

④ Eigen Values: $|sI_{3 \times 3} - A| = 0$

③

$$\begin{vmatrix} \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 125 & 0 & -125 \\ 0 & 41.67 & 83.33 \\ 500 & -500 & 0 \end{bmatrix} & = & \begin{bmatrix} s-125 & 0 & 125 \\ 0 & s-41.67 & -83.33 \\ -500 & 500 & s \end{bmatrix} \end{vmatrix}$$

$$\begin{aligned} \det(A) &= (s-125)[s(s-41.67) - (500)(-83.33)] + (500)[(s-41.67)(125)] \\ &= s^3 - 166.67s^2 + 109374s - 7.8125 \times 10^4 \\ &= (s-76.2348)(s^2 - 90.4352s + 102479) \end{aligned}$$

$$\lambda_1 = 76.2348, \lambda_2 = 45.2176 - j316.915, \lambda_3 = 45.2176 + j316.915$$

Eigen Vector:

$$(A - \lambda I)\vec{x} = 0 \Rightarrow \left(\begin{bmatrix} 125 & 0 & -125 \\ 0 & 41.67 & 83.33 \\ 500 & -500 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 125-\lambda & 0 & -125 \\ 0 & 41.67-\lambda & 83.33 \\ 500 & -500 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 76.2348$$

$$\begin{bmatrix} 48.77 & 0 & -125 \\ 0 & -34.56 & 83.33 \\ 500 & -500 & -76.23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 48.77 & 0 & -125 \\ 0 & -34.56 & 83.33 \\ 500 & -500 & -76.23 \end{bmatrix} \xrightarrow{-10.2522R_1 + R_3 \rightarrow R_3}$$

$$\begin{bmatrix} 48.77 & 0 & -125 \\ 0 & -34.56 & 83.33 \\ 0 & -500 & 781.525 \end{bmatrix} \xrightarrow{14.4676R_2 - R_3 \rightarrow R_3}$$

$$\begin{bmatrix} 48.77 & 0 & -125 \\ 0 & -34.56 & 83.33 \\ 0 & 0 & 424.06 \end{bmatrix}$$

$$\|x\| = \sqrt{1086.89^2 + 1022.48^2 + 424.06^2} = 1551.33$$

$$\begin{aligned} 48.77x_1 - 125x_3 &= 0 \Rightarrow x_1 = 1086.89 \\ -34.56x_1 + 83.33x_3 &= 0 \Rightarrow x_2 = 1022.48 \\ 424.06 &= x_3 \end{aligned} \left\{ \begin{array}{l} x_1 = 1086.89 \\ x_2 = 1022.48 \\ x_3 = 424.06 \end{array} \right\} \Rightarrow \hat{x}_1 = \frac{x_1}{\|x\|} = \begin{bmatrix} 0.700619 \\ 0.659099 \\ 0.273353 \end{bmatrix}$$

4 cont... $\lambda = 45.2174 - j316.915$

(4)

$$\begin{bmatrix} 79.7824 + j316.915 & 0 & -125 \\ 0 & -3.5476 + j316.915 & 83.33 \\ 500 & -500 & -45.2176 + j316.915 \end{bmatrix} \begin{matrix} \\ \\ (-0.373512 + j1.48368)R_1 + R_3 \rightarrow R_3 \end{matrix}$$

$$\begin{bmatrix} 79.7824 + j316.915 & 0 & -125 \\ 0 & -3.5476 + j316.915 & 83.33 \\ 0 & -500 & 1.47139 + j131.455 \end{bmatrix} \begin{matrix} \\ \\ (0.017659 + j1.57751)R_2 - R_3 \rightarrow R_3 \end{matrix}$$

$$\begin{bmatrix} 79.7824 + j316.915 & 0 & -125 \\ 0 & -3.5476 + j316.915 & 83.33 \\ 0 & 0 & 0.000133 - j0.000937 \end{bmatrix}$$

$$(79.7824 + j316.915)x_1 - 125x_3 = 0$$

$$(-3.5476 + j316.915)x_2 - 83.33x_3 = 0$$

$$x_3 = 0.000133 - j0.000937$$

$$x_2 = -0.000247 - j0.000032$$

$$x_1 = -0.000335 - j0.000137$$

$$x_2 = \begin{bmatrix} -0.000335 - j0.000137 \\ -0.000247 - j0.000032 \\ 0.000133 - j0.000937 \end{bmatrix}$$

$$\|x_2\| = \sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2}$$

$$\hat{x}_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} -0.321161 - j0.131245 \\ -0.236471 - j0.030947 \\ 0.127764 - j0.898015 \end{bmatrix}$$

$$\begin{bmatrix} -0.0847 - j0.3364 & -0.0847 + j0.3364 & -0.7007 \\ -0.0027 + j0.2385 & -0.0027 - j0.2385 & -0.6590 \\ -0.9071 & -0.9071 & -0.2734 \end{bmatrix}$$