



total mass = M Mass per length = m

1. $F = m a$ (force = mass \times accel.)

2. $a = r \omega^2$

3. $\therefore F = M r \omega^2$

4. Assuming straight shaft.

$$r = \frac{\Delta}{L} \times l \quad (\text{radius @ given distance})$$

5. $\therefore F = m \cdot \left(\frac{\Delta l}{L} \right) \cdot \omega^2$

6. $\Sigma F = \int_0^L \left(m \cdot \frac{\Delta l}{L} \cdot \omega^2 \right) dl$

Removing Constants:

7. $\Sigma F = m \cdot \omega^2 \cdot \frac{\Delta}{L} \int_0^L l \, dl$

8. $\Sigma F = m \cdot \omega^2 \cdot \frac{\Delta}{L} \left(\frac{l^2}{2} \right)$

9. $\Sigma F = \frac{m \cdot \omega^2 \cdot \Delta \cdot l^2}{2L}$

UNITS (S.I.)

10. $F = \frac{\text{kg}}{1} \times \left(\frac{\text{rad}}{\text{s}} \right)^2 \times \frac{\text{m}}{1} \times \left(\frac{\text{m}}{1} \right)^2 \times \frac{1}{\text{m}}$

11. $= \frac{\text{kg} \cdot \text{rad}^2 \cdot \text{m} \cdot \text{m}^2}{\text{s}^2 \cdot \text{m}^2}$

12. $= \frac{\text{kg} \cdot \text{m}^3}{\text{s}^2 \cdot \text{m}^2}$

13. $= \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$

14. $= \text{N} \cdot \text{m}$ (TORQUE)

15. Divide by shaft length, L

$$F = \frac{\text{N} \cdot \text{m}}{\text{m}}$$

$$= \text{N}$$

16. Divide by shaft length, L

17. $F = \frac{\text{N}}{\text{m}}$

Giving Equivalent U.D.L

18. Applying to line (9).

$$F = \frac{m \cdot \omega^2 \cdot \Delta \cdot l^2}{2L} \times \frac{1}{L^2}$$

19. $F = \left(\frac{m \cdot \omega^2 \cdot \Delta \cdot l^2}{2L^3} \right) \text{N/m}$

20. Applying cantilever & UDL deflection equation:

$$\delta_{\text{max}} = \frac{W L^4}{8 E I}$$

where $W = \text{line (19)}$. $E = \text{Young's Modulus}$ $I = \text{Moment of Inertia}$.

21. $\delta_{\text{max}} = \frac{\left(\frac{m \cdot \omega^2 \cdot \Delta \cdot l^2}{2L^3} \right) \times L^4}{8 \cdot E \cdot I}$