The Feynman Path Integral Technique

on one page (or less)

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Suppose a particle starts at point a(x,t). We wish to calculate the probability that our measurement device will find it at point b(x,t).

Step 1: Draw every path φ from a to b. In general, there will be a truly infinite number of paths. However, it is possible to approximate the path integral technique using the "K most representative" paths, where K is a (suitably large but finite) integer. So when we approximate, the paths are labelled with the variable k, where $k = 1, 2, \ldots, K$. In the general case (with an infinite number of paths), we use the continuous variable z instead of the discrete variable k.

Step 2: For each path φ_k, calculate the lagrangian L_k:

$$L_k(t) = KE(x,t) - PE(x,t) \tag{1}$$

where KE is the kinetic energy and PE is the potential energy of the particle.

Step 3: For each path φ_k, calculate the classical action S_k:

$$S_k = \int L_k \, dt. \tag{2}$$

Step 4: For each path φ_k, calculate the amplitude ϕ_k:

$$\phi_k = A e^{-\imath S_k / h} \tag{3}$$

where A is a normalization constant.
Step 5: Calculate the wavefunction Ψ_b that the particle will be detected at b:

$$\Psi_b = \sum_{k=1}^{K} \phi_k \tag{4}$$

Step 6: Calculate the relative probability P_b that the particle will be detected at b:

$$\mathrm{P_b} = |\Psi_b|^2 \tag{5}$$

Step 7: For each individual point on the detector, repeat steps 1 through 6.

Summary:

$$\mathrm{P_b} = |A|^2 \left| \sum_{k=1}^{K} e^{(-\imath/h)\int_{k^{th}{}_{path}} KE(x,t) - PE(x,t) dt} \right|^2 \tag{6}$$

If we let the sum become an integral (so the paths are labelled by z instead of k), we have

$$\mathrm{P_b} \quad = \quad |A|^2 \left| \int_{-\infty}^{\infty} e^{(-\imath/h)\int_{z^{th}{}_{path}} KE(x,t) - PE(x,t) dt} dz \right|^2 \tag{7}$$