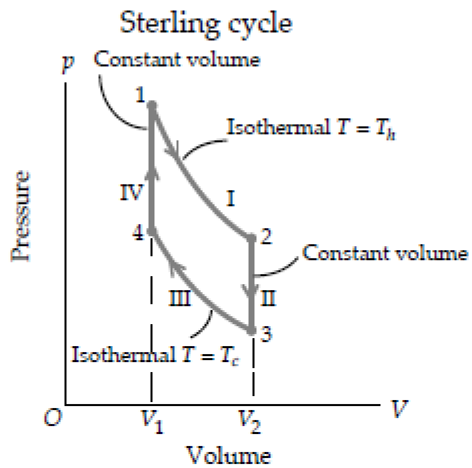
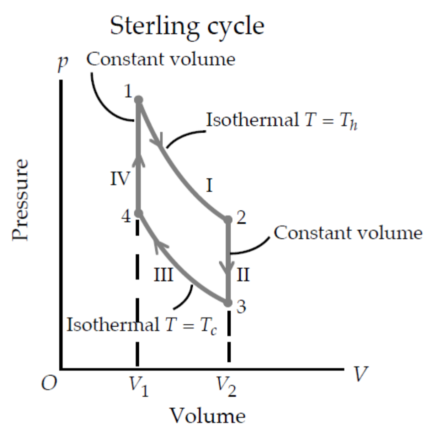
3. Consider the ideal Stirling cycle working between a maximum temperature T_h and a minimum temperature T_c, and a minimum volume V_1 and a maximum volume V_2. Suppose that the working gas of the cycle is 0.1 mol of an ideal gas with $c_V' = 5R/2$. (a) What are the heat flows to the cycle during each leg? Be sure to give the sign. For which legs is the heat flow positive? (b) What work is done by the cycle during each leg? Again, be sure to include the sign. (c) If, in the definition of the efficiency of this cycle, $\eta = W/Q_{pos}$, where Q_{pos} is the total positive heat flow to the engine, what is the efficiency of the cycle when $T_h = 700\text{K}$ and $T_c = 400\text{K}$? $V_1 = 0.5\text{ L}$ and $V_2 = 1.5\text{ L}$. Compare this efficiency to the efficiency of a Carnot cycle that operates between the same temperature extremes.



Problem 3.

A P-V Diagram of the Stirling cycle described in problem 3 is shown to the right.



In each leg, **I**, **II**, **III**, and **IV**, we wish to calculate the work during each step in the cycle, the heat gained from the hot reservoir, and the heat dissipated to the cold reservoir.

Each leg is labeled to the right and the tables below show all relevant calculations for the particular Stirling cycle.

Leg I

The gas in the engine undergoes isothermal expansion at constant temperature T_H. In order to maintain constant temperature, the gas must absorb heat from the hot reservoir.

The work done may be calculated:

$$W_{iso} = \int_{V_i}^{V_f} P\,dV = nRT\,ln(\frac{V_f}{V_i})$$

$\Delta T = 0 \rightarrow Q_c = 0$

$W = Q_H - Q_C \rightarrow W = Q_H \rightarrow Q_H = nRT\,ln(\frac{V_f}{V_i})$

Work is positive and heat is coming in from the hot reservoir.

Supposing T_h is 700 K , T_c is 400 K, V_i is 0.5 L , and V_f is 1.5 L:

$W = (0.1\ mol)\ (8.314 \frac{J}{mol * K})\ (700\ K)\ ln(\frac{1.5\ L}{0.5\ L}) = 639\ Joules$

$Q_H = -(639\ Joules)$

Work = 639 Joules, Q_C = 0 Joules, Q_H = 639 Joules

Leg II

The gas in the engine is contained within volume V_f, which stays constant during Leg II. During this isochoric process, the temperature of the gas falls from T_h to T_c.

The work done is zero as $dV = 0$.

Heat must be removed and $Q_H - Q_C = 0 \rightarrow Q_h = Q_c = mC_v\Delta T$