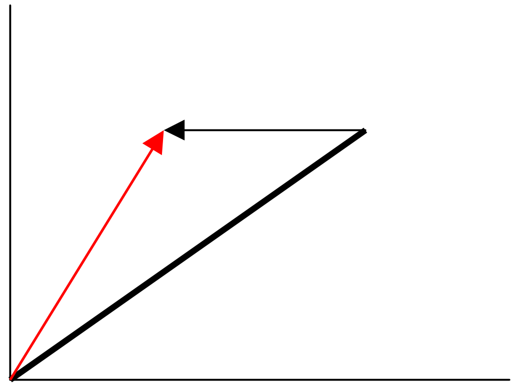

$$c\tau' = \sqrt{l^2 \sin^2\theta + \left(\frac{l\cos\theta}{\gamma} - v\tau'\right)^2}$$

$$c^2\tau'^2 = l^2\left(1-\cos^2\theta\right) + \frac{l^2\cos^2\theta}{\gamma^2} - 2\frac{l\cos\theta}{\gamma}v\tau' + v^2\tau'^2$$

$$c^2\tau'^2 = l^2\left(1-\cos^2\theta + \frac{\cos^2\theta}{\gamma^2}\right) - 2\frac{l\cos\theta}{\gamma}v\tau' + v^2\tau'^2$$

$$0 = \left(c^2 - v^2\right)\tau'^2 + 2\frac{l\cos\theta}{\gamma}v\tau' - l^2\left(1-\cos^2\theta\left(1-\frac{1}{\gamma^2}\right)\right)$$

$$0 = \left(1-\beta^2\right)c^2\tau'^2 + 2\frac{l\cos\theta}{\gamma}\beta c\tau' - l^2\left(1-\beta^2\cos^2\theta\right)$$

$$0 = \frac{c^2\tau'^2}{\gamma^2} + 2l\beta\cos\theta\frac{c\tau'}{\gamma} - l^2\left(1-\beta^2\cos^2\theta\right)$$

$$\frac{c\tau'}{\gamma} = \frac{-2l\beta\cos\theta \pm \sqrt{\left(2l\beta\cos\theta\right)^2 + 4l^2\left(1-\beta^2\cos^2\theta\right)}}{2}$$

$$\frac{c\tau'}{\gamma} = -l\beta\cos\theta \pm l\sqrt{\beta^2\cos^2\theta + 1 - \beta^2\cos^2\theta}$$

$$\frac{c\tau'}{\gamma} = -l\beta\cos\theta \pm l = l - l\beta\cos\theta = l\left(1-\beta\cos\theta\right)$$

$$\tau' = \left(1-\beta\cos\theta\right)\gamma\frac{l}{c}$$