$

It is convenient to derive a relationship between F_N and F_B of the form

$$F_B = CF_N \qquad (12\text{–}10)$$

where C = constant to be determined

$$C = \frac{F_B}{F_N} = \frac{F_1 + F_2}{F_1 - F_2} \qquad (12\text{–}11)$$

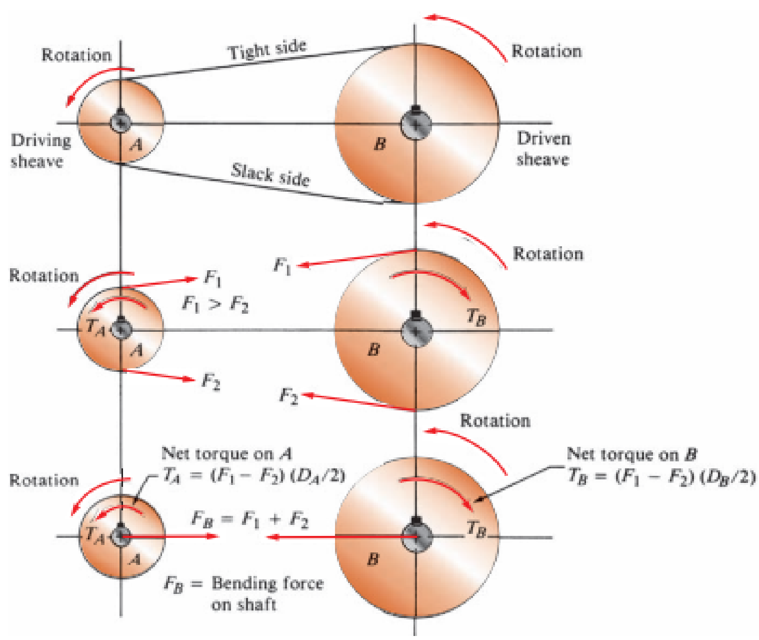
But from Equation (12–9), $F_1 = 5F_2$. Then

$$C = \frac{F_1 + F_2}{F_1 - F_2} = \frac{5F_2 + F_2}{5F_2 - F_2} = \frac{6F_2}{4F_2} = 1.5$$

Equation (12–10) then becomes, for V-belt drives,

➪ Bending Force on Shaft for V-Belt Drive

$$F_B = 1.5\,F_N = 1.5T/(D/2) \qquad (12\text{–}12)$$



Design Example 12–2

The shaft shown in Figure 12–13 receives 110 hp from a water turbine through a chain sprocket at point *C*. The gear pair at *E* delivers 80 hp to an electrical generator. The V-belt sheave at *A* delivers 30 hp to a bucket elevator that carries grain to an elevated hopper. The shaft rotates at 1700 rpm. The sprocket, sheave, and gear are located axially by retaining rings. The sheave and gear are keyed with sled runner keyseats, and there is a profile keyseat at the sprocket. Use SAE 1040 cold-drawn steel for the shaft. Compute the minimum acceptable diameters D_1 through D_7 as defined in Figure 12–13.

Solution

First, the material properties for the SAE 1040 cold-drawn steel are found from Appendix 3:

$$s_y = 71\,000 \text{ psi} \qquad s_u = 80\,000 \text{ psi}$$

Then from Figure 5–11, $s_n = 30\,000$ psi. Let's design for a reliability of 0.99 and use $C_R = 0.81$. The shaft size should be moderately large, so we can assume $C_s = 0.85$ as a reasonable estimate. Then the modified endurance strength is

$$s'_n = s_n\, C_s\, C_R = (30\,000)(0.85)(0.81) = 20\,650 \text{ psi}$$

This application is fairly smooth: a turbine drive and a generator and a conveyor at the output points. A design factor of $N = 2$ should be satisfactory.

Torque Distribution in the Shaft: Recalling that all of the power comes into the shaft at *C*, we can then observe that 30 hp is delivered down the shaft from *C* to the sheave at *A*. Also, 80 hp is delivered down the shaft from *C* to the gear at *E*. From these observations, the torque in the shaft can be computed:

$$T_A = T_{AC} = (63\,000)(30 \text{ hp})/1700 \text{ rpm} = 1112 \text{ lb}\cdot\text{in} \qquad \text{from } A \text{ to } C \text{ in shaft}$$

$$T_E = T_{CE} = (63\,000)(80 \text{ hp})/1700 \text{ rpm} = 2965 \text{ lb}\cdot\text{in} \qquad \text{from } C \text{ to } E \text{ in shaft}$$

Figure 12–14 shows a plot of the torque distribution *in the shaft* superimposed on the sketch of the shaft. When designing the shaft at *C*, we will use 2965 lb·in *at C and to the right*, but we can use 1112 lb·in *to the left of C*. Notice that no part of the shaft is subjected to the full 110 hp that comes into the sprocket at *C*. The power splits into two parts as it enters the shaft. When analyzing the sprocket itself, we must use the full 110 hp and the corresponding torque:

$$T_C = (63\,000)(110 \text{ hp})/1700 \text{ rpm} = 4076 \text{ lb}\cdot\text{in} \qquad \text{(torque on the sprocket)}$$

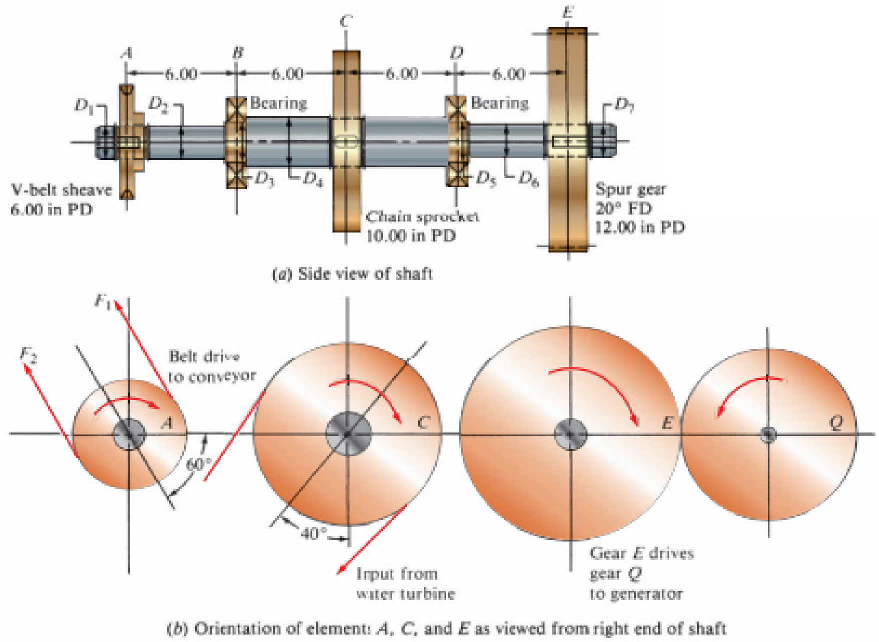


FIGURE 12–13 Shaft design for Example Problem 12–2

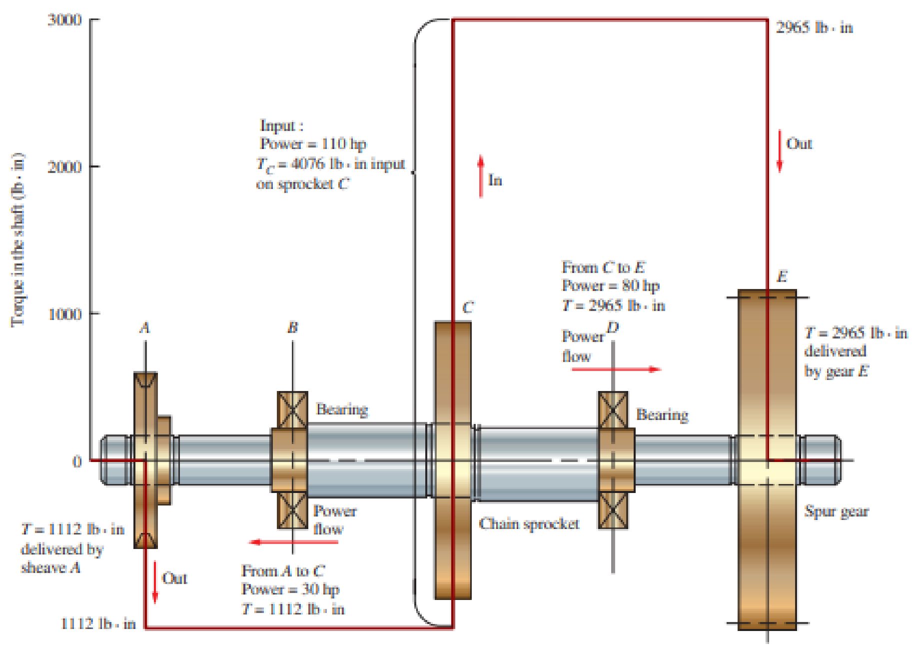


FIGURE 12–14 Torque distribution in the shaft

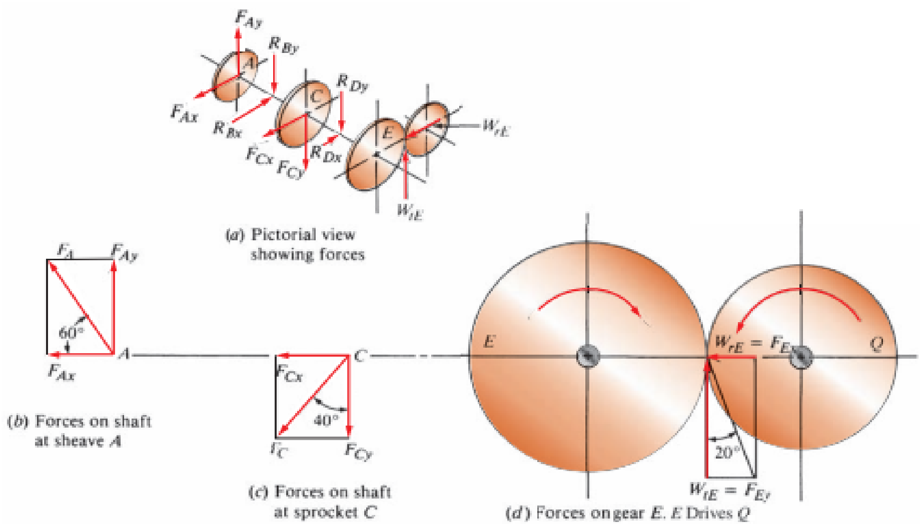


FIGURE 12–15 Forces resolved into x- and y-components

Forces: We will compute the forces at each element separately and show the component forces that act in the vertical and horizontal planes, as in Design Example 12–1. Figure 12–15 shows the directions of the applied forces and their components for each element.

1. ***Forces on sheave A:*** Use Equations (12–7), (12–8), and (12–12):

$$F_N = F_1 - F_2 = T_A/(D_A/2) = (1112 \text{ lb}\cdot\text{in})/3.0 \text{ in} = 371 \text{ lb} \qquad \text{(net driving force)}$$

$$F_A = 1.5\,F_N = 1.5(371 \text{ lb}) = 556 \text{ lb} \qquad \text{(bending force)}$$

The bending force acts upward and to the left at an angle of 60° from the horizontal. As shown in Figure 12–15, the components of the bending force are

$$F_{Ax} = F_A\cos(60°) = (556 \text{ lb})\cos(60°) = 278 \text{ lb} \leftarrow \text{(toward the left)}$$

$$F_{Ay} = F_A\sin(60°) = (556 \text{ lb})\sin(60°) = 482 \text{ lb} \uparrow \text{(upward)}$$

2. ***Forces on sprocket C:*** Use Equation (12–6):

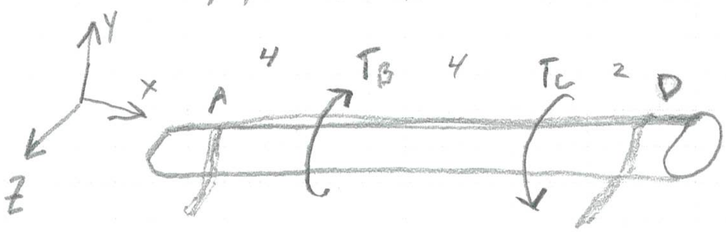
$$F_C = T_C/(D_C/2) = (4076 \text{ lb}\cdot\text{in})/5.0 \text{ in} = 815 \text{ lb}$$

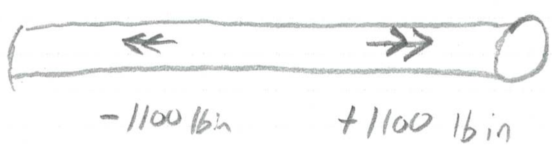
This is the bending load on the shaft. The components are

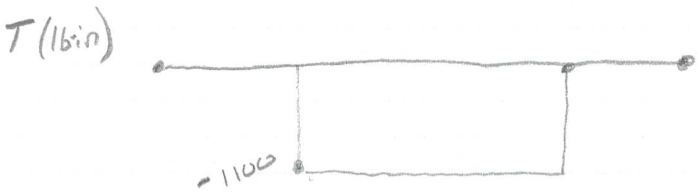
$$F_{Cx} = F_C\sin(40°) = (815 \text{ lb})\sin(40°) = 524 \text{ lb} \leftarrow \text{(to the left)}$$

$$F_{Cy} = F_C\cos(40°) = (815 \text{ in})\cos(40°) = 624 \text{ lb} \downarrow \text{(downward)}$$

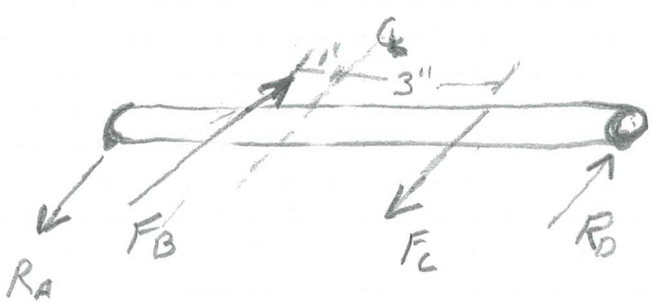
4-1 Mott







FORCES FREE BODY



$T_{MAX} = 1100$ lb in

$D = 1.25$ in

$$F_{NET} = T/(D/2) = 1100/(1.25/2) = 1760 \text{ lbf.}$$

BENDING FORCE ON SHAFT

$$F_B = 1.5\, F_{NET} = 2640 \text{ lbf.}$$