Cosmology

What does it mean to say the universe is expanding?

Let's say that you were floating in empty, intergalactic space. However, you also brought a friend. Your friend also happened to be standing a very large distance away from you. In cosmological terms, you and your friend each constitute **bound systems**. If the universe was filled with water, then the individual H2O molecules would be bound systems. In the modern day, galaxies in our universe make up bound systems.

Let's also say that you and your friend agreed to stay 1 kilometer away from each other. You took very precise measurements to ensure that this was true, each set up your own laboratories at both points, and knew for a fact that the distance in between you was 1 kilometer. In fact, it was so important to you to that you were exactly 1 kilometer apart that you decided to send lasers in between the two of you. We know that light travels at a fixed speed *c*, so we can use elementary physics to calculate the distance in between you. If the laser took *t* seconds to reach you, then you can multiple this by the speed of light to get the distance. You measure $3.33564095 \times 10^{-6}$ seconds, and then multiply this by the speed of light (299,793.458 km/s) and find the distance to be one kilometer. Simple.

Or not? After a while, you notice something strange – the distance is slightly *larger* now. In order to make sure this isn't a measurement error, you ask your friend to perform the measurement also, and he finds the same conclusion. You even take out an enormous measuring tape, and confirm that the distance is larger. Let's say you now measure it be 1.1 kilometers.

You also happen to know another astronaut who has a lab set up exactly 1 kilometer in the other direction. So, you send a laser there and back. You find that the distance in now 1.1 kilometers. Strange.

What is happening is what is referred to as the **expansion of the universe**. From our above analogy, we can make a few conclusions about expansion:

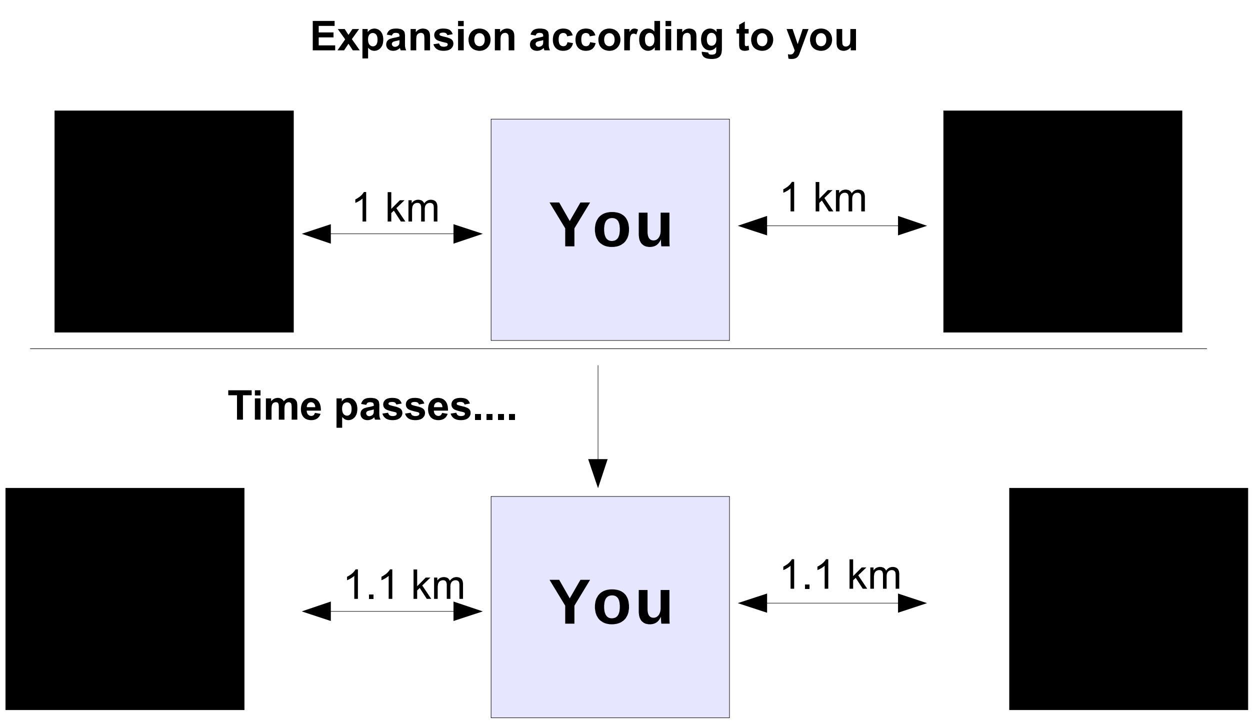
*It is the increase in distance between bound systems.

*It occurs in between *all* bound systems.

*No one is actually moving, distances are just getting larger.

From these three properties, we can make an important observation – the expansion is **isotropic**. This is a consequence of all of the above. Since the distances in between all bound systems increase, and you are still at rest (since nothing is moving, only distances are growing), you see everyone moving away from you. So, you appear to be at the center of the universe. This applies to everyone – your friend also says that your lab is moving away from him, so he claims that he is at the center of expansion.

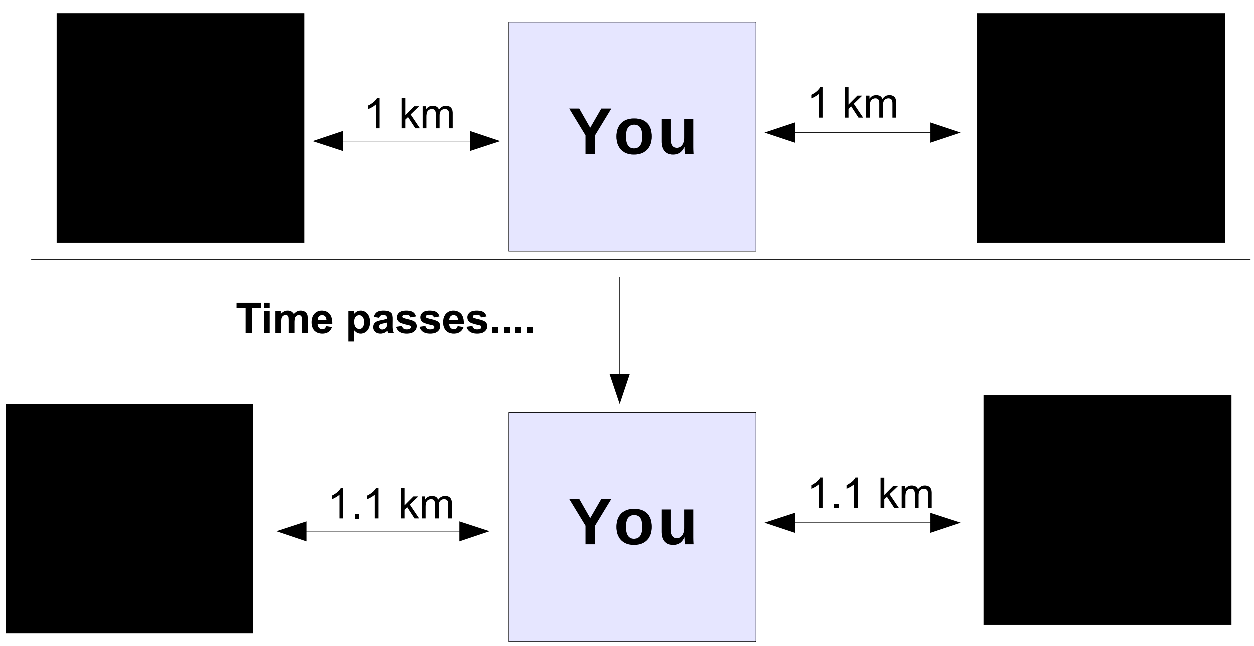
This also leads to another property of the expansion – **homogeneity**. This means that there are no special places in the universe – if you are surrounded by four objects, each equally distant, the will always remain equally distant from you – the expansion has no preferred direction.



So, even though nothing is really moving, the growth of distances makes it **look like** other bound systems are moving away. In reality, they aren't – it's the distance that's growing.

So, we'll call this apparent velocity **recessional velocity**. Before we get into it, let's examine something else.   What does the above diagram look like from the perspective of your friend on the right. Well, he sees you moving away with a recessional velocity. But since you also say the friend on the left moving with recessional velocity, then your friend must say he is moving away **faster** than you.

Expansion according to your Friend



I simply moved the black square to the right a little bit, to represent the fact that we are looking from his perspective. And from the isotropy property, we know that this means he considers himself to be at rest, and everyone else to be moving away. So, we can clearly see that the black square on the left has moved more than you have according to him. You appear to be .1 kilometer further away, but the black square on the left appear to be .2 kilometers further. This leads to the above conclusion – since he sees the farther object moving away faster, we can conclude that **apparent recessional velocity is proportional to distance**. That is, the farther a bound object is away from you, the faster it looks like it is receding.

This also makes sense from another perspective – consider property number 2 above. Since distances grows from every point, and there is more expanding space in between you and the further galaxy, so more distance growth, and larger recessional velocity.

Particularly, there is a mathematical formula for this, called **Hubble's Law**:

$\mathbf{V = H_0 D}$

Where **V** is the recessional velocity, and **D** is **proper distance**. Proper distance is the distance you would measure to the object if you could stop the expansion of the universe and then measure the intervening distance. $\mathbf{H_0}$ is called **Hubble's constant**. So, Hubble's constant determines how fast other bound systems (galaxies for our universe, you friends' labs in our analogy) appear to be moving away from you.

But, we've established many times that this velocity is only apparent, the systems aren't actually moving, distances are just getting larger. If so, then how are we to interpret the real physical meaning of the Hubble constant? In order to write down an expression for the Hubble constant that makes more sense, we must first introduce a concept called the **scale factor**.

Once again, let's return to you in intergalactic, expanding, space. We know when you first set up your equipment, the distance in between you and your friend was 1 km. Let's call this distance $\mathbf{d_0}$. Then, at a later time *t*, we measure the distance to be 1.1 km. Let's call this **d(t)**. We can now relate these two distances with an equation:

$\mathbf{d(t) = a(t)d_0}$

Where a(t) is some positive constant. So, we see that this constant controls how fast the distance in between *any* two arbitrarily defined points. We can think of it as the rate of expansion. If your bank account was the distance in between the labs, then a(t) would be like the interest rate – a very high interest rate means your bank account grows very fast. Similarly, a high a(t) means distances grow very fast. Because it essentially 'scales' distances, we call a(t) the **scale factor**. What's the scale factor for our example? Well, using the above equation,

a(t) = (d(t))/(d(0) = (1.1 km)(1.0 km) = 1.1

So, we notice something interesting – the scale factor is dimensionless, it has no units. If it remained at this same value forever, then the universe wouldn't be expanding – if the distance is larger at an even later time, then the scale factor must be growing. So, we can finally get a very good definition for what it means to say the universe is expanding: **the universe is expanding if the scale factor is increasing with time**. Mathematically speaking, the time derivative (how the scale factor changes in time) is positive. What is a time derivative on any quantity? Well, it's how that quantity changes over time. What is the time derivative of position? Velocity. What about the time derivative of velocity? Acceleration. We write the time derivative of the scale factor as a'(t) (read 'a prime'), but you also see it written as an a with a dot over it. So, the time derivative of the scale factor represents how fast distances are growing.

Now, back to Hubble's constant. We can re-arrange Hubble's Law to say that **H_o = V/D.** Now, we know that recessional velocity is dependent on how faster distances grow. From the last section, we know this is the time derivative of the scale factor. We also know that distance can be associated with the scale factor. Using those facts, we now have an expression for the Hubble constant:

H = (a'(t))/(a(t))

That is, Hubble's constant equals time tine derivative of the scale factor divided by the scale factor.