The Poynting vector is,

$$\mathbf{S} = \frac{\mu_0}{16\pi^2 c}\frac{(\hat{\mathbf{r}}\times\ddot{\mathbf{p}})^2}{|\mathbf{r}|^2}\hat{\mathbf{r}} \qquad \text{[I.1]}$$

Consider two charged particles, one at the origin (charge q1 and mass m1) and the other (charge q2 and mass m2) passing by with a large speed v, large enough that the trajectory is a straight line. The distance of closest approach is a. Find the Poynting vector in the *<u>center of mass frame</u>, with use of* [I.1].

Lab frame: we have location of each charge as a function of time,

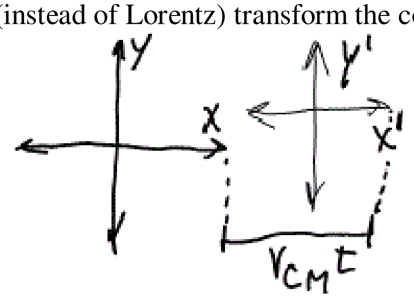
$$\mathbf{r}_1 = \mathbf{0} \qquad \text{[I.2]}$$

$$\mathbf{r}_2 = vt\hat{\mathbf{x}} + a\hat{\mathbf{y}} \qquad \text{[I.3]}$$

Transform to centre of momentum. The center of momentum coordinate is moving at a velocity,

$$\mathbf{v}_{CM} = \frac{\mathbf{p}_{CM}}{m_1+m_2} = \frac{m_1 v\hat{\mathbf{x}}+0}{m_1+m_2} = \frac{m_1}{m_1+m_2}v\hat{\mathbf{x}} \qquad \text{[I.4]}$$

Semiclassical approximation: can Galilean (instead of Lorentz) transform the coordinates to CM-frame,



[I.5]

$$x' \rightarrow x = x' - v_{CM,x}t \qquad \text{[I.6]}$$

$$y' \rightarrow y = y' - v_{CM,y}t = y - 0 = y \qquad \text{[I.7]}$$

Putting the transformations [I.6] and [I.7] into [I.2] and [I.3],

$$\mathbf{r}_1' = 0 - v_{CM}t = -\frac{m_1}{m_1+m_2}vt \qquad \text{[I.8]}$$

$$\mathbf{r}_2' = \left(vt - \frac{m_1}{m_1+m_2}vt\right)\hat{\mathbf{x}} + a\hat{\mathbf{y}} = \left(1 - \frac{m_1}{m_1+m_2}\right)vt\cdot\hat{\mathbf{x}} + a\hat{\mathbf{y}} = \frac{m_2}{m_1+m_2}vt\cdot\hat{\mathbf{x}} + a\hat{\mathbf{y}} \qquad \text{[I.9]}$$

Looking at [I.8] and [I.9], it seems that we still have "constant velocity" motion, and there's no way the dipole moment is accelerating,

$$\ddot{\mathbf{p}} = \frac{d^2}{dt^2}\left(q_1\mathbf{r}_1' + q_2\mathbf{r}_2'\right) = q_1\mathbf{0} + q_2\mathbf{0} = \boxed{\mathbf{0}} \qquad \text{[I.10]}$$

Putting [I.10] into [I.1], we instantly get,

$$\boxed{\mathbf{S} = 0} \qquad \text{[I.11]}$$

There's no need to even transform to the CM-frame; this [I.11] is true in all inertial (nonaccelerating) frames. Even if there is a field on the point charges exerting a force on them, it specifically says in the problem statement that the trajectory is a straight line, along which the particle flies with constant velocity.