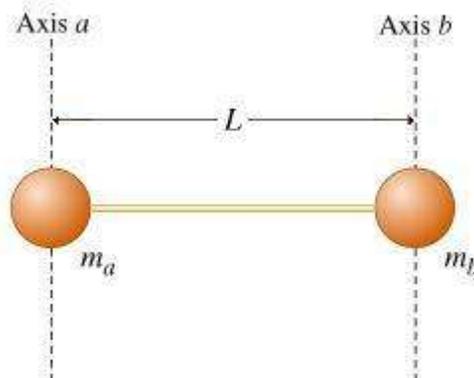


### Moment of Inertia and Center of Mass for Point Particles

Ball a, of mass  $m_a$ , is connected to ball b, of mass  $m_b$ , by a massless rod of length  $L$ . The two vertical dashed lines in the figure, one through each ball, represent two different axes of rotation, axes  $a$  and  $b$ . These axes are parallel to each other and perpendicular to the rod. The moment of inertia of the two-mass system about axis  $a$  is  $I_a$ , and the moment of inertia of the system about axis  $b$  is  $I_b$ . It is observed that the ratio of  $I_a$  to  $I_b$  is equal to 3:

$$\frac{I_a}{I_b} = 3$$



Assume that both balls are pointlike; that is, neither has any moment of inertia about its own center of mass.

#### Part A

Find the ratio of the masses of the two balls.

##### Hint A.1 How to approach the problem

Find an expression for  $I_a$  and for  $I_b$  in terms of the masses  $m_a$  and  $m_b$ . Substitute these expressions into the formula given in the problem introduction and then solve for the ratio of the masses.

##### Part A.2 Find $I_a$

Find  $I_a$ , the moment of inertia of the system about axis  $a$ .

##### Hint A.2.a Formula for the moment of inertia

The formula for the moment of inertia  $I$  of an object consisting of particles with masses  $m_i$ , at distances  $r_i$  from the rotation axis is

$$I = \sum_i m_i r_i^2 .$$

Express your answer in terms of any or all of the following quantities:  $L$ ,  $m_a$ , and  $m_b$ .

ANSWER:  $I_a = m_a * L^2$

Express your answer numerically.

ANSWER:  $\frac{m_a}{m_b} = \text{Answer not displayed}$

#### Part B

Find  $d_a$ , the distance from ball A to the system's center of mass.

**Hint B.1 How to approach the problem**

To find  $d_a$ , compute the position of the center of mass of the system, using a coordinate system in which ball a is at the origin. In these coordinates, ball a is at distance zero from the origin, ball b is at distance  $L$  from the origin, and the center of mass is at distance  $d_a$  from the origin. Using these values in the equation for center of mass, you obtain

$$d_a = \frac{0 \cdot m_a + Lm_b}{m_a + m_b}.$$

Use the result from Part A to eliminate  $m_a$  and  $m_b$  from this equation and obtain  $d_a$  in terms of  $L$ .

**Part B.2 Find  $m_b$  in terms of  $m_a$**

What is  $m_b$  in terms of  $m_a$ ?

ANSWER:  $m_b = \text{Answer not displayed}$

**Express your answer in terms of  $L$ , the length of the rod.**

ANSWER:  $d_a = \text{Answer not displayed}$