Weighted Least Squares Regression ($b = 0$) :

$$\chi^2 = -\frac{1}{2}\sum_{i=1}^{N}\left(\frac{y_i - ax_i}{\sigma_i^2}\right)^2 \tag{1}$$

$$\frac{\partial\chi^2}{\partial a} = \sum_{i=1}^{N}\frac{x_i}{\sigma_i^2}(y_i - ax_i) = 0 \tag{2}$$

$$\sum_{i=1}^{N}\frac{x_i y_i}{\sigma_i^2} = a\sum_{i=1}^{N}\frac{x_i^2}{\sigma_i^2} \tag{3}$$

$$a = \sum_{i=1}^{N}\frac{x_i y_i}{x_i^2} \tag{4}$$

$$\sigma_a^2 = \sum_{i=1}^{N}\frac{x_i^2}{\sigma_i^2}S^2 \tag{5}$$

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}\frac{(y_i - ax_i)^2}{\sigma_i^2} \tag{6}$$

$$\sigma_a^2 = \frac{1}{N-1}\left(\sum_{i=1}^{N}\frac{x_i^2}{\sigma_i^2}\right)\left(\sum_{i=1}^{N}\frac{(y_i - ax_i)^2}{\sigma_i^2}\right) \tag{7}$$

$$\tag{8}$$

Applied to our experiment :

$$r_n^2 = \left(n - \frac{1}{2}\right)\lambda R \tag{9}$$

$$a = \sum_{n=1}^{15}\frac{r_n^2\left(n - \frac{1}{2}\right)}{(r_n^2)^2} \tag{10}$$

$$\sigma_a^2 = \frac{1}{14}\left(\sum_{n=1}^{14}\frac{(r_n^2)^2}{\sigma_r^2}\right)\left(\sum_{i=1}^{N}\frac{(r_n^2 - a\left(n - \frac{1}{2}\right))^2}{\sigma_r^2}\right) \tag{11}$$

$$\Psi(x) = x^2 \tag{12}$$

$$\sigma_\Psi^2 = (\frac{\partial}{\partial x}(x^2)\Delta x)^2 = 4x^2(\Delta x)^2 \tag{13}$$

$$\sigma_{r_n^2} = 4r_n^2(\Delta r)^2 \tag{14}$$

$$\sigma_a^2 = \frac{1}{14}\left(\sum_{n=1}^{14}\frac{(r_n^2)^2}{4r_n^2}\right) / \left(\sum_{i=1}^{N}\frac{(r_n^2 - a\left(n - \frac{1}{2}\right))^2}{4r_n^2}\right) \tag{15}$$