MA 315, Spring 2012, Optional Extra Credit
Optional Extra Credit
Due Monday, April 9, 2012

You will receive 2 points for each correct answer up to a maximum of 8 points. At my discression, you may receive 1 point for an answer that involves the tiniest slip-up. You will receive nothing for a problem that is not correct. Only hand in answers that you believe are correct; if you hand in incorrect answers, it devalues the work you do that is correct. Please note that, since typical homeworks are between 8 and 16 points, an addition of 8 points to your grade can be significant. If your current homework grade (after seven homeworks) is 66%, receiving 8 points of extra credit will raise your homework grade to 76%. This extra credit assignment will be graded before the drop deadline, April 13, 2012.

You must work alone on these problems. You may talk in a general way about these problems with your classmates, however you may not work together to obtain answers. I will not grade papers that show a great deal of collaboration.

Provide complete proofs for each problem. If you are asked to prove a problem that is not true, provide a counterexample; if the problem is partially true, prove a more limited result that is true.

1. Let $p(x) = x^2 + x + 4$. Determine a set S such that $S = \{p(n) : n \in \mathbb{Z}\}$. Your definition of S should not refer to the polynomial $p(x)$.

2. Prove that if $\mathcal{P}(A) = \mathcal{P}(B)$ then $A = B$.

3. Assume $A \subset B$. It is easy to see that $\mathcal{P}(A) \subset \mathcal{P}(B)$; you do not have to prove that. Prove that $\mathcal{P}(A^c) = (\mathcal{P}(A))^c$. In the previous equation, the left hand complement is taken relative to B and the right hand complement is taken relative to $\mathcal{P}(B)$.

4. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

5. If A is a set that contains a finite number of elements, we say A is a finite set. If A is a finite set, we write $|A|$ to denote the number of elements in the set A. We also write $|B| < \infty$ to indicate that B is a finite set. Define the sets X and Y by

$$X = \{T : T \subset \mathcal{P}(\mathbb{Z}) \wedge |T| < \infty\}, \quad Y = \{T \in X : T \neq \emptyset\}.$$

   Prove or disprove the following:

$$(\exists R \in X)(\emptyset \in R \wedge (\forall S \in Y)(|R| \leq |S|)).$$