For their conclusions, all four students indicated that the bob's trajectory was different each time it was released, which supports sub-hypothesis H2a (limited prediction). ED and AB wrote, "It depends how you release it," evidence for sub-hypothesis H2c (sensitivity to initial conditions). JQ could not let go of the notion that there must be some predictability in the system, incorrectly assuming that the bob "will always go to the opposite magnet" based on two trials, evidence for hypothesis H1 (prior predictability). However, there is also significant qualitative evidence that JQ's views were altered during the intervention ("it didn't do that"), so his comments support H2a as well. [19]

January 18, 2006

On Wednesday, Groups 2 (FR, PA, EZ, and KC) and 6 (MT, CP, IP, JCh) began the computer portion of "chaos project." Students observed an evolving a phase space plot of a chaotic pendulum on the Internet.[20] Answering the question, "Does the pendulum's motion ever repeat itself?" EZ wrote, "yes and no because it never actually goes on the same line twice." CP observed, "There is a pattern, but it changes a little bit each time. It could never go through the middle or the corners." Thus, students observed strange attractors in phase space, and could see how they are bounded and ergodic. Because of time constrains, such technical terms were not introduced, but students gained some understanding of these concepts because they had already been introduced to phase space graphs when working with the simple pendulum.

Next, students were shown a computer simulation of the magnetic pendulum written in the Boxer programming language by Dr. Andrea A. diSessa, Boxer's inventor. The software helped students perform the experiment of mapping starting points ("initial conditions") with the magnet at which the bob eventually ends up. To facilitate this experiment, one magnet was colored red and the other was colored green. Once the bob icon stops moving, a red or green dot is left on the screen showing at which magnet the bob ended up when it started from that point. Students were given a worksheet (see Appendix) with a blank graphics box just like that in the Boxer pendulum simulation and asked to "make a hypothesis about what you think the graphics box would look like if all starting positions (initial conditions) were tried." JCh predicted "it's random," and KC drew interwoven regions of red and regions of green throughout the box. [21]

Using the Boxer software, EZ explored the boundary between a red region and a green region. He found that the boundary has "hooks," much like the pictures of fractals passed out in class. The concept that zooming in on the border region produces a greater and greater level of detail was facilitated through a magnifying tool in Boxer. Students

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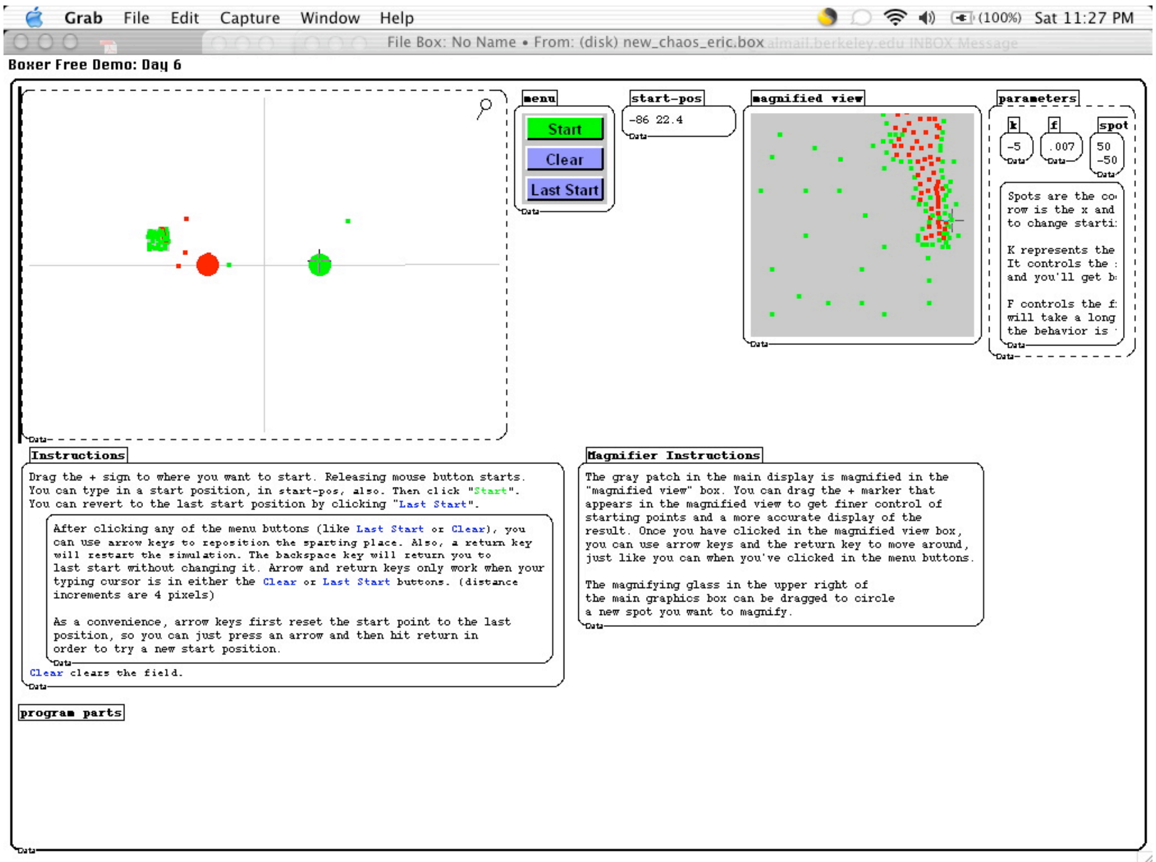
[19] ED observed that the magnets did not seem to effect the bob when it was dropped from a large height because the bob had a large momentum by the time it came close to the magnets. In future work, instructions should clearly indicate that students should only release the bob from small initial displacements.

[20] See www.myphysicslab.com/pendulum2.html for this educational resource.

[21] Students should be given red and green pens to draw their hypotheses. Scaffolding was required to get students to understand how to use the zooming in tool built into the software.

could click on a magnifying glass icon to see a magnified portion of the graphics box in another box, as shown in the picture below:



Picture 2. The results of EZ’s experimentation with the Boxer simulation. The large red and green dots are the two magnets. The small red and green dots represent positions from which the bob’s trajectory ends at the red or green magnet, respectively. On the right is a magnified view of what appears to be a mostly green square to the upper left of the red magnet.

Students commented on their findings:
FR: As you look at smaller scales, you see more things
PI: Recall the dueling calculator activity...
KC: When you get closer, you see more detail
PI: Do you see a connection between round off and...
FR: it’s getting smaller as you go on...there’s something beyond that
PI: When you round it, you’re eliminating the small detail
PI: EZ is seeing the shape of the fractal boundary...it’s a hook shape
With the Boxer simulation, students could actively construct a visual map of the chaotic and non-chaotic regions of starting positions, a reproducible benefit of using computers to supplement the teaching of modern physics concepts in classrooms.

January 19, 2006

On Thursday prior to class, a magnetic pendulum was set up for students to observe. Groups 3 (WL, DN, BT, and AC) and 7 (JG, CS, BS, and MSt) were instructed to describe the behavior of the magnetic pendulum. 50% of students (MSt, DN, BS, and WL) wrote "chaotic and unpredictable," with MSt adding "speratic [sic]"— evidence for H2a (limited prediction), and H2d (examples of chaos).

Students in Groups 3 and 7 then read a handout with five pieces of information about the project:

1. As the system approaches equilibrium and stops moving, its mechanical energy dissipates, becoming thermal energy.
2. The boundaries between regions of initial conditions from which the pendulum reaches equilibrium at magnet "A," and regions from which it reaches equilibrium at the other magnet "B," can also be fractals.
3. Altering the system's parameters (bob mass, string length, friction) can make it either chaotic, periodic, or quasi-periodic on its way to equilibrium.
4. When it is chaotic, "windows" of ordered, periodic behavior can occur (called "dissipative structures").
5. The system becomes truly chaotic after repeated period doubling, or "bifurcation."[22] Cycles of two become cycles of four, eight, 16, 32, 64, and on to infinity (true chaos), in less and less time. Chaos is when the same pattern never repeats. The way in which period doubling occurs is similar in all chaotic systems.[23]

Below this information was a series of pictures of the Mandelbrot Set from Burger and Starbird (2000). A sequence of smaller scale, zoomed in images embedded within larger images showed how increasing magnification power produces new intricate details on smaller and smaller scales. [24]

The handout also included a picture from Gleick (1987) of a possible hypothesis as to which initial conditions would end in the bob being suspended over the red magnet, and which would result in it ending up at the green magnet. In this picture, the "red" and "green" regions swirl around each other. Students were instructed to draw their own hypotheses. Most drew something similar to the example hypothesis; some drew more simplistic pictures. They then used the Boxer software to produce evidence supporting or disproving their hypotheses.

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[22] In the study of dynamical systems, a "bifurcation" occurs when a small smooth change made to the parameter values of a system will cause a sudden *qualitative* or topological change in the system's long-term dynamical behavior. This can be seen mathematically in the logistic map. Bifurcations in the pendulum's trajectory could not be detected by visual observation alone. However, it is noticeable that when released twice from approximately the same position, the pendulum starts out moving the same way before trajectories bifurcate and diverge.

[23] This information was probably not well understood by many of the students. In future work, the information should be rephrased using fewer technical terms, and given out at the end of the project.

[24] Mandelbrot (1977) wrote, "in the final analysis, fractal methods can serve to analyze any 'system,' whether natural or artificial, that decomposes into 'parts' articulated in a self-similar fashion, and such that the properties of the parts are less important than the rules of articulation."

When a bug in the software caused some problems, the instructor opened a file containing EZ's work from the day before. Comparing this information with their own experimental explorations, students saw how the boundary between "red" initial positions and "green" ones was very complex, and did not look anything like the picture from Gleick (1987). Since no students had predicted such a level of intricacy, students saw how the Boxer representation provided evidence against their initial hypotheses. A discussion was then held about whether or not it is appropriate to apply evidence from a simulation to make conclusions about a real world system:

PI: Are we disproving our hypothesis? Why do you think this simulation might be different from what we found in the lab?

AC: It's on the computer, it's not in real life.

PI: Would it be possible to create a computer program where it would be the same?

AC: No.

PI: Why not?

AC: It's not random.

PI: Are you saying that you can't use a computer to model behavior that in nature is random?

AC: yeah, exactly

Interestingly, AC expressed an epistemological belief that computers cannot accurately model nature, because nature has more inherent randomness than computers are able to handle. This is qualitative evidence supporting sub-hypothesis H2b (modeling uncertainty). BS, however, expressed an opposing view:

BS: The computer knows enough to calculate what's going to happen, and so it eliminates some of the variables, because it knows how to compute them out and it can do it

PI: Are you talking about round-off error like we saw in the dueling calculators?

BS: Something like that.

PI: Are you saying that you think it's impossible to get a computer simulation to behave like an actual system?

BS: Not necessarily, I'm saying it can

PI: Do you think you can see some of the same patterns that exist in nature using the computer simulation?

BS: Yeah, you probably could program it so you could

PI: In this particular simulation, there is no gravity. Would this effect the shape of the map?

BS: maybe...

BS believes that computers can accurately model nature. He is unsure if the lack of gravity in the simulation may have produced inaccurate results, a non-trivial question.

In their conclusions, students compared their results with their hypotheses. Six out of eight students explicitly stated that their hypotheses were "disproved" based on the evidence that the boundaries (between the red and green regions) were "fractal." A seventh student (BT) described the fractal nature of the boundary without using the term explicitly: "There were some points that were in between green and red." Thus, 87.5% of students successfully learned about the nature of fractal boundaries, regions where the pendulum's behavior is unpredictable and is extremely sensitive to its initial conditions. This is qualitative evidence for sub-hypotheses H2a (limited prediction) and H2c (sensitivity to initial condition). BS even wrote, "there are fractal points on the y axis," showing an understanding that starting positions on the y-axis result in unpredictable behavior.

January 20, 2006

On January 20, 2006, Groups 8 (ED, JC, AB, JQ) and 4 (MS, DS, MV, MP) began by observing a magnetic pendulum set up prior to class. Students wrote down descriptions of its behavior and discussed their ideas:
AB: It's getting hyphy,[25] going crazy, spinning in circles.
PI: would you say it's unpredictable?
AB: yes
MP: It's not predictable / it's chaotic.
PI: what do you mean by chaotic?
MP: It's crazy, it's moving around like everywhere
MS noted, "It seems as if the situation of the magnets affect the bob," a true statement acknowledging that bob's trajectory is infinitely sensitive to the placement of the magnets. AB wrote, "The pendulum is not predictable because any small force can change the pendulum." MS's and AB's comments are in keeping with sub-hypothesis H2c (sensitivity to initial conditions). DS wrote, "there is chaos from the two forces," recognizing that the two competing magnetic forces create an unstable system that gives rise to chaos. DS's remark is in keeping with H2d (examples of chaos). Altogether, four students or 50% (AB, JC, MP, and MS) described the system as "unpredictable," qualitative evidence for H2a (limited prediction):
PI: Do you think that if we start it from the same starting position twice, it will end up at the same magnet every time?
Ss: no
PI: why not?
ED: You can't control how much push or pull you give it when you release it
PI: So if we use the computer simulation and start it from the same position, would it be the same?
AB: yes, because all the numbers would be the same
DS: do they have the same magnet force?
PI: yes, they do

Students then drew their hypotheses as to what a mapping of initial positions to final magnet positions might look like, shading in the region(s) from which the bob ends up at the left magnet. This day, two example hypotheses were provided.[26] The instructor mentioned that a third hypothesis: that the boundary could be fractal, showing students a series of pictures of a fractal in which zooming in produces more and more detail.

Next, students moved to the computers and worked in Boxer, seeing how the program leaves a green or a red dot at the bob's starting point depending on whether it

[25] "Hyphy" is a slang term from hip hop meaning "out of control" or "crazy."
[26] Pictures were taken from http://dept.physics.upenn.edu/courses/gladney/mathphys/subsection3_2_5.html and Gleick (1987), p. 235.

ends up at the green or the red magnet. The instructor noted that the simulation could either support, disprove, or be irrelevant to their hypotheses. Students saw that the bob icon follows the same path if it is released from the same point multiple times. In the Boxer simulation, the bob's path is deterministic because the initial conditions are exactly the same. At this point, MP asked, "what will happen if you release it from the middle? maybe it will stay in the middle?" AB found that if started from the y-axis, the bob moves up and down and eventually stops at the origin rather than one of the magnets. The instructor explained that the middle is a line of unstable equilibrium like a pencil balanced at its point; one cannot predict which way it will fall.

Students brought up a concern that consecutive trials might depend on previous trials, and the instructor said that they do not. They attempted to find starting points for which the outcome is not deterministic. Due to the nature of the program, some points near boundary regions appeared to do this. However, these points were not exactly the same, they were just too close to be distinguished given the resolution of the graphics:

MS: there's a 50/50 probability which magnet it ends up at
DS: in two trials, it went to two different magnets from the same point
PI: are you sure it was exactly the same point?
DS: no

The instructor then explained how to use the magnifying glass feature to examine a region in finer detail. Students saw that each unique point did yield a deterministic trajectory—until, after zooming in many times, the program's numerical resolution was exhausted.

In their conclusions, six out of eight students (75%) wrote that their data disproved their hypotheses because the computer simulation did not produce a pattern similar to their drawing. DS said that he had not thought that there would be regions of mixed colors. MP had hypothesized that the bob would end up on the same side most of the time, and ED that bob would always go to the magnet near its starting point. Making sensible inferences, these students realized their experimentation disproved their predictions. This is evidence for sub-hypotheses H2a (limited prediction) and H2g (limited to probability). [27]

January 23, 2006

Monday, January 23, 2006 was the final day of the educational intervention. Groups 1 (QP, JM, MR, AP, and LR) and 5 (CC, TL, IC, and SS) began by observing a magnetic pendulum and writing down what they had already learned about it.

MR: It moves in different ways.
LR: not really
MR: every time it gets on top of it [a magnet], it moves in chaos
PI: ok, what is chaos?
MR: it does that [pointing], I don't know how to say it in words though

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[27] For an unknown reason, two students indicated that the simulation data did support their hypotheses. Perhaps these students thought they would receive a higher grade for such a response.

AP: It reminds me of a ouija board[28]
MR: every time it gets on top of it, it does its random things
PI: why is it random?
LR: the magnet attracts it
MR: the magnet will just do things to it
LR: the magnet attracts loose strings
PI: Say you release it from the same point twice?
AP: so there's no way to determine where it will go?
MR: Unless we have a very precise way to do it the same every time...we have only our hand and our hand is not that precise.
PI: exactly
These students' written responses indicated an understanding of sensitivity to initial conditions in keeping with sub-hypothesis H2c, and the deterministic nature of chaos, in keeping with hypothesis H4. For example, MR wrote, "unless you have a very precise way of measuring, every time we drop the pendulum it will follow a different path b/c our hand [sic] are not that precise." AP wrote, "Without precision to determine the exact position to start from every time we only assume that the activity occuring [sic] is chaotic because our hands are not precise." AP did not see that chaotic behavior can be deterministic, but realized that a lack of precision in the experiment was the cause of the discrepant results. LR wrote that "precision is the only way to determine the exact direction of the pendulum," indicating an understanding of deterministic chaos as predicted by hypothesis H4 (deterministic chaos).

Students in Group 5 engaged in dialog:
TL: when it's done swinging, it favors one of the magnets. it's arbitrary selection
PI: Say you released it from the same point twice, would it follow the same path?
TL: no
CC: yes
TL: It's a possibility
PI: Are the laws of physics based on probability?
TL: you can't say that things will be a certain way all the time; there will always be exceptions
SS: I think it's sensitive...if there's no air resistance, the motion could be very different.
These students' written responses provide evidence for sub-hypothesis H2c (sensitivity to initial conditions). For example, CC noted, "It is sensitive to many things. If we blow on it, it will move a little or a lot," and IC wrote, "at the end of the screw's trajectory, it favors one magnet and stays with it, not the same one b/c sensitive to the slightest change."

Students then moved over to the computer simulations. The instructor noted an experimental advantage to using them: a far greater degree of precision than existed when students were using their hands to release the bob. Students began the experiment of starting the Boxer simulation from the same initial position twice in a row. For her hypothesis, AP indicated that the bob will follow the same path "because it is starting from the exact same position; making the experiment precise and without the error of our hands." Her group's results concurred with this hypothesis, and LR noted, "unless you

[28] AP's comment is more evidence for "magical thinking."

start at the EXACT same beginning point, it will not follow the same path." Thus, students demonstrated an understanding of the concept of *deterministic* chaos in keeping with hypothesis H4 (deterministic chaos).

Next, students in Group 5 were asked if there were any regions from which one could be sure the bob icon would end up at a one particular magnet. Such regions were located:

TL: I found a region
PI: How many data points do you think you would need to plot to be sure?
TL: I think five.

Then, students found regions from which one could not predict where the bob would land:

(0:24) PI: is it like the real magnetic pendulum?
AP: In a sense
PI: why do you say that?
AP: there are areas where you don't know where it's going or where it'll end up...we figured out that from up here, it will go to red. If it's a little bit closer, it will go to green
PI: there's got to be a boundary between the regions of the red dots and the regions of the green dots

Students used the magnifying glass feature to zoom in on a "chaotic region":

AP: we found a border
PI: what do you think is happening there?
AP: we can't zoom in anymore
PI: you're at maximum magnification

At maximum magnification, resolution limits prevented students from exploring the fractal structure of the boundary any further. AP wrote, "at first we thought there would be a definite border then there were no definite pattern." CC concluded, "An area closer to the green magnet is more likely to be green, however it may be red too. You can predict for some areas, however, there are some places that are undetermined. The closer to the center, the easier to predict." Thus, students saw that probabilistic information is all that can be known, in keeping with sub-hypothesis H2g (limited to probability).

Students in Group 1 claimed to have found a clear-cut border:

(0:40) PI: So you're finding that there is a definite cut-off?
AP: It should be like 7 million decimal places
PI: If you could keep on zooming in, you might find that the green dots and the red dots are mixed together...you might not necessarily have found what you found in other regions.

AP wrote, "in our experiment we tried to find the exact point of the boundary. So far we figured that 131.299 is green and 131.300 is red. Somewhere in between that region could be the boundary."

Students in Group 5, however, noted that there was "really no defined boundary line. There was a weird area where the dots could be a boundary, but not" (CC). SS wrote, "I found that there is something more complicated, there isn't just a simple boundary. There are green dots mixed in w/ the red dots and visa versa." IC wrote, "I saw that the results can vary. If it is closer to green it goes to green, red—goes to red. If it is in the middle, it is random, like a pattern. There are some regions where you can predict where, there is some that have arbitrary selection."

At the end of the period, students compared their results with those displayed on other computers. Students in Group 5 were interviewed about their conclusions:
(0:45) PI: There's some regions for example here where it's unclear whether it's going to end up and the red or the green. it's complicated
CC: TL and I were just talking about that.
TL and PI pointing at the computer screen.
PI: what if you're right here where the red and the green dots are very close
TL: That's why I called it random selection because you don't know where it goes...unless you measure it by something... i don't know
[dialog skipped]
PI: [pointing at a region] So there's regions like here where there is no arbitrary selection...
TL: So, right, there is *some* arbitrary selection
PI: So there's some starting points where there's no arbitrary selection, but then there are other regions where there is?
TL: um hum, yeah. because it's not definite at all that's why. There are exceptions.
PI: So sometimes it's definite or determined, and then for other conditions or other locations it's not?
TL: yeah. It's not always definite.
TL's comments are in keeping with sub-hypotheses H2a (limited prediction), H2c (sensitivity to initial conditions), and H2g (limited to probability). Toward the end of the interview, she states that there are some regions of starting points where outcomes are "arbitrary," and some that are not, reflecting an understanding that nonlinear systems can behave predictably or unpredictably depending on their parameters. This is evidence for hypothesis H3 (complexity).

Results

Hypotheses	Likert	True or False	Written Responses	Interviews and Comments
H1: prior predictability	n/a	A2: ns B2: ns	pre: A3 & B4: 89% A4 & B3: 54% same, 67% same or maybe during: hypothesis1:100% hypothesis2: 91%	Jan. 13 Jan. 17: JQ
H2a: limited prediction	A5/E14: p=0.06 D6/F15: p=0.007	A2/E2: ns B2/F2: p=0.05	during: observation1: DS, FR, KC, CP, JCh, MT, IP hypothesis2: BS observation2: CC,	Jan. 11 Jan. 13: MSt, JM Jan. 17: JCh, AB, JQ Jan. 23: TL

			SS, TL, IC, BS, ED, AB, JQ, JC observation3: MSt, DN, BS, WL Jan. 19: 88% Jan. 20: AB, JC, MP, MS post: E5 & F5: 8 Ss E9/F8: 53%	
H2b: modeling uncertainty	C8/E17: p=0.03 D8/F17: ns C6/E15: ns	n/a	calculator1: 43% calculator2: 54%	Jan. 11 Jan. 19: AC
H2c: sensitivity to initial conditions	C7/E16: ns D7/F16: ns	n/a	pre: A3 & B4: 11% A4 & B3: 46% during: calculator1: 61% calculator2: 25% hpothesis2: SS, IC, BS observation2: PA, BS, MS, ED, AB Jan. 19: 88% Jan. 23: MR, LR, AP, QP, JM, CC, TL, IC, SS post: E5 & F5: 5 Ss E8/F7: 80% F11 & E12: 43%	Jan. 11 Jan. 13 Jan. 17: JCh, MT, IP, AB Jan. 19 Jan. 20: MS, AB Jan. 23: MR, SS, TL
H2d: examples of chaos	n/a	n/a	pre: A3 & B4: 7% during: observation2: KC Jan 19: MSt, DN, BS, WL post: E9/F8: 83% F12 & E13: 77%	Jan. 13 Jan. 16 Jan. 19 Jan. 20: DS

H2e: holistic view	C9/E18: ns B5/F14: ns	n/a	n/a	
H2f: few variables	n/a	E1: 33% F1: 75% Ave: 55%	n/a	
H2g: limited to probability	D9/F18: ns	E4: 80% F4: 69% Ave: 74%	observation1: CP, JCh, MT, IP observation2: CC, SS, TL, IC Jan. 20: 75% Jan. 23: CC, TL, IC, SS	Jan. 17 Jan. 20: MS Jan. 23: CC, TL
H3: complexity	n/a	E3 & F3: 53%	n/a	Jan. 23: TL
H4: deterministic chaos	n/a	n/a	Jan 23: MR, AP, LR, QP, JM	Jan. 11 Jan. 23: MR
H5: phase space	n/a	n/a	PA,EZ, FR, DS, KC, AP, MR, QP, JM, SS, TL, CC, IC, DN, MV, MP, MS, JG, LR, MS, CS, BS	Jan. 13: QP, MR, AP, JM Jan. 17: CC, TL, IC, SS Jan. 18: EZ, CP

Table 2. The results of quantitative and qualitative data analysis. All results listed are included in text of this thesis. It should not be taken as an exhaustive list.
ns = no significant result
n/a = not asked

Hypotheses

Hypothesis H1 (prior predictability): "After learning traditional high school level classical mechanics and prior to the modern physics educational intervention, students believe that all physical systems are predictable." Answers to true or false questions A2 and B2 seem to disprove this hypothesis, but data from A1 and B1 indicate that students did not have a clear concept of a physical system prior to the curricular intervention. Although students thought that small changes can produce different answers, when they were actually presented with a chaotic system, no students hypothesized that the magnetic pendulum would behave unpredictably. Interestingly, in the act of doing science, students did not think that unpredictability was likely, even though they had advocated for it in the pre-assessments.

Sub-hypothesis H2a (limited prediction): "Students will learn that in some systems ("chaotic" systems), there are limits to what it is possible to know the system's future behavior, no matter how much information one has about the system initially. In other words, some systems are not predictable." This sub-hypothesis was supported with statistical significance by pre-assessment questions B2, A5, and D6 and post-assessment questions F2, E14, and F15, respectively (p=0.05, p=0.06, p=0.007). It was also supported by post-assessment questions E5, F5, E9, and F8, and by numerous student interviews and written responses.

Sub-hypothesis H2b (modeling uncertainty): "Students will have a greater tendency to disagree with the view that given the right measuring devices, *all* systems can be accurately modeled with certainty." This sub-hypothesis was somewhat supported by the data. Given the specific example of a coin toss (C8 and E17) students moved away from a belief in inherent predictability (p=0.06). However, questions D8/F17 ("It is always possible to know something with absolute certainty, if one has the right tools or measuring devices") and C6/E15 ("Anything in nature can be accurately modeled with computers") produced average student responses of "unsure" both before and after the educational intervention. During the dueling calculators activity, about half of class explicitly indicated that different calculators do not always give the same results, thus placing constraints on modeling.

Sub-hypothesis H2c (sensitivity to initial conditions): "During the intervention, students will learn the concept of "sensitivity to initial conditions," and afterward they will have a greater tendency to believe that small influences in a system can sometimes produce large changes in the future behavior of the system." Data from A3 and B4, and A4 and B3, in conjunction with E5 and F5, E8/F7, and E12 and F11, provide evidence for this hypothesis. In pre/post question C7/E16, students remained unsure, and in D7/F16, students tended to agree that small influences can produce large effects *both before and after* the intervention. However, numerous student interviews and qualitative responses support H2c.

Sub-hypothesis H2d (examples of chaos): "Before the intervention, students will have difficulties providing examples of systems in which sensitivity to initial conditions occurs (A3, B4). After, students will be able to provide physics definitions of "initial conditions," "chaos," and examples of chaotic systems. Some students will be able to demonstrate their comprehension of these concepts in writing." This sub-hypothesis was supported by the data from pre-assessment questions A3 and B4 in conjunction with post-assessment data from E9/F8, E13 and F12, and by several student interviews and written responses.

Sub-hypothesis H2e (holistic view): "The educational intervention will cause students to move away from reductionist epistemologies and adopt a more holistic view of physical systems." This sub-hypothesis was tested by questions C9/E18 and B5/F14. Data did not support this sub-hypothesis. Students tended to agree with or were unsure of a statement advocating reductionism (B5/F14) both before and after the intervention. Interestingly, students also tended to agree with or were unsure about a statement advocating holism (C9/E18) both before and after the intervention. This hypothesis requires further investigation.

Sub-hypothesis H2f (few variables): "Before the intervention, students believe chaotic systems must have many variables. After interacting with physical systems with few variables, students realize that chaotic systems with few variables can exist." This hypothesis was tested by post-assessment questions E1, F1. In these questions, 55% of students indicated that many variables are not a prerequisite for chaos. The investigator assumed that before the intervention, students would not be aware of the difference between high-dimensional systems and nonlinear systems, so this hypothesis was not tested on the pre-assessment, perhaps in error. However, pre-assessment question C10 did show that students did not understand the meaning of "chaos" as it is used in scientific academic language, thus it is difficult to formulate the question pre-intervention. This hypothesis requires further investigation.

Sub-hypothesis H2g (limited to probability): "The intervention will show students that sometimes in physics, it is only possible to know the probability that something will happen." Results from E4, F4, and F18 indicate that after the intervention, most students agreed with the epistemological view that, "In some systems, it is only possible to know the probability." A comparison of pre- and post-responses to D9 and F18 show that the intervention did not make students more likely to agree. However, in E4 and F4, 74% of students indicated that sometimes, a probability is all that physics can predict. H2g was also supported by student interviews and written responses.

Hypothesis 3 (complexity): "During the intervention, students will see how non-linear systems exhibit both order and chaos, with windows of key variable ranges that result in periodicity mapped within variable ranges resulting in chaos." In post-assessments E3 and F3, 53% of students indicated that a chaotic system can exhibit order or periodicity. An interview with TL on January 23 also provides evidence for this hypothesis.

Hypothesis 4 (deterministic chaos): "The computer simulation will allow students to see examples of *deterministic* chaos. When two starting positions are infinitely identical, trajectories will be the exactly same, even where arbitrarily close starting points produce drastically different results." This hypothesis was supported by qualitative evidence collected on January 23.

Hypothesis 5 (phase space): "Students will learn about graphical representations in phase space." With scaffolding, most students (79%) were able to draw phase space plots of the simple pendulum. No conclusions could be drawn regarding the degree to which students understood computer generated phase space plots of a chaotic pendulum.

True or False Questions

In pre-assessment question A2, all students circled true, indicating an awareness of unpredictability in physics that was not suggested by this experiment's primary hypothesis. However, student answers to questions A1 and B1 indicated that many students did not have a good understanding of what a physical system is. It is possible some students thought that social interactions could be an example of a physical system. Social interactions appear to be unpredictable, but they are not physical.

Seven students indicated that "some" physical systems are unpredictable; one circled "about half." Five indicated "most," one gave "almost all," and no students indicated that "none" or "a few" are unpredictable. The average response was "about half," but the modal answer was "some."

In pre-assessment question B2, four students answered "true" and nine indicated "false." Of the latter group, three indicated that "a few" systems are unpredictable, two gave "some," one answered "about half," two indicated "most," and one gave "all." The average response was coded as 2.67, in between "some" and "about half."

In post-assessment item F2, students were given the same question. This time, fifteen students answered false and one student, DS, answered true. (DS also indicated true in pre-assessment B2.) This change was statistically significant (t-statistic = -

1.666; r=27; p=0.054 in lower-tailed test),[29] giving support to sub-hypothesis H2a (limited prediction).

There were five false and ten true responses to E1, and four false and twelve true responses to F1, indicating that the wording of the question strongly effected the results; students were more likely to answer true in both cases. Altogether, 17 students indicated that systems with few variables can exhibit chaos (55%); 14 felt that many variables are necessary. Thus, H2f (few variables) was supported by a thin majority of students.

In post-assessment questions E3 and F3, phrasing did not effect students' responses, as the class was divided in their answers to both positive and negative phrasings. 16 students indicated that chaotic systems can exhibit both chaos and order; 14 said that they can never exhibit "ordered or periodic" behavior. Thus, a majority of students (53%) understood that a "chaotic system" and "chaos" are not the same thing, and that chaotic systems can exhibit order—evidence hypothesis H3 (complexity). It is unclear if the remaining students were confused about the semantic similarities between "chaos" and "chaotic system," or never realized that nonlinear systems can also exhibit order.

To post-assessment question E4, twelve students answered true and three answered false; to question F4, eleven students answered true and five gave false. In keeping with H2g (limited to probability), 74% indicated that probability is inherent in physics, since it is not always possible to give anything more accurate than a probabilistic answer.

Likert Question Results

The Likert questions asked in pre-assessments were compared to the same Likert questions in post-assessments E and F. In their responses to post-assessment F, four students (JG, AC, MT, and EZ) gave responses of 4, 5, and 6 only, indicating that they did not strongly agree or disagree with any of the questions. All other students in both post-assessments included at least one response greater than 6 or less than 4. In class, these students had English difficulties, and it is possible that they could not comprehend the questions due to their poor English skills. MT answered 5, or unsure, for all questions, and AC answered 6 for all questions. Because of these concerns, the post-assessment F data was reanalyzed with these four students removed. These results are referred to below as the n=11 post-assessment responses. No students were removed from post-assessment E data sets.

Pre-assessment question A5 and post-assessment E14 were statistically compared to test sub-hypothesis H2a (limited prediction). In the n=13 A5 responses, the average score was 4.15 with a standard deviation of 1.91. In the n=15 E14 responses, the average score was 5.20 with a standard deviation of 1.47. Thus, students tended to disagree more with the view that predictability is always possible after the educational intervention, as suggested by sub-hypothesis H2a. This result was statistically significant with p=0.06 in a lower-tailed t-test.

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[29] The following websites were used for the calculation of p values:
http://www.stat.sc.edu/~ogden/javahtml/pvalcalc.html
http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm

Sub-hypothesis H2a (limited prediction) was also test by contrasting pre- and post-questions D6 and F15, respectively. In the n=14 pre-assessment responses (D6), the average score was 4.57 with a standard deviation of 1.16. In the n=15 post-assessment responses (F15), the average score was 3.67 with a standard deviation of 1.63. In the n=11 post-assessment responses (F15), the average score was 3.09 with a standard deviation of 1.51. Thus, after the educational intervention, students agreed more with this statement, as suggested by sub-hypothesis H2a. This result was statistically significant, with p=0.007 in a lower-tailed t-test. This result is similar to the one obtained from the question's analog, A5/E14.

Pre-assessment question C8 and post-assessment E17 were compared to test sub-hypothesis H2b (modeling uncertainty) using the example of a coin toss. In the n=13 pre-assessment responses (C8), the average score was 4.69 with a standard deviation of 2.18. In the n=15 post-assessment responses (E17), the average score was 6.27 with a standard deviation of 2.05. Thus, students moved toward strong disagreement after the educational intervention, as predicted by sub-hypothesis H2b. This result was statistically significant with p=0.03 in an upper-tailed t-test.

Pre- and post-questions D8 and F17 were also statistically contrasted to test sub-hypothesis H2b (limited prediction). In the n=14 pre-assessment responses (D8), the average score was 5.79 with a standard deviation of 1.85. In the n=15 post-assessment responses (F17), the average score was 5.60 with a standard deviation of 2.03. In the n=11 post-assessment responses (F17), the average score was 5.82 with a standard deviation of 2.32. Students were largely unsure of this statement both before and after the intervention. In addition, pre-assessment question C6 and post-assessment E15 were compared statistically to test sub-hypothesis H2b (modeling uncertainty). In the n=13 pre-assessment (C6) responses, the average score was 5.69 with a standard deviation of 1.44. In the n=15 post-assessment (E15) responses, the average score was 5.60 with a standard deviation of 1.55. Students remained unsure regarding whether computers could accurately model nature both before and after the intervention. The experimental hypothesis suggested that students would move toward disagreement. However, since the chaos project itself involved computer modeling, this question was likely to confuse students, and probably should not have been included.

Pre- and post-questions D7 and F16 were compared statistically to test sub-hypothesis H2c (sensitivity to initial conditions). In the n=14 pre-assessment responses (D7), the average score was 3.71 with a standard deviation of 1.20. In the n=15 post-assessment responses (F16), the average score was 3.73 with a standard deviation of 1.67. In the n=11 post-assessment responses (F16), the average score was 3.36 with a standard deviation of 1.75. Students tended to agree that small changes can produce large changes both before and after the intervention. These results are similar to those obtained from the question's analog, E16. Pre-assessment question C7 and post-assessment E16 were also compared to test sub-hypothesis H2c (sensitivity to initial conditions). The n=13 pre-assessment responses (C7), the average score was 5.38 with a standard deviation of 2.06. In the n=15 post-assessment responses (E16), the average score was 5.20 with a standard deviation of 2.24. Students remained unsure if small changes "cannot produce large changes" both before and after the intervention. It is possible that students misread this question as "can produce large changes" rather than "cannot."

Pre-assessment question C9 and post-assessment question E18 were compared to test H2e (holistic view). In the n=13 pre-assessment responses (C9), the average score was 3.46 with a standard deviation of 1.76. In the n=15 post-assessment responses (E18), the average score was 3.57 with a standard deviation of 1.50. Thus, students tended to agree with the holistic paradigm both before and after the educational intervention.

H2e (holistic view) was also tested by comparing pre- and post- questions B5 and F14, respectively. In the n=13 pre-assessment responses (B5), the average score was 3.54 with a standard deviation of 1.39. In the n=15 post-assessment responses (F14), the average score was 3.87 with a standard deviation of 1.36. In the n=11 post-assessment responses (F14), the average score was 3.36 with a standard deviation of 1.20. Ironically, students also tended to agree with the reductionist paradigm both before and after the educational intervention.

Pre- and post-questions D9 and F18 were statistically compared to test sub-hypothesis H2g (limited to probability). In the n=14 pre-assessment responses (D9), the average score was 3.43 with a standard deviation of 1.50. In the n=15 post-assessment responses (F18), the average score was 4.53 with a standard deviation of 1.64. In the n=11 post-assessment responses (F18), the average score was 4.27 with a standard deviation of 1.79. Thus, the educational intervention made students more unsure or more likely to disagree with this statement, the opposite of what is suggested by this experiment's hypothesis, but this result was not statistically significant ($p=0.22$ in a two-tailed t-test). Perhaps students thought that in chaotic systems, it is not even possible to know a probability.

Open Response Questions

Data from the pre-assessments A1 and B1 indicated that students could not come to a consensus on the definition of a physical system, although the concept had already been covered in prior instruction during units on the conservation of momentum and the conservation of energy. At this time the instructor defined a system as "a defined collection of objects." He also stated that a "closed" system "does not gain or lose mass" and an isolated system is "a closed system where the net external force is zero."

In the pre-assessment, two students used the "system" definition given in class. Five students' responses were coded as "a group of objects," and another five as those mentioning "forces" or "energy." Three students wrote about objects "inside an area," three mentioned "solid," "matter," or "Earth," and three more defined physical systems as "anything" or "everything." Two said systems "change in time" and another two mentioned "equations." Four students did not know or did not give a response.

Students' vague concepts of a physical system should be taken into consideration when interpreting responses to the true or false questions that followed them about predictability in physics and unpredictability in physical systems. One student's definition of a physical system was a "living organism" and "everything" includes human beings. If students' concepts of a physical system included themselves, it is of little surprise that most answered that some systems are unpredictable. No matter where they stand on the free will debate, many people feel that their lives are somewhat unpredictable. In future research, survey tools should ask students if they consider themselves or their lives to be physical systems.

In the post-assessment, seven students mentioned “a group of objects,” an increase of two. Surprisingly, six students included “experimentation” in their definitions. The only explanation for the appearance of this term post-intervention is that the students had recently conducted experiments of the magnetic pendulum. Five mentioned “forces” or “energy,” and another five wrote “inside an area.” Seven students said they did not know or did not give a response. Overall, these responses indicate that the majority of students were aware that the computer simulation represented a system. Most likely, they also conceived the magnetic pendulum as a system, although they may not have been aware that, by itself, it is not an isolated system. [30]

In pre-assessment questions A3 and B4 (n=27), students were asked to provide examples of systems in which a small change in a variable could produce a completely different future outcome. Before the intervention, seven students mentioned something covered previously in the physics course. Most examples students gave were not examples of sensitivity to initial conditions at all, but two did mention unbounded mathematical operations like “squaring, cubing” and “exponents.” Two students (7%) did provide possibly chaotic examples, both meteorological: “earthquakes, hurricanes, and tsunamis – natural disasters” and “a tornado.” One student mentioned $E=mc^2$: an interesting answer, since the factor c^2 is usually extremely large compared to the change in mass. Four students provided mathematical examples, two gave chemical examples, and single students mentioned examples from astronomy and biology. One student mentioned a plane crash, and eight students did not provide an answer.

In one version of the pre-assessment, fourteen students were asked if they would always get the same answer if they solved the same physics problem using “v = 1.0000000000 m/s” and solved it again using “v = 1.00000000001 m/s,” (A4). Nine respondents indicated yes, four gave maybe, and only one wrote no. In another version, thirteen were asked, “Is there any chance that the new answer could be very different? Why or why not?” (B3). Here, five indicated “no” and seven gave “yes.” A yes answer to the first phrasing corresponds to a no answer on the second phrasing. For both phrasings, students tended to answer “yes,” which may be because survey respondents are more likely to give affirmative answers to complex questions, a common problem in survey design. However, the combined result of fourteen students indicating that answers would be the same and eight that they may be different is significant. Out of 26 students, fourteen (54%) held this view; including the four “maybe” responses, 67% did, supporting hypothesis H1 (prior predictability).

[30] Even the magnetic pendulum-Earth system is not isolated system. Since small influences matter, one must also include the wind currents in the room and all minor forces as significant.

In the post-assessments, students were asked, "What happens when a system's behavior is very sensitive to initial conditions?" (F11) and to provide an example (E12). In a follow-up question, they were asked to give examples of chaotic systems that were not studied during the project and to explain why they are chaotic (E13, F12). Survey versions E and F were coded separately. Thirteen students (43%) could answer this question successfully, with eleven providing examples from meteorology like "wind speed & direction." Eight students cited examples from the traditional physics curriculum that are not chaotic, such as "rolling balls from a high surface" and simple pendulums.[31] Three gave chemical examples, such as "A flame of fire is chaotic; it flickers where it wants." Two students mentioned "earthquakes." These students may have been drawing on prior knowledge that the moment an earthquake strikes cannot be accurately predicted.

Before the intervention, 54-67% believed that a small change in a variable could produce a completely different outcome, but only 11% could provide an example of such a system. After, 43% of students could provide a real world example of sensitivity to initial conditions. Students' written and verbal comments provide many examples of students understanding the concept behind H2c (sensitivity to initial conditions). In conjunction with the data supporting hypothesis H1 (prior predictability), these results indicate that, overall, students moved toward an epistemological view in which there are systems in which small changes can have large effects, the position taken in sub-hypothesis H2c. However, pre- and post- results from Likert questions D7/F16 and C7/E16 do not provide further evidence for or against this sub-hypothesis, since students tended to either agree that small influences can produce large changes both before and after the intervention.

Although the survey indicated that "answers will not be graded," students' responses in E12, E13, F11, and F12 were also coded based on the grade they would have received if they were graded by the teacher and conductor of this research. Ten students' writing samples used modern physics terms appropriately in their answers (A=33%); eight students' responses indicated that they could comprehend and adequately answer questions about nonlinear physics concepts (B=27%); five students showed some understanding (C=17%); three students demonstrated misconceptions or misunderstandings (D=10%); and four did not give answers (F=13%). This coding scheme indicates that roughly 23 out of 30 students (77%) showed understanding of chaos theory concepts in their writing. Since only two students (7%) could give examples of chaos in the pre-assessment A3 and B4, these results clearly support hypothesis H2d (examples of chaos).

In the post-assessment, fifteen students were asked for the most important thing they learned from the chaos project (E5). Four responses were coded as "some things are unpredictable or not controllable," evidence for H2a, and two as "small change can have

[31] A somewhat surprising result was that twelve students' examples were somewhat psychological in that they included a physical agent: "We could...," "...you don't know." For unknown reasons, no students gave physical agent responses in the pre-assessment. A possible reason for this finding is that since students had recently engaged in experimentation, they were in a more active frame of mind, and thus more likely to write about situations in which they themselves could perform experiments.

big effects," in keeping with H2c. Six students mentioned a specific detail of the project, and three stated that pendulums cannot be predicted, which is not generally true. Three students wrote something wrong or irrelevant. Fifteen students were asked for the most interesting thing they learned from the chaos project (F5). Four responses were coded as "some things are unpredictable or not controllable," in keeping with H2a, and three as "small change can have big effects," evidence for H2c. Ten students mentioned specific details of the project. Since these results were difficult to understand, in future research, the "interesting" phrasing will be eliminated in favor of the "important" phrasing. Significantly, 13 out of 30 students (43%) volunteered one of the primary concepts of chaos physics without scaffolding.

Post-assessment questions F7 and E8 were used to provide data for testing sub-hypothesis H2c (sensitivity to initial conditions). All 30 students were asked, "What is an initial condition?" (F7/E8). 24 students (80%) provided an adequate definition, mentioning "beginning," "starting," or "before." This is not that surprising, since the terms "initial position" and "initial velocity" had been used throughout the semester in the traditional physics curriculum. Four students demonstrated misunderstandings, and three did not answer, but these data provide evidence that most students understood this concept.

"What is chaos?" was also posed to 30 subjects in post-assessments E9 and F8. Sixteen, or 53%, of answers were coded as "unpredictability" – strong evidence in support of H2a (limited prediction). Surprisingly, no students mentioned sensitivity to initial conditions, but seven gave "no repeating pattern" and five indicated something that is "not controllable." Four students mentioned "going crazy,"[32] and three, "randomness."[33] Altogether, 25 students, or 83%, gave responses that overlap with the scientific definition of chaos, strong evidence for sub-hypothesis H2d (examples of chaos). Of the remaining 19%, four did not answer, two wrote "confusion," and one thought chaos was "a danger in the system."[34]

Discussion

This project was motivated by prior research indicating that high school students can learn modern physics concepts through their own experimentation. The question of whether such concepts are worth teaching is not worth asking, for there is no reason why students should only have the opportunity to learn about physics discovered before the 20th century. It is the responsibility of physics educators to keep up with current trends in the field. Of course, many modern physics concepts may be too difficult to teach at the high school level. The results of this research show that classroom experimentation in nonlinear systems is quite doable, especially if done in conjunction with computer simulations.

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[32] "Going crazy," though certainly not academic language, may be an example of students attempting to devise their own terminology for scientific phenomena. This has been observed before in the Patterns Research Group when a student accurately described the process of temperature equilibration using the phrase "freaking out."
[33] Technically, chaos is not randomness, since is it usually deterministic.
[34] Chaos is not necessarily dangerous, and it is not a psychological state like confusion.

Results clearly indicate that most students learned that chaotic systems are unpredictable and that small influences can have large effects. However, the degree to which students came into the intervention believing that everything is predictable in physics cannot be ascertained. Initially, a majority of students indicated that some physical systems are unpredictable, but did not think the magnetic pendulum would be unpredictable. Perhaps survey instruments could not distinguish between students' notions of unpredictability in their daily lives and their ideas about the nature of physics.

It is clear that at the end of the intervention, most students realized that unpredictable physical systems are possible. However, *some* students robustly held on to predictable epistemologies well into or throughout the intervention. Further research is necessary to see the degree which advocacy of predictability can be correlated with beliefs about the nature of physics, or a subject's level of physics education. It is even possible that physics undergraduates may be more likely to believe in predictability and that small influences can be neglected than high school physics students.

Unfortunately, analysis of the Likert questions showed that students tended to answer them the same way on post- and pre-assessments. This may be due to a psychological effect observed in survey research: when presented with the same question twice, subjects tend to recall their old answer and provide a similar response (Feldman & Lynch, 1988). Despite this effect, three of the Likert questions did return statistically significant changes. In support sub-hypothesis H2a, students tended to disagree more with the view that predictability is always possible "if enough information is known" after the intervention ($p=0.06$), and agree more with the view that nature has limits to what it is possible to know ($p=0.007$). These results are strong evidence that the intervention changed students' epistemological beliefs.

Another statistically significant change was a greater tendency to disagree with the view that scientific instruments could be used to predict the outcome of a coin toss ($p= 0.03$). Ironically, this statement is true; a coin toss is not a good example of a nonlinear system. In an analysis of coin flipping, Stanford physicist Persi Diaconis (1988) found that a mechanical coin flipper that imparts approximately the same initial conditions for every toss has a highly predictable outcome. In other words, the phase space is fairly regular, more like a simple pendulum than a chaotic one. This experimental finding is not intuitively obvious, so it is unreasonable to expect that students would know it. Since the practical effect of a coin toss is unpredictability, it is not surprising that students tended to frame it as an example of chaos. Students probably connected the perceived unpredictability of a coin flip with the unpredictability of the magnetic pendulum and inferred that a coin toss is chaotic as well.

Conclusion

The chaos project is highly recommended to all high school physics teachers. In hindsight, the dueling calculators activity was a good way to start the project, as it serves to introduce the concept that small changes—such as rounding off a number—can produce large effects. However, it is basically mathematics, not physics. It would not be useful as a stand-alone activity in a physics course.

The faucet and bowl activities did not work well. Students seemed confused as to what they were supposed to be learning from the activities. In the bowl activity, exploring stable and unstable equilibrium conditions seemed too obvious to them; they did not feel is if they were learning anything new. In future work, it should probably be replaced with a Boxer simulation used in the Patterns Project. The faucet activity may be useful if it is restructured with more instructor scaffolding. In hindsight, too little scaffolding was provided. Students should have been told not to worry about the water pressure in the faucet itself, but instead to focus on how the shape of the water stream changes as the water pressure is increased. For unknown reasons, students did not tend to pay attention to this. Perhaps students tend to connect physics with engineering applications like plumbing, so the nature of turbulence in the water stream was overlooked. However, these activities did serve to get students into the habit of simple experimentation by asking them to write hypotheses beforehand, make observations, and write conclusions.

The explorations with the real magnetic pendulum were the heart of this project. In hindsight, too much time was spent having students analyze the energy dynamics of the simple pendulum. Although this activity connected with the topic taught most recently in the regular curriculum of the course, it did not connect conceptually with the chaos concepts. An alternative activity suggested by Oliver (1999) is to analyze the energy dynamics of the magnetic pendulum, including magnetic potential energy along with gravitational potential energy and kinetic energy. A drawback of such an approach is that detracts from students' understandings of modern physics concepts like unpredictability and sensitivity to initial conditions.

Students study graphing in mathematics courses, but seem to have difficulties in physics when axes represent physical quantities. Students struggled with graphical representations throughout the semester. The difficulties students had in constructing phase space plots of the simple pendulum show how graphical representations can confuse students again when the quantities represented on the axes are different from those to which they are accustomed. More research into fruitful ways to teach students about graphical representations in physics should be conducted, starting with a literature search, in order to figure out how the phase space activity could be better scaffolded. The activity should not be omitted from the chaos project. Graphical representations are important in understanding physics, and asking students to draw a phase space plot is an excellent way to get students thinking critically. It also presents students with a different kind of graphical representation, and an extremely powerful one.

Observation of phase space plots of a chaotic pendulum available on the Internet was a good use of web-based resources, but it was unclear how well students could connect the phase space plots they made of the simple pendulum with the computer generated chaotic plots. Students could see that the chaotic pendulum did not display periodicity, but the phase space representation may have been too abstract for them to truly understand. It is unclear whether this portion of the project should be included in further work, or how it should be better scaffolded.

The Boxer simulation provided students with a means to conduct experiments that cannot be conducted in with a real world pendulum. Students enjoyed this portion of the project, expressing enthusiasm when they realized that the magnetic pendulum could be modeled with a computer. The Boxer representation allowed students to explore a chaotic pattern, and their results surprised them. Explorations of the fractal boundary where arbitrarily close starting positions produce radically different outcomes were especially interesting to students. Unfortunately, resolution limitations in the software—or perhaps in the computer itself—were an obstacle to students' understanding. One of the main concepts students were supposed to grasp was that the boundary is not a line with finite length; rather, it is a fractal with an infinite length and detail. When resolution limits were reached by some students, they falsely concluded that there is a define boundary line. It is unclear if these students understood the instructor's comments about resolution limits. However, most students never encountered the resolution limits, and thus correctly concluded that the boundary was complex.

The simulation also served to provide a deeper understanding into the deterministic nature of chaos. In the physical experiment, it was impossible for students to see that chaotic trajectories are deterministic, because it was not possible to release the bob from the exact same position in multiple trials. The simulation enabled students to do such an experiment, and provided further insight into what students think is happening in chaotic systems.

In hindsight, important questions were left out of the pre-assessments that could have shed more light on sub-hypotheses H1 (prior predictability) and H2f (few variables). To test H1 more accurately, students could have been asked, "Is it possible for a pendulum to be unpredictable?" To test H2f in the pre-assessment, the question, "Can a system with few variables be unpredictable?" could have been included.

More activities focused around hypothesis H3 (complexity) could have been included in the curricular intervention. There are various web resources about the Logistic Map that show how ordered, periodic behavior can exist within chaos. Such activities were left out of this curriculum because they were too mathematical, and the goal of this project was for students to explore chaos in physical systems. Unfortunately, it was difficult for students to see complexity in the magnetic pendulum. When it did occur, students could have dismissed it as an anomaly. Periodic behavior is much more obvious in other nonlinear systems like the Lorenzean water wheel and the double pendulum. A water wheel may be difficult to obtain or transport, but a double pendulum is not difficult to construct, and it can exhibit both periodic and chaotic behavior. In future work, students could explore the dynamics of a double pendulum—which, for small angular displacements, is clearly periodic—in addition to those of a magnetic pendulum.

Another avenue of investigation for future work is to see if students are able to understand that the unstable equilibrium created by two competing magnets is what gives rise to chaos. This approach was taken by Duit and Komorek (1997). In the chaos project, one student, DS, did conclude that the conflicting pulls of the two magnets gave rise to chaos (January 20), indicating that high school students are capable of understanding the concept.

There are also connections between chaotic behavior and time scales. One can predict the weather a few seconds from now fairly accurately, but weather prediction becomes less and less accurate the further into the future one chooses to look. A magnetic pendulum behaves in much the same way: on short time scales, one can see where its trajectory is taking it, but the bob's long term trajectory is a mystery. Further research into students' views about the connection between time scale length and unpredictability in chaotic patterns is warranted.

Survey instruments are never definite measures of students' epistemological and conceptual views, so all results obtained in this project are preliminary. However, the investigator feels that results demonstrate that some concepts in chaos theory are possible for students to comprehend, and that the intervention altered some students' epistemological views regarding predictability in nature, and the nature of prediction itself. The significance of these findings should not be taken lightly.

In diSessa's view, a pattern is the interaction among qualities (2006). In the magnetic pendulum, gravity, magnetic forces, and string tension all interact to produce a chaotic pattern. But unlike any other patterns, the chaotic pattern is also influenced by the multitude of small, even infinitesimally small, forces around us all the time, like air currents and the earth's magnetic field. In most traditional physics problems, these small forces can be safely ignored. But for nonlinear systems acting chaotically, unknown minor forces can produce dramatically different outcomes. Hopefully, students came away from their work in this project with a newly found respect for the unknown.

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