

instead of e , so we replace every factor of e^2 in the wave functions and the energy levels by Ze^2 . In particular, the energy levels are given by Eq. (41.21) with e^4 replaced by Z^2e^4 :

$$E_n = -\frac{1}{(4\pi\epsilon_0)^2} \frac{m_r Z^2 e^4}{2n^2 \hbar^2} = -\frac{Z^2}{n^2} (13.6 \text{ eV}) \quad (41.43)$$

This approximation is fairly drastic; when there are many electrons, their interactions with each other are as important as the interaction of each with the nucleus. So this model isn't very useful for quantitative predictions.

The Central-Field Approximation

A less drastic and more useful approximation is to think of all the electrons together as making up a charge cloud that is, on average, *spherically symmetric*. We can then think of each individual electron as moving in the total electric field due to the nucleus and this averaged-out cloud of all the other electrons. There is a corresponding spherically symmetric potential-energy function $U(r)$. This picture is called the **central-field approximation**; it provides a useful starting point for understanding atomic structure.

In the central-field approximation we can again deal with one-electron wave functions. The Schrödinger equation for these functions differs from the equation for hydrogen, which we discussed in Section 41.3, only in that the $1/r$ potential-energy function is replaced by a different function $U(r)$. Now, Eqs. (41.20) show that $U(r)$ does not appear in the differential equations for $\Theta(\theta)$ and $\Phi(\phi)$. So those angular functions are exactly the same as for hydrogen, and the orbital angular momentum *states* are also the same as before. The quantum numbers l , m_l , and m_s have the same meanings as before, and Eqs. (41.22) and (41.23) again give the magnitude and z -component of the orbital angular momentum.

The radial wave functions and probabilities are different than for hydrogen because of the change in $U(r)$, so the energy levels are no longer given by Eq. (41.21). We can still label a state by using the four quantum numbers (n , l , m_l , m_s). In general, the energy of a state now depends on both n and l , rather than just on n as with hydrogen. (Due to fine-structure effects, the energy can also depend on the total angular momentum quantum number j . These effects are generally small, however, so we ignore them for this discussion.) The restrictions on the values of the quantum numbers are the same as before:

Allowed values of quantum numbers for one-electron wave functions:	Principal quantum number $n \geq 1$	$0 \leq l \leq n - 1$	Orbital magnetic quantum number $ m_l \leq l$	Spin magnetic quantum number $m_s = \pm \frac{1}{2}$	(41.44)
		Orbital quantum number			

The Exclusion Principle

To understand the structure of many-electron atoms, we need an additional principle, the *exclusion principle*. To see why this principle is needed, let's consider the lowest-energy state, or *ground state*, of a many-electron atom. In the one-electron states of the central-field model, there is a lowest-energy state (corresponding to an $n = 1$ state of hydrogen). We might expect that in the ground state of a complex atom, *all* the electrons should be in this lowest state. If so, then we should see only gradual changes in physical and chemical properties when we look at the behavior of atoms with increasing numbers of electrons (Z).

Such gradual changes are *not* what is observed. Instead, properties of elements vary widely from one to the next, with each element having its own distinct personality. For example, the elements fluorine, neon, and sodium have 9, 10, and 11 electrons, respectively, per atom. Fluorine ($Z = 9$) is a *halogen*; it tends strongly to form compounds in which each fluorine atom acquires an extra electron.

Sodium ($Z = 11$) is an *alkali metal*; it forms compounds in which each sodium atom *loses* an electron. Neon ($Z = 10$) is a *noble gas*, forming no compounds at all. Such observations show that in the ground state of a complex atom the electrons *cannot* all be in the lowest-energy states. But why not?

The key to this puzzle, discovered by the Austrian physicist Wolfgang Pauli (Fig. 41.20) in 1925, is called the **exclusion principle**. This principle states that **no two electrons can occupy the same quantum-mechanical state** in a given system. That is, **no two electrons in an atom can have the same values of all four quantum numbers (n, l, m_l, m_s)**. Each quantum state corresponds to a certain distribution of the electron “cloud” in space. Therefore the principle also says, in effect, that no more than two electrons with opposite values of the quantum number m_s can occupy the same region of space. We shouldn’t take this last statement too seriously because the electron probability functions don’t have sharp, definite boundaries. But the exclusion principle limits the amount by which electron wave functions can overlap. Think of it as the quantum-mechanical analog of a university rule that allows only one student per desk. This same exclusion principle applies to all spin- $\frac{1}{2}$ particles, not just electrons. (We’ll see in Chapter 43 that protons and neutrons are also spin- $\frac{1}{2}$ particles. As a result, the exclusion principle plays an important role in the structure of atomic nuclei.)

CAUTION The meaning of the exclusion principle Don’t confuse the exclusion principle with the electrical repulsion between electrons. While both effects tend to keep electrons within an atom separated from each other, they are very different in character. Two electrons can always be pushed closer together by adding energy to combat electrical repulsion, but *nothing* can overcome the exclusion principle and force two electrons into the same quantum-mechanical state. ■

Table 41.2 lists some of the sets of quantum numbers for electron states in an atom. It’s similar to Table 41.1 (Section 41.3), but we’ve added the number of states in each subshell and shell. Because of the exclusion principle, the “number of states” is the same as the *maximum* number of electrons that can be found in those states. For each state, m_s can be either $+\frac{1}{2}$ or $-\frac{1}{2}$.

As with the hydrogen wave functions, different states correspond to different spatial distributions; electrons with larger values of n are concentrated at larger distances from the nucleus. Figure 41.8 (Section 41.3) shows this effect. When an atom has more than two electrons, they can’t all huddle down in the low-energy $n = 1$ states nearest to the nucleus because there are only two of these states; the exclusion principle forbids multiple occupancy of a state. Some electrons are forced into states farther away, with higher energies. Each value of n corresponds roughly to a region of space around the nucleus in the form of a spherical *shell*. Hence we speak of the *K* shell as the region that is occupied by the electrons in the $n = 1$ states, the *L* shell as the region of the $n = 2$ states, and so on. States with the same n but different l form *subshells*, such as the $3p$ subshell.

41.20 The key to understanding the periodic table of the elements was the discovery by Wolfgang Pauli (1900–1958) of the exclusion principle. Pauli received the 1945 Nobel Prize in physics for his accomplishment. This photo shows Pauli (on the left) and Niels Bohr watching the physics of a toy top spinning on the floor—a macroscopic analog of a microscopic electron with spin.



TABLE 41.2 Quantum States of Electrons in the First Four Shells

n	l	m_l	Spectroscopic Notation	Number of States	Shell
1	0	0	1s	2	<i>K</i>
2	0	0	2s	2	<i>L</i>
2	1	-1, 0, 1	2p	6	
3	0	0	3s	2	<i>M</i>
3	1	-1, 0, 1	3p	6	
3	2	-2, -1, 0, 1, 2	3d	10	
4	0	0	4s	2	<i>N</i>
4	1	-1, 0, 1	4p	6	
4	2	-2, -1, 0, 1, 2	4d	10	
4	3	-3, -2, -1, 0, 1, 2, 3	4f	14	

The Periodic Table

We can use the exclusion principle to derive the most important features of the structure and chemical behavior of multielectron atoms, including the periodic table of the elements. Let's imagine constructing a neutral atom by starting with a bare nucleus with Z protons and then adding Z electrons, one by one. To obtain the ground state of the atom as a whole, we fill the lowest-energy electron states (those closest to the nucleus, with the smallest values of n and l) first, and we use successively higher states until all the electrons are in place. The chemical properties of an atom are determined principally by interactions involving the outermost, or *valence*, electrons, so we particularly want to learn how these electrons are arranged.

Let's look at the ground-state electron configurations for the first few atoms (in order of increasing Z). For hydrogen the ground state is $1s$; the single electron is in a state $n = 1$, $l = 0$, $m_l = 0$, and $m_s = \pm\frac{1}{2}$. In the helium atom ($Z = 2$), *both* electrons are in $1s$ states, with opposite spins; one has $m_s = -\frac{1}{2}$ and the other has $m_s = +\frac{1}{2}$. We denote the helium ground state as $1s^2$. (The superscript 2 is not an exponent; the notation $1s^2$ tells us that there are two electrons in the $1s$ subshell. Also, the superscript 1 is understood, as in $2s$.) For helium the K shell is completely filled, and all others are empty. Helium is a noble gas; it has no tendency to gain or lose an electron, and it forms no compounds.

Lithium ($Z = 3$) has three electrons. In its ground state, two are in $1s$ states and one is in a $2s$ state, so we denote the lithium ground state as $1s^2 2s$. On average, the $2s$ electron is considerably farther from the nucleus than are the $1s$ electrons (Fig. 41.21). According to Gauss's law, the *net* charge Q_{encl} attracting the $2s$ electron is nearer to $+e$ than to the value $+3e$ it would have without the two $1s$ electrons present. As a result, the $2s$ electron is loosely bound; only 5.4 eV is required to remove it, compared with the 30.6 eV given by Eq. (41.43) with $Z = 3$ and $n = 2$. Chemically, lithium is an *alkali metal*. It forms ionic compounds in which each lithium atom loses an electron and has a valence of $+1$.

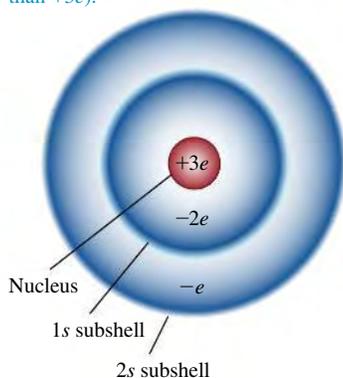
Next is beryllium ($Z = 4$); its ground-state configuration is $1s^2 2s^2$, with its two valence electrons filling the s subshell of the L shell. Beryllium is the first of the *alkaline earth* elements, forming ionic compounds in which the valence of the atoms is $+2$.

Table 41.3 shows the ground-state electron configurations of the first 30 elements. The L shell can hold eight electrons. At $Z = 10$, both the K and L shells are filled, and there are no electrons in the M shell. We expect this to be a particularly stable configuration, with little tendency to gain or lose electrons. This element is neon, a noble gas with no known compounds. The next element after neon is sodium ($Z = 11$), with filled K and L shells and one electron in the M shell. Its “noble-gas-plus-one-electron” structure resembles that of lithium; both are alkali metals. The element *before* neon is fluorine, with $Z = 9$. It has a vacancy in the L shell and has an affinity for an extra electron to fill the shell. Fluorine forms ionic compounds in which it has a valence of -1 . This behavior is characteristic of the *halogens* (fluorine, chlorine, bromine, iodine, and astatine), all of which have “noble-gas-minus-one” configurations (Fig. 41.22).

Proceeding down the list, we can understand the regularities in chemical behavior displayed by the **periodic table of the elements** (Appendix D) on the basis of electron configurations. The similarity of elements in each *group* (vertical column) of the periodic table is the result of similarity in outer-electron configuration. All the noble gases (helium, neon, argon, krypton, xenon, and radon) have filled-shell or filled-shell plus filled p subshell configurations. All the alkali metals (lithium, sodium, potassium, rubidium, cesium, and francium) have “noble-gas-plus-one” configurations. All the alkaline earth metals (beryllium, magnesium, calcium, strontium, barium, and radium) have “noble-gas-plus-two” configurations, and, as we just mentioned, all the halogens (fluorine, chlorine, bromine, iodine, and astatine) have “noble-gas-minus-one” structures.

41.21 Schematic representation of the charge distribution in a lithium atom. The nucleus has a charge of $+3e$.

On average, the $2s$ electron is considerably farther from the nucleus than the $1s$ electrons. Therefore, it experiences a net nuclear charge of approximately $+3e - 2e = +e$ (rather than $+3e$).



41.22 Salt (sodium chloride, NaCl) dissolves readily in water, making seawater salty. This is due to the electron configurations of sodium and chlorine: Sodium can easily lose an electron to form an Na^+ ion, and chlorine can easily gain an electron to form a Cl^- ion. These ions are held in solution because they are attracted to the polar ends of water molecules (see Fig. 21.30a).



TABLE 41.3 Ground-State Electron Configurations

Element	Symbol	Atomic Number (Z)	Electron Configuration
Hydrogen	H	1	1s
Helium	He	2	1s ²
Lithium	Li	3	1s ² 2s
Beryllium	Be	4	1s ² 2s ²
Boron	B	5	1s ² 2s ² 2p
Carbon	C	6	1s ² 2s ² 2p ²
Nitrogen	N	7	1s ² 2s ² 2p ³
Oxygen	O	8	1s ² 2s ² 2p ⁴
Fluorine	F	9	1s ² 2s ² 2p ⁵
Neon	Ne	10	1s ² 2s ² 2p ⁶
Sodium	Na	11	1s ² 2s ² 2p ⁶ 3s
Magnesium	Mg	12	1s ² 2s ² 2p ⁶ 3s ²
Aluminum	Al	13	1s ² 2s ² 2p ⁶ 3s ² 3p
Silicon	Si	14	1s ² 2s ² 2p ⁶ 3s ² 3p ²
Phosphorus	P	15	1s ² 2s ² 2p ⁶ 3s ² 3p ³
Sulfur	S	16	1s ² 2s ² 2p ⁶ 3s ² 3p ⁴
Chlorine	Cl	17	1s ² 2s ² 2p ⁶ 3s ² 3p ⁵
Argon	Ar	18	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶
Potassium	K	19	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s
Calcium	Ca	20	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ²
Scandium	Sc	21	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d
Titanium	Ti	22	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ²
Vanadium	V	23	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ³
Chromium	Cr	24	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ¹ 3d ⁵
Manganese	Mn	25	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ⁵
Iron	Fe	26	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ⁶
Cobalt	Co	27	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ⁷
Nickel	Ni	28	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ⁸
Copper	Cu	29	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ¹ 3d ¹⁰
Zinc	Zn	30	1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 3d ¹⁰

A slight complication occurs with the M and N shells because the $3d$ and $4s$ subshell levels ($n = 3, l = 2$, and $n = 4, l = 0$, respectively) have similar energies. (We'll discuss in the next subsection why this happens.) Argon ($Z = 18$) has all the $1s$, $2s$, $2p$, $3s$, and $3p$ subshells filled, but in potassium ($Z = 19$) the additional electron goes into a $4s$ energy state rather than a $3d$ state (because the $4s$ state has slightly lower energy).

The next several elements have one or two electrons in the $4s$ subshell and increasing numbers in the $3d$ subshell. These elements are all metals with rather similar chemical and physical properties; they form the first *transition series*, starting with scandium ($Z = 21$) and ending with zinc ($Z = 30$), for which all the $3d$ and $4s$ subshells are filled.

Something similar happens with $Z = 57$ through $Z = 71$, which have one or two electrons in the $6s$ subshell but only partially filled $4f$ and $5d$ subshells. These *rare earth* elements all have very similar physical and chemical properties. Another such series, called the *actinide* series, starts with $Z = 91$.

Screening

We have mentioned that in the central-field model, the energy levels depend on l as well as n . Let's take sodium ($Z = 11$) as an example. If 10 of its electrons fill its K and L shells, the energies of some of the states for the remaining electron are found experimentally to be

- 3s states: -5.138 eV
- 3p states: -3.035 eV
- 3d states: -1.521 eV
- 4s states: -1.947 eV

BIO Application Electron Configurations and Bone Cancer Radiotherapy

The orange spots in this colored x-ray image are bone cancer tumors. One method of treating bone cancer is to inject a radioactive isotope of strontium (^{89}Sr) into a patient's vein. Strontium is chemically similar to calcium because in both atoms the two outer electrons are in an s state (the structures are $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2$ for strontium and $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$ for calcium). Hence the strontium is readily taken up by the tumors, where calcium turnover is more rapid than in healthy bone. Radiation from the strontium helps destroy the tumors.



The $3s$ states are the lowest (most negative); one is the ground state for the 11th electron in sodium. The energy of the $3d$ states is quite close to the energy of the $n = 3$ state in hydrogen. The surprise is that the $4s$ state energy is 0.426 eV below the $3d$ state, even though the $4s$ state has larger n .

We can understand these results by using Gauss's law (Section 22.3). For any spherically symmetric charge distribution, the electric-field magnitude at a distance r from the center is $Q_{\text{encl}}/4\pi\epsilon_0 r^2$, where Q_{encl} is the total charge enclosed within a sphere with radius r . Mentally remove the outer (valence) electron atom from a sodium atom. What you have left is a spherically symmetric collection of 10 electrons (filling the K and L shells) and 11 protons, so $Q_{\text{encl}} = -10e + 11e = +e$. If the 11th electron is completely outside this collection of charges, it is attracted by an effective charge of $+e$, not $+11e$. This is a more extreme example of the effect depicted in Fig. 41.21.

This effect is called **screening**; the 10 electrons *screen* 10 of the 11 protons in the sodium nucleus, leaving an effective net charge of $+e$. From the viewpoint of the 11th electron, this is equivalent to reducing the number of protons in the nucleus from $Z = 11$ to a smaller *effective atomic number* Z_{eff} . If the 11th electron is *completely* outside the charge distribution of the other 10 electrons, then $Z_{\text{eff}} = 1$. Since the probability distribution of the 11th electron does extend somewhat into those of the other electrons, in fact Z_{eff} is greater than 1 (but still much less than 11). In general, an electron that spends all its time completely outside a positive charge $Z_{\text{eff}}e$ has energy levels given by the hydrogen expression with e^2 replaced by $Z_{\text{eff}}e^2$. From Eq. (41.43) this is

$$E_n = -\frac{Z_{\text{eff}}^2}{n^2} (13.6 \text{ eV}) \quad (41.45)$$

Effective (screened) atomic number

Principal quantum number

Energy levels of an electron with screening

CAUTION Different equations for different atoms Equations (41.21), (41.43), and (41.45) all give values of E_n in terms of $(13.6 \text{ eV})/n^2$, but they don't apply in general to the same atoms. Equation (41.21) is for hydrogen *only*. Equation (41.43) is for only the case in which there is no interaction with any other electron (and is thus accurate only when the atom has just one electron). Equation (41.45) is useful when one electron is screened from the nucleus by other electrons. **|**

DATA SPEAKS

Many-Electron Atoms and Electron States

When students were given a problem involving quantum-mechanical states in many-electron atoms, more than 32% gave an incorrect response. Common errors:

- **Confusion about quantum numbers.**

There are limits on the values of the four quantum numbers n , l , m_l , and m_s . For a given n value, l can be no greater than $n - 1$; for a given l value, m_l can be no greater than l and no less than $-l$; and m_s has only two possible values, $+\frac{1}{2}$ and $-\frac{1}{2}$.

- **Confusion about electron subshells.**

A *subshell* corresponds to a given value of n and l . The total number of electrons that can be present in a given subshell is $2(2l + 1)$ (that is, two possible values of m_s multiplied by $2l + 1$ possible values of m_l , from l through 0 to $-l$).

Now let's use the radial probability functions shown in Fig. 41.8 to explain why the energy of a sodium $3d$ state is approximately the same as the $n = 3$ value of hydrogen, -1.51 eV . The distribution for the $3d$ state (for which l has the maximum value $n - 1$) has one peak, and its most probable radius is *outside* the positions of the electrons with $n = 1$ or 2 . (Those electrons also are pulled closer to the nucleus than in hydrogen because they are less effectively screened from the positive charge $11e$ of the nucleus.) Thus in sodium a $3d$ electron spends most of its time well outside the $n = 1$ and $n = 2$ states (the K and L shells). The 10 electrons in these shells screen about ten-elevenths of the charge of the 11 protons, leaving a net charge of about $Z_{\text{eff}}e = (1)e$. Then, from Eq. (41.45), the corresponding energy is approximately $-(1)^2(13.6 \text{ eV})/3^2 = -1.51 \text{ eV}$. This approximation is very close to the experimental value of -1.521 eV .

Looking again at Fig. 41.8, we see that the radial probability density for the $3p$ state (for which $l = n - 2$) has two peaks and that for the $3s$ state ($l = n - 3$) has three peaks. For sodium the first small peak in the $3p$ distribution gives a $3p$ electron a higher probability (compared to the $3d$ state) of being *inside* the charge distributions for the electrons in the $n = 2$ states. That is, a $3p$ electron is less completely screened from the nucleus than is a $3d$ electron because it spends some of its time within the filled K and L shells. Thus for the $3p$ electrons, Z_{eff} is greater than unity. From Eq. (41.45) the $3p$ energy is lower (more negative)

than the $3d$ energy of -1.521 eV. The actual value is -3.035 eV. A $3s$ electron spends even more time within the inner electron shells than a $3p$ electron does, giving an even larger Z_{eff} and an even more negative energy.

This discussion shows that the energy levels given by Eq. (41.45) depend on both the principal quantum number n and the orbital quantum number l . That's because the value of Z_{eff} is different for the $3s$ state ($n = 3, l = 0$), the $3p$ state ($n = 3, l = 1$), and the $3d$ state ($n = 3, l = 2$).

EXAMPLE 41.9 DETERMINING Z_{eff} EXPERIMENTALLY



SOLUTION

The measured energy of a $3s$ state of sodium is -5.138 eV. Calculate the value of Z_{eff} .

SOLUTION

IDENTIFY and SET UP: Sodium has a single electron in the M shell outside filled K and L shells. The ten K and L electrons partially screen the single M electron from the $+11e$ charge of the nucleus; our goal is to determine the extent of this screening. We are given $n = 3$ and $E_n = -5.138$ eV, so we can use Eq. (41.45) to determine Z_{eff} .

EXECUTE: Solving Eq. (41.45) for Z_{eff} , we have

$$Z_{\text{eff}}^2 = -\frac{n^2 E_n}{13.6 \text{ eV}} = -\frac{3^2(-5.138 \text{ eV})}{13.6 \text{ eV}} = 3.40$$

$$Z_{\text{eff}} = 1.84$$

EVALUATE: The effective charge attracting a $3s$ electron is $1.84e$. Sodium's 11 protons are screened by an average of $11 - 1.84 = 9.16$ electrons instead of 10 electrons because the $3s$ electron spends some time within the inner (K and L) shells.

Each alkali metal (lithium, sodium, potassium, rubidium, and cesium) has one more electron than the corresponding noble gas (helium, neon, argon, krypton, and xenon). This extra electron is mostly outside the other electrons in the filled shells and subshells. Therefore all the alkali metals behave similarly to sodium.

EXAMPLE 41.10 ENERGIES FOR A VALENCE ELECTRON



SOLUTION

The valence electron in potassium has a $4s$ ground state. Calculate the approximate energy of the $n = 4$ state having the smallest Z_{eff} , and discuss the relative energies of the $4s$, $4p$, $4d$, and $4f$ states.

SOLUTION

IDENTIFY and SET UP: The state with the smallest Z_{eff} is the one in which the valence electron spends the most time outside the inner filled shells and subshells, so that it is most effectively screened from the charge of the nucleus. Once we have determined which state has the smallest Z_{eff} , we can use Eq. (41.45) to determine the energy of this state.

EXECUTE: A $4f$ state has $n = 4$ and $l = 3 = 4 - 1$. Thus it is the state of greatest orbital angular momentum for $n = 4$, and thus the state in which the electron spends the most time outside the electron charge clouds of the inner filled shells and subshells. This makes Z_{eff} for a $4f$ state close to unity. Equation (41.45) then gives

$$E_4 = -\frac{Z_{\text{eff}}^2}{n^2}(13.6 \text{ eV}) = -\frac{1}{4^2}(13.6 \text{ eV}) = -0.85 \text{ eV}$$

This approximation agrees with the measured energy of the sodium $4f$ state to the precision given.

An electron in a $4d$ state spends a bit more time within the inner shells, and its energy is therefore a bit more negative (measured to be -0.94 eV). For the same reason, a $4p$ state has an even lower energy (measured to be -2.73 eV) and a $4s$ state has the lowest energy (measured to be -4.339 eV).

EVALUATE: We can extend this analysis to the *singly ionized alkaline earth elements*: Be^+ , Mg^+ , Ca^+ , Sr^+ , and Ba^+ . For any allowed value of n , the highest- l state ($l = n - 1$) of the one remaining outer electron sees an effective charge of almost $+2e$, so for these states, $Z_{\text{eff}} = 2$. A $3d$ state for Mg^+ , for example, has an energy of about $-2^2(13.6 \text{ eV})/3^2 = -6.0$ eV.

TEST YOUR UNDERSTANDING OF SECTION 41.6 If electrons did *not* obey the exclusion principle, would it be easier or more difficult to remove the first electron from sodium? **I**