A variational problem with the constraint that the function be decreasing

Context in which the problem arose (not relevant to understanding the mathematical problem):

There increasingly is congestion at sea straights through which ships pass. This increases the frequency of collisions and their damaging effects such as environmental pollution by oil tankers. It has been proposed[1] that a tax be levied on each passage of a ship through a sea straight and that the revenue be used to fight AIDs, Malaria and TB. The tax would be charged for each ship that passes, irrespective of the ship's size. It has been raised as an objection to the tax that it will penalize smaller ships more than larger ships relative to their size. It is therefore useful to investigate whether market forces will lead ship sizes that are below or above the social optimum (there is a trade-off: on the one hand, there is a value in high frequency of transport, on the other hand larger ships are more fuel efficient.) The problem arose in the model that I developed for that end. It just has one parameter, namely θ, which is a positive constant that is estimated to be 0.8. If it turns out that the maximal value of the problem is smaller than θ then this would mean that the free market leads to an inefficiently small ship size. Such a result could provide support for the proposal.

The mathematical problem:

θ is a constant that equals 0.8.

We consider the set

$$S = \{(F,h): F\ is\ a\ decreasing\ function\ from\ R^+\ to\ R^+, \qquad h\epsilon R, \qquad 0 = 1 - \frac{\theta+1}{\theta}\frac{\int_{y=1}^{1} F(y)\ dy}{F(0)}\frac{F(0)-\frac{1}{2}F(h)}{\big(F(0)-F(h)\big)}\}$$

The function L is defined on S by

$$L(F,h(F)) = \frac{\int_{x=0}^{h}\left(\int_{y=x}^{h} F(y)dy\right)dx}{\int_{x=0}^{h}\left(\int_{y=0}^{h} F(y)\ dy\right)dx}h$$

We want to fnd the maximal value of L.

[1] http://www2.weed-online.org/uploads/International_Taxes.pdf