

I have corrected your original bending curve by inserting the factor of $1/EI$ where EI is the bending stiffness of the beam I assumed.

$$w'(x) = \frac{1}{EI} \left[\frac{q_0}{6} \langle x-l \rangle^3 - \frac{q_0 l}{4} \langle x-l \rangle^3 - \frac{3q_0 l}{4} \langle x-l \rangle^2 + \frac{9q_0 l^3}{24} \right]$$

To find the position along the beam at which the deflection is maximum, $w'(x) = 0$, NOT $w'(0) = 0$.

Hence, expanding the unitstep or heaviside functions as if they were normal polynomial functions gives

$$\begin{aligned} w'(x) &= \frac{1}{EI} \left[\frac{q_0}{6} \langle x-l \rangle^3 - \frac{q_0 l}{4} \langle x-l \rangle^3 - \frac{3q_0 l}{4} \langle x-l \rangle^2 + \frac{9q_0 l^3}{24} \right] \\ &= \frac{1}{EI} \left[\frac{q_0}{6} (x^3 - l^3 + 2l^2 x - 2lx^2) - \frac{q_0 l}{4} (x^3 \dots\dots) \dots\dots \right] \\ &= \dots\dots \end{aligned}$$

As you can imagine, it is very complicated. I doubt you would have time solving for x in the examination.

However, you can predict the position along the beam at which the deflection is maximum by observing the moment diagram $M(x)$. Just look at the stationary point(s) and the corresponding position x yields the maximum deflection.

Anyway, our professor has told us that this kind of problem will not be asked in the examination. If it were asked, you just need to write

$$w'(x) = \frac{1}{EI} \left[\frac{q_0}{6} \langle x-l \rangle^3 - \frac{q_0 l}{4} \langle x-l \rangle^3 - \frac{3q_0 l}{4} \langle x-l \rangle^2 + \frac{9q_0 l^3}{24} \right] = 0$$

solving for x gives the position along the beam at which the max deflection occurs.

for the examination is to test your knowledge and skills in mechanics, not elementary pure mathematics (leave this task to the mathematicians).