$$\psi(x) = Be^{kx} \qquad \forall \qquad x < -a \tag{1}$$

$$\psi(x) = Ce^{-kx} + De^{kx} \qquad \forall \qquad x < -a \tag{2}$$

$$\psi(x) = Fe^{kx} \qquad \forall \qquad x > a \tag{3}$$

From the earlier equations,

$$kCe^{ka} = \frac{m\alpha}{\hbar^2} Be^{-ka} \tag{4}$$

$$kDe^{ka} = \frac{m\alpha}{\hbar^2} Fe^{-ka} \tag{5}$$

it is obvious that

$$\frac{D}{C} = \frac{F}{B} = \lambda \tag{6}$$

where λ is some scalar constant. Hence $D = \lambda C$ and $F = \lambda B$. Substituting into the original system,

$$Be^{-ka} = Ce^{ka} + De^{-ka} \tag{7}$$

$$Fe^{-ka} = Ce^{-ka} + De^{ka} \tag{8}$$

dividing (7) by (8), we get,

$$\frac{1}{\lambda} = \frac{e^{2ka} + \lambda}{1 + \lambda e^{2ka}} \tag{9}$$

which is easily solved to get $\lambda = \pm 1$. Hence the two solutions are $D = \pm C$ and $F = \pm B$.

Plugging back into either of (7) or (8) yields

$$D = C = \left(\frac{e^{-ka}}{e^{ka} + e^{-ka}} \right) B \tag{10}$$

for the positive sign, and,

$$D = -C = \left(\frac{e^{-ka}}{e^{ka} - e^{-ka}} \right) B \tag{11}$$

for the negative sign.

Hence for the positive sign,

$$\psi(x) = Be^{kx} \qquad \forall \qquad x < -a \tag{12}$$

$$\psi(x) = \left(\frac{Be^{-ka}}{e^{ka} + e^{-ka}}\right)\left(e^{kx} + e^{-kx}\right) \qquad \forall \qquad -a < x < a \tag{13}$$

$$\psi(x) = Be^{-kx} \qquad \forall \qquad x > a \tag{14}$$

And for the negative sign,

$$\psi(x) = Be^{kx} \qquad \forall \qquad x < -a \tag{15}$$

$$\psi(x) = \left(\frac{Be^{-ka}}{e^{ka} - e^{-ka}}\right)\left(e^{-kx} - e^{kx}\right) \qquad \forall \qquad -a < x < a \tag{16}$$

$$\psi(x) = -Be^{-kx} \qquad \forall \qquad x > a \tag{17}$$

Consider (+) sign. From equations (4) and (10), we get

$$\left(\frac{\hbar^2}{m\alpha}\right) k = 1 + e^{-2ka} \tag{18}$$

again a transcedental equation.

Issues:

1. Solution of an equation like (18).
2. What is the significance of the sign of α? If we chose (16) for example, then the probability of finding the particle at $x = 0$ is zero, whereas it is nonzero from (13).