COMPUTATIONAL MATHEMATICS 1

Assignment 3
Due Tuesday October 3

1. Let $f(x) = \ln(1+x)$.

   (a) Find a **third** degree **Taylor** polynomial, $P_3(x)$, about $a = 0$ for $f(x)$, together with the remainder term.

   (b) Plot $f(x)$ and $P_3(x)$ on the same graph for $0 \leq x \leq 2$.

   (c) Use $P_3(x)$ to approximate $\ln 2$. Calculate an error bound of this interpolation value. Compare the theoretical eror bound with the actual error.

   (d) Compute the following limit:

$$\lim_{x \to 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2}{x^3}.$$

2. The **error function**

$$\texttt{erf(x)} = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

   plays an important role in the solution of heat conduction problems. A table for various values of $erf(x)$ is given by

x	0.2	0.6	1.0	1.4	1.8
`erf(x)`	0.2227	0.6039	0.8427	0.9523	0.9891

   (a) Construct a second degree **Lagrangian** interpolating polynomial using the first 3 table values.

   (b) Use the result of (b) to approximate `erf`(0.5).

   (c) Construct the **divided-difference** table.

   (d) Find **Divided-difference** polynomials of degree **two** and **three** using $x_0 = 0.2$. Comment on the results.

   (e) Use the results of (d) to approximate `erf`(0.5).

(f) Construct the **forward difference** table.

(g) Find the **Newton-Gregory** forward polynomials of degree **two** and **three** again using $x_0 = 0.2$.

(h) Use the results of (g) to approximate `erf`(0.5).

(i) Use the **Newton-Gregory** backward polynomial of degree **three** to approximate `erf(0.5)` using $x_0 = 1.4$

(j) Approximate `erf(0.9)` using a **central** polynomial of degree 2 with $x_0 = 1.0$.

The **central** polynomial is given by

$$f(x_s) \approx f_0 + \binom{s}{1}\frac{\Delta f_{-1} + \Delta f_0}{2} + \frac{\binom{s+1}{2} + \binom{s}{2}}{2}\Delta^2 f_{-1} + \binom{s+1}{3}\frac{\Delta^3 f_{-2} + \Delta^3 f_{-1}}{2}.$$