NEW TYPE OF UNDULATORY MOTION FOR CHARGED PARTICLES

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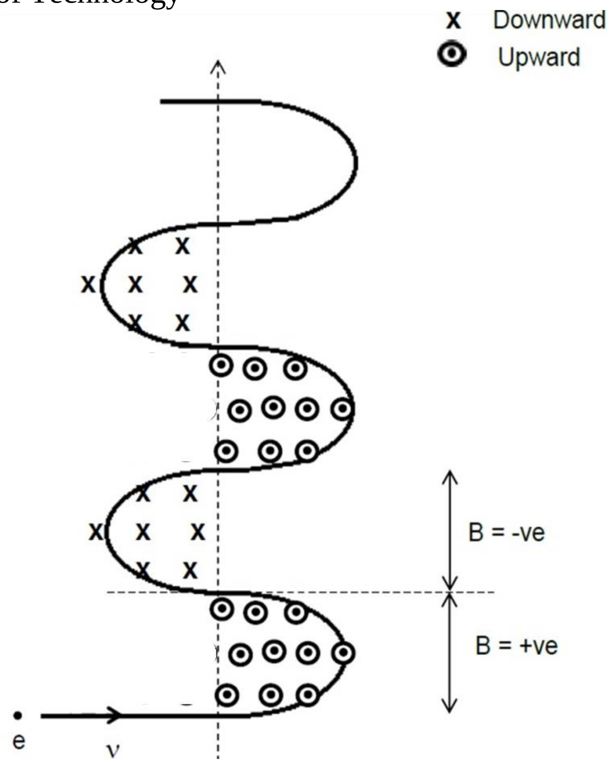
1. INTRODUCTION

The motion of charged particles in magnetic field has been studied in great detail and is widely applicable in the field of Plasma Physics and Accelerator and Beam Physics. Various methods are employed to manipulate the trajectory of ions like the 'undulator/wiggler' to make the electrons travel in wave like motion in order to get coherent radiation by making it undergo acceleration. In this document I try to introduce a new type of wave-like motion for ions under the influence of time-varying magnetic field. The gyromotion of ions/electrons in constant or grad-B drift where the drift velocity is non-zero has been studied where the trajectory of ion/electrons has been cycloidal in nature. The time taken by electron to complete one cycle and its trajectory also depends on the initial velocity with which the electrons/ions were injected with [1]. The equation of drift velocity in grad-B drift also show that for one cycle of particle motion, the time taken to complete one cycle also depends on velocity of the particle. The proposed theory does not show such case and causes the particle to complete one cycle in fixed time-period independent of the gyro velocity or drift velocity. This is similar to how cyclotron causes the particle to complete one cycle in fixed time independent of the velocity of the particle.



Undulators in FEL provide a narrow beam coherent radiation whose solid angle depends inversely with the number of cycles/periods the particle has undergone in the undulator. In order to get a narrow beam as well as short wavelength radiation the undulator poles are placed very close to one another in mm. to cm. range. The physical restriction in construction of extremely short spaced poles in undulator is cited as a limitation of the undulator[2]. The proposed trajectory of particle can cause the particle to travel in much smaller wavelength than any undulator thereby narrowing the beam angle. However, this new motion causes the particle to move with constant frequency in one cycle even with its velocity decreasing, unlike with constant wavelength in an undulator. For coherence to occur new set of formulas have to be derived for constructive interference of the radiation.

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2. QUALITATIVELY

2.1.Background Theory

The magnetic field is applied in z-axis only and is time varying. The angular frequency of the time varying magnetic field is ω_0. The electron is projected in the x-y plane with initial velocity v and at the instant when magnetic field is beginning to rise from zero. The electron experiences Lorentz Force corresponding to the induced electric field and magnetic field: $\vec{F} = \mathrm{q}\,E\overrightarrow{e_\theta} + \mathrm{q}\,(vB)\overrightarrow{e_r}$ -(1). Since the magnetic field is zero at the beginning, the Larmor Radius corresponding to it is very large.

The induced electric field follows the curl along the larmor orbit corresponding to the magnetic field and velocity of the particle [3] Fig[1].

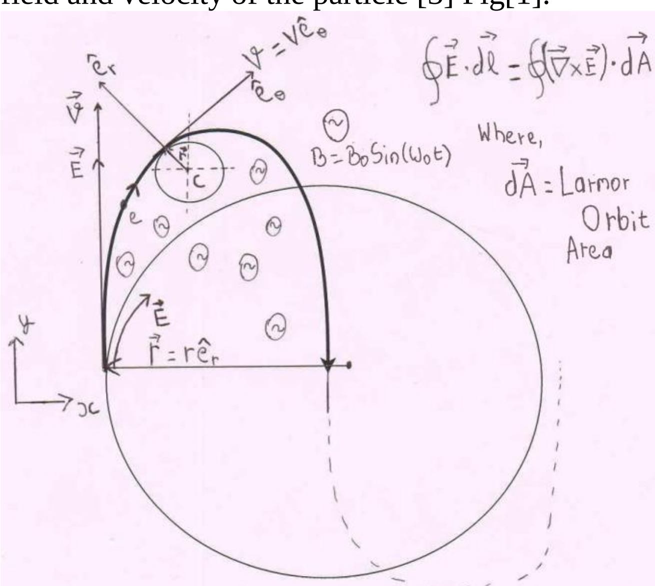

Figure 1: Curl and velocity vector

Since the tangent to the curl is in the direction of particle velocity as well as the electric field we can write acceleration as:

$$\vec{a} = (\frac{dv}{dt}\overrightarrow{e_\theta} + v\frac{d\theta}{dt}\overrightarrow{e_r}) \qquad (2)$$

Where the velocity at any instant is $\vec{v} = v\overrightarrow{e_\theta}$. (3)

Equating (1) and (2) to their corresponding vector components we can say that:

$$qE\overrightarrow{e_\theta} = m\frac{dv}{dt}\overrightarrow{e_\theta} \qquad (2.1)$$

And, $qvB\overrightarrow{e_r} = v\frac{d\theta}{dt}\overrightarrow{e_r}$. (2.2)

Let the centre of our Larmour orbit be our origin of reference, hence the position of the particle on the orbit $\vec{r} = r\overrightarrow{e_r}$ is in $\overrightarrow{e_r}$ as the radius lies perpendicular to the tangent on the circle along the normal. The differentiation of position gives us the velocity: $\vec{v} = \frac{dr}{dt}\overrightarrow{e_r} + r\frac{d\theta}{dt}\overrightarrow{e_\theta}$ - (3.1). Since we already know that velocity is $\vec{v} = v\overrightarrow{e_\theta}$, hence even though $\dot{r}$ is non zero, its value along $\frac{dr}{dt}\overrightarrow{e_r}$ is zero. Using (3) and (3.1) we get the required case $v = r\frac{d\theta}{dt}$ -(3.2). This can further be clarified by taking the following example:

Let us assume that there is region with two different strength of magnetic field present, each in one half as shown in Fig[2].

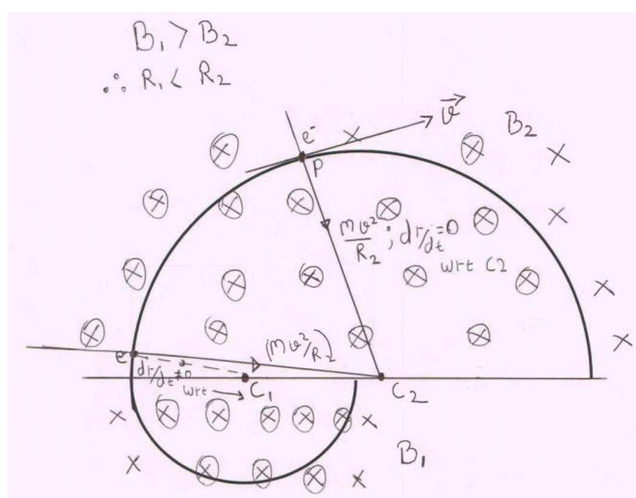

Figure 2: Shifting of origin

When the electron moves in first region take its centre of curvature as C1. As soon as it completes its semicircular arc in B1 field and just enters B2 field, let its centre of curvature be C2. If we keep observing from C1 as our origin, then we will have a non-zero $\frac{dr}{dt}\vec{e_r}$ term with respect to C1. But if we instantly observe from C2 as our origin, then there is no $\frac{dr}{dt}\vec{e_r}$ term. Our switch of origin is not the same as observing from non inertial frame of reference with some acceleration as our origin is not traversing with some acceleration, but we are creating new origin as soon as the magnetic field changes. If there was a huge difference in magnitude of magnetic field, then the acceleration of our frame of reference would be almost infinite as time interval dt is infinitesimally small, yet the distance between C1 and C2 is finite and large, giving d(C2-C1)/dt almost infinite.

Our case of time varying magnetic field is similar to this example, where the centre of curvature is instantly changing Fig[3].

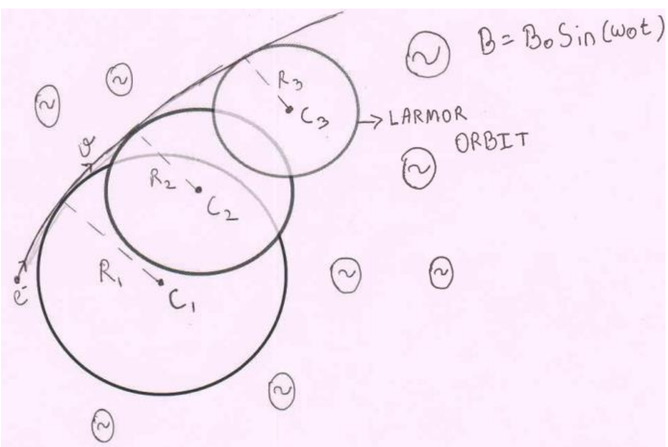

Figure 3: Larmor Orbits

Since the electric field has no radial component with respect to the origin at the centre of curvature corresponding to the magnetic field and velocity value, which is the centre for the Larmor orbit at that instant also, our radial force can be written as $mv\frac{d\theta}{dt}$. This can further be equated to $q(v \times B)$, giving $\frac{d\theta}{dt} = \frac{q}{m}B_0 Sin(\omega_0 t)$, as our B = $B_0 Sin(\omega_0 t)$ - (5).

3. DERIVATION

Since we already know that $\frac{d\theta}{dt} = \frac{q}{m}B_0 Sin(\omega_0 t)$ from (5), we take the condition such that when the magnetic field completes its one half cycle, i.e. it completes its positive half cycle in T_0 /2 time, then correspondingly the particle should have traversed covering π radians along the Larmor orbit as shown in Fig[4].

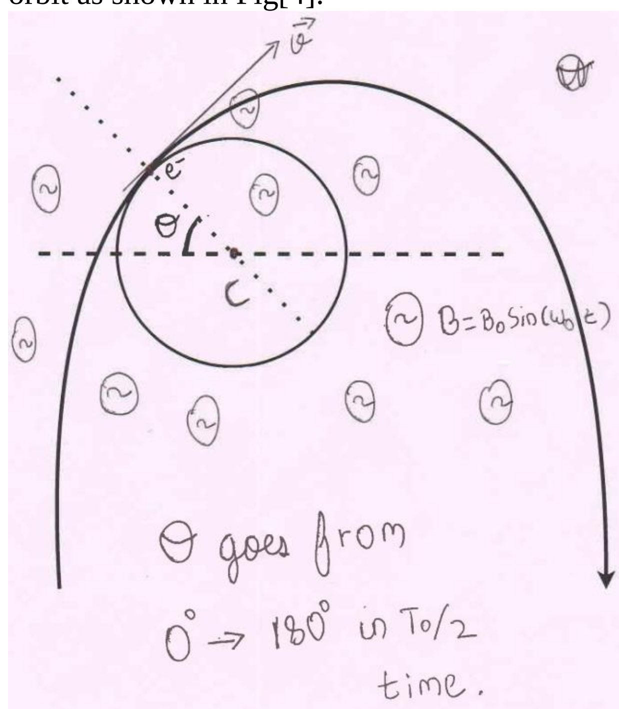

Figure 4: Angle swept by the particle

This makes our integration:

$$\int_0^{\pi} d\theta = \int_0^{T_0/2} \frac{q}{m} B_0 Sin(\omega_0 t)dt \qquad (6)$$

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$$\pi = \frac{2qB_0}{m\omega_0} \tag{7}$$

Therefore, we get the necessary relationship between the peak magnetic field strength and the frequency $\omega_0 / 2\pi$ at which the particle is undulating as well as the magnetic field is varying. Since the magnetic field is sinusoidal in nature, it can be produced by using electromagnets. The high impedance owing to its high frequency can be reduced by using a tank capacitor in an LC oscillator type circuit used in devices like induction cooker. This use of oscillating magnetic field instead of permanent magnets has several advantages such as achieving greater magnetic strength with ease.

The magnetic field strength for frequency of 2.5 Ghz comes out to be approx. 0.1428 Tesla for electron as our particle. Therefore, our magnetic field function turns out to be: B $= 0.1428 Sin(2\pi \times 2.5 \times 10^9 t)$. We shall further see that the wavelength produced by this undulatory motion is very small.

4. SIMULATION

The following simulation has been performed using the formula (7). The mass of the particle is $6.6422*10^{-26}$ Kg. The charge being $1.602*10^{-19}$ C. Taking the value of B_0= 2 Tesla, we get the following angular frequency as ω_0 = 3071203.006 Hz. The following plots have been created for the following calculated inputs, but it is to be noted that the effect of induced electric field has not been taken into account in the simulation. The role of electric field is to change the velocity of the particle from equation (2.1), yet there is no radial component of the electric force, hence demonstrating the independence of the velocity of the particle from its time required to complete one cycle is necessary. At the beginning the larmor radius will be infinite, therefore the Emf will be the integral $\oint \vec{E} \cdot \vec{dl}$. As this electric field follows the curl by Larmor orbit, the circumference $2\pi R$ will be very large, therefore the value of Electric field will be almost zero at the beginning. The value of electric field keeps increasing until it further decreases till 1/4th of the cycle where it becomes zero, i.e. when the value of magnetic field is at peak which is at 90 degrees. The electric field will do positive work on the particle till then, later from the interval T/4 to T/2 the electric field is opposing direction of motion, therefore will do negative work on it, thereby the net work done by the electric field in one cycle is zero.

In order to prove that the time taken for each cycle is independent of the velocity of the particle, we take various input velocity V for the particle in our simulation and try to see its plot. Below in the figures you can see that for the given time period $T_0 = 2.0458 \times 10^{-6}$ sec. the particle completely covers its cycle.

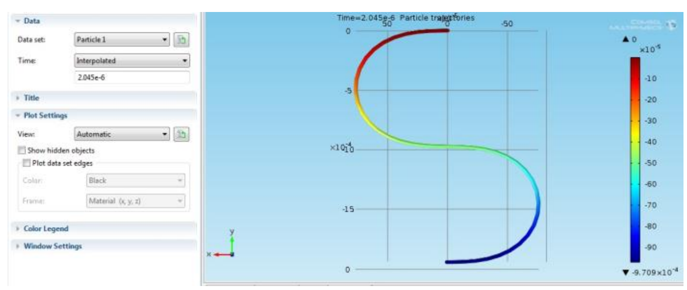

Figure 5: Initial velocity 2*10^3 m/sec

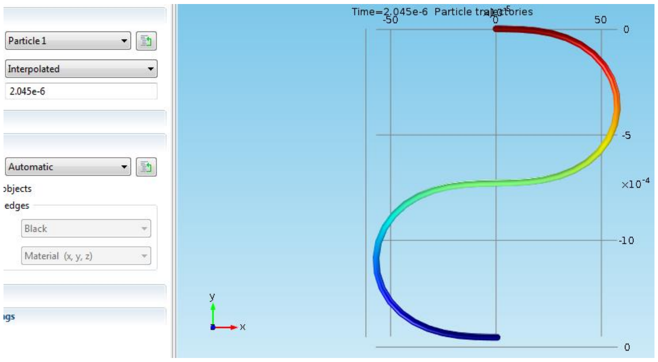

Figure 6: Initial velocity 1.5*10^3 m/sec

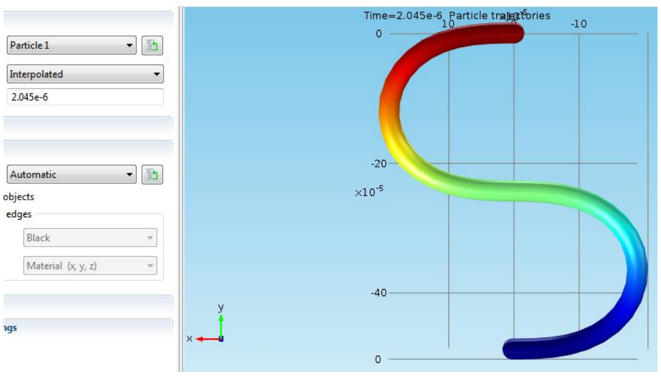

Figure 7: Initial velocity 0.5*10^3 m/sec

4.1.Amplitude calculation

From equation (5) we can see that the angle can be represented as: $\int_0^{\theta} d\theta = \int_0^{t} \frac{q}{m} B_0 Sin(\omega_0 t)dt$, therefore we can write the angle for time between 0-T/4 is:

$$\theta = \frac{qB_0}{m\omega_0}[1 - Cos(\omega_0 t)] \tag{8}$$

As we know from equation (7), we can write equation (8) as,

$$\theta = \frac{\pi}{2}[1 - Cos(\omega_0 t)]. \tag{9}$$

This allows us to calculate the amplitude as the y component of the velocity integrated over time. Amplitude A = $\int_0^{T_0/4} V_y \cdot dt$. Refer Fig[8] to see that $V_y = VCos(\theta)$, where V is the velocity of the particle at that instant.

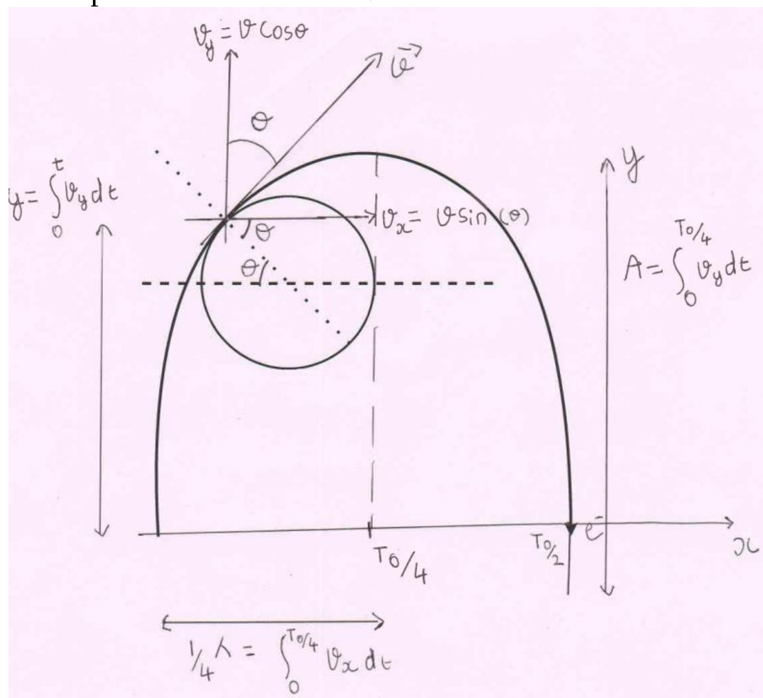

Figure 8: Wavelength and Amplitude

We take the sample case of 2.5 Ghz frequency and the velocity of electron as 4.2*10^7 m/sec by accelerating it in a potential difference of 5 KV. We have also neglected the effect of electric field which will be causing the amplitude to further increase by a small factor. The integrated result gives us the Amplitude of it as A = $3.156 \times 10^{-3} m$.

4.2.Wavelength Calculation

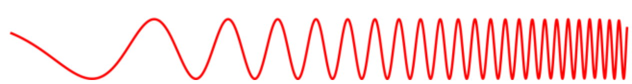

Figure 9: Approximate trace of varying wavelength (amplitude should also be decreasing)

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The wavelength is calculated in a similar way where its written as $\lambda = 4 \times \int_0^{T_0/4} V\, Sin(\theta) \cdot dt$. Here too we have taken the magnitude of velocity as constant throughout for simplicity. Taking the same case of 2.5 Ghz and 4.2*10^7 m/sec velocity, we get the wavelength as $\lambda = 7.91 \times 10^{-3} m$. For higher frequencies the wavelength becomes even smaller. As the velocity of the particle decreases, the wavelength too decreases. After a certain time when the velocity is very low, the electron moves in x direction with even smaller velocity component, and in y direction with higher velocity component.

5. APPLICATIONS

The following property of electron to move with constant frequency despite losing/gaining its kinetic energy can be utilized in many ways. When the electrons are injected in pulses, each pulse occurring after any integral multiple of the time period T_0 which is $1/f_0$ where f_0 is the frequency at which magnetic field oscillates, we get an electron density which oscillates and moves forward in phase with each other, much like how wave travels in a string. When two parallel plates are put in between the electron oscillation as shown in Fig[10], we can get induced charges at the two outer ends of the parallel plates. These two ends can then be used to excite a resonant cavity and thereby obtain high energy electromagnetic waves by using a pick-up loop. The advantage of such vacuum tube device is that the net distance travelled by the electron is much more than standard vacuum tube devices like klystron/TWT, yet the length of the tube is smaller in size. This is due to the fact that the electrons travel wave like path, therefore taking longer time to travel the same length of the tube, giving us more time to extract energy and resulting in electrons losing almost all of its initial kinetic energy.

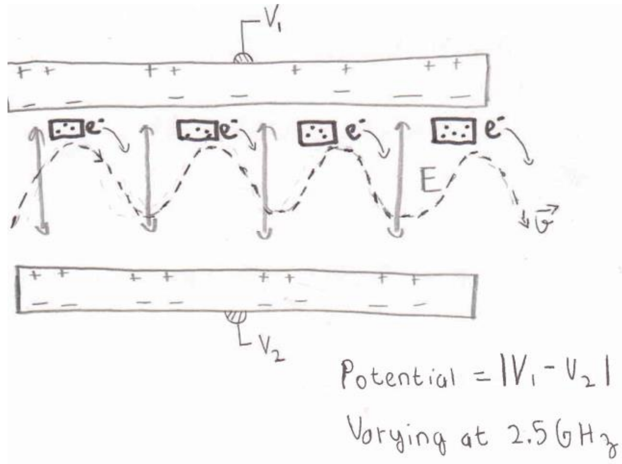

Figure 10: Induced Potential at the two ends of plates

Another use is in the study of Surface Plasmon Resonance which observes the electron moving inside the conductor with very high frequency in wave like pattern.
Undulators/wigglers are known devices which are used to produce bright beams off accelerated particles. The following equation can help us overcome many of the limitations which undulators have, giving us more number of cycles and shorter wavelength.
Plasma can also be used as they too are charged particles.

6. REFERENCES

[1] Jr. Gilmour, *Principles of travelling wave tubes* (Pg. No. 38).
[2] J. Rossbach, E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, "Performance Limitations of an X-Ray FEL,", EPAC'96, Sitges, June 1996, WEP059G
[3] F.F. Chen, *Introduction to Plasma Physics and Controllable Fusion* (Pg. No. 41 & 42).