

# The Calculated Value of the Fine Structure Constant From Gravitational Potential

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## ABSTRACT

In previous papers [1], [2], we have postulated a relation between the gravitational potential and the Fine Structure Constant, ( $\alpha$ ) necessary for the validity of those papers. The Scalar Gravitational paper tests the relation with respect to local orbital mechanics for comparison with GR. In this paper will explore the absolute magnitude of  $\alpha$  in regard to the cosmological induced potential generated by the total mass in the system. Although the value of the Fine Structure Constant is known to a very high precision, current QM and GR do not offer an explanation for the value, and it must be determined experimentally. Extremely precise relations between  $\alpha$  and the Gyromagnetic ratio make this possible. (.7ppb) This paper offers an explanation of, and calculates the value, within the error bars, of current of experimental data. Since there is a temporal variability predicted in  $\alpha$ , and because of the current precision of measurements of  $\alpha$ , a test of the change predicted by this conjecture is possible.

## INTRODUCTION

In the previous papers we have shown a relation between the gravitational potential and the value of the Fine Structure Constant which produces both the proper value of the electric charge, and the proper gravitational interaction for orbital mechanics. This paper exploits the functional relation to integrate the value of the mass over the Universe Hypersphere, and compare the value with those arrived at by luminosity and WIMAP methods.

In earlier papers, [1], [2], we presented and used the relation between the observable mass and what we have designated as the gravitationally field free mass:

$$M_F = M / \alpha , \quad (1)$$

where  $\alpha$  is a scalar variable at a particle in space ( $r = \mu$ ), and related to the gravitational interaction by:

$$1 = \sum_n^N \frac{\alpha_n \mu_n}{r_n}, \quad (2)$$

This is asserted to be the fundamental relation between the Gravitational Constant and the Fine Structure Constant.

For the value at the locus of a mass particle this is:

$$\alpha = \left( 1 - \sum_n^N \frac{\alpha_n \mu_n}{r_n} \right). \quad (3)$$

### CALCULATIONS

Since the subscript in  $\alpha_n$  is only necessary because of local variations cut to local mass effects, for global purposed it can be dropped and can be factored out of the sum. Rearranging, and noting that the first term is also a part of the sum, alpha can be evaluated as the integral of the gravitational potential over the particle masses in the universe.

$$\left( \frac{1}{\alpha} \right) = \frac{\mu}{r_0} + \sum_n^N \frac{\mu_n}{r_n} = \int_0^{\mathfrak{R}} \frac{\mu}{r} \rho dv \quad (4)$$

Integrating over a hypersphere is a bit tricky, but a modified Milne model will do for the purposes.

We will form the volume integral as a spherical surface with thickness  $dr$ . In a hypersphere as  $r$  increases the volume of the universe decreases, and thus the surface area of a sphere decreases as a function of  $r$  such that:

$$sdr = 4\pi \left[ r \frac{(\mathfrak{R} - r)}{\mathfrak{R}} \right]^2 dr, \quad (5)$$

where  $\mathfrak{R}$  is the current radius of the expanding universe.

As  $r$  increases the universe is smaller and thus the number density of particles is not constant but dependent of the volume of the universe at the distance of the sphere. The number density of protons as a function of  $r$  is then:

$$\rho = \frac{N}{V} = \frac{N}{\frac{4}{3}\pi[(\mathfrak{R} - r)]^3} \quad (6)$$

Eq. (4) can now be restated as:

$$\frac{1}{\alpha} = 3 \frac{\mu N}{\mathfrak{R}} \int_0^{\mathfrak{R}-\mathfrak{R}_0} \frac{r dr}{(\mathfrak{R} - r)}, \quad (7)$$

Note that the upper limit on the integral is not the total radius, which would make the value infinite, but is short of the radius of the universe, by a distance associated with the Planck radius.  $N$  is the number of protons in the universe and  $\mu$  is the gravitational radius of the proton. The electron mass should be included also, however it is not significant in these calculations.

Integrating and putting in the experimental value of  $\alpha$  gives:

$$\frac{1}{\alpha} = (137.035) = \frac{3\mu N}{\mathfrak{R}} [\ln(\mathfrak{R}/\mathfrak{R}_0) - 1], \quad (8)$$

where  $\mathfrak{R}_0$  is on the order of the Planck radius.

The first thing noticeable about the expression is that the right coefficient is very close to a value of one, and if the value of  $\mathfrak{R}$ , the radius of the universe, should be doubled, the value of the log function would only increase by a half percent.

It is not likely that in the current universe this value is coincidentally now equal to one. We will presume that via Occam's razor, Eq. (8), implies that value of the coefficient must **always** be equal to one, i.e. time invariant. This diverges at this point from the Milne model, since the mass in the standard Milne view is constant, and this term could only be equal to one in the current universe. We will refer to this as a Modified Milne model, which has properties similar to the “tired photon” model. In this case the photon received at a later point in time has a redshift because the mass of the receiving atom has increased and its frequency is greater than the photon emitted at an earlier time. The measured results are indistinguishable but the conceptual structures are quite different.

This brings on another issue, since  $\mathfrak{R}$  is time dependent, and if the right coefficient is constant, then the particle mass must also be time dependent. This dependence however does provide an opportunity for insight into the cutoff point,  $\mathfrak{R}_0$  for the integral. If we express  $\mu$  in the coefficient in terms of the Planck mass and radius, using:

$$r_{\text{pl}} = \frac{\hbar}{M_{\text{pl}}c} \quad r_{\text{pl}}^2 = \frac{G\hbar}{c^3} \quad (9)$$

the coefficient can be expressed as:

$$1 = \frac{3\mu N}{\mathfrak{R}} = \frac{3r_{\text{pl}}^2 c}{\hbar} \frac{NM}{\mathfrak{R}} = \frac{3r_{\text{pl}} N}{M_{\text{pl}}} \frac{M}{\mathfrak{R}} \quad (10)$$

and:

$$\mathfrak{R}_0 = \frac{3r_{\text{pl}} N}{M_{\text{pl}}} M_0 \quad (11)$$

Next we have to make an assumption necessary for the values of the integral to have approximately the correct value, though at the same time the assumption must be consistent with our knowledge of particle physics. We will assume that: *In the initiation of the universe a Planck mass internally breaks up into  $3N$  quarks each with a  $1/3$  baryon number, and each proton is composed of three of the proper combination.* Thus we can calculate the initial radius in terms of the Planck mass:

$$\mathfrak{R}_0 = \frac{3r_{\text{pl}} N}{M_{\text{pl}}} (M_0) = \frac{3r_{\text{pl}} N}{M_{\text{pl}}} \left( \frac{3M_{\text{pl}}}{N} \right) = 9r_{\text{pl}} \quad (12)$$

The initial radius of the universe then becomes 9 Planck radii.

Putting this value of the initial radius into Eq. (8), gives:

$$\left( \frac{1}{\alpha} \right) = \left[ \ln(\mathfrak{R}/9r_{\text{pl}}) - 1 \right] = 137.03470 \pm 0.02858 \quad (13)$$

where the uncertainty of one part in 5000, is caused by the uncertainty in the (WMAP) Hubble value of, 71 (km/sec)/Mpc, (+0.04/-0.03) in the radius of the universe [3]. The exact value of  $1/\alpha$ , (137.03599) is near the center of the error bars, however, and differs from this by only one part in 100,000.

ASIDES

From Eq. (1), Eq. (10) , and Eq. (13) , we can eliminate the current mass and solve for the current value of the free mass  $M_F$  of Eq. (1), which is a function of the radius of the universe.

$$M_F = \frac{M}{\alpha} = \frac{M_{pl} \Re}{3r_{pl} N} \frac{1}{\alpha} = \left( \frac{M_{pl} \Re}{3r_{pl} N} \right) [\ln(\Re / \Re_0) - 1], \quad (14)$$

The first term in the right side of this expression is the temporal redshift term that is equivalent in this model to the Hubble redshift. The second term should be measurable, although quite small. The mass  $M_F$  is a temporal variable, but not directly measurable since the mass of the instruments used to take measurements is also temporal.

having a current value of  $M_F = M \times 137.03599$ . Although this is the free mass in the current universe and not a function of local mass, it is an increasing function of the expansion. This defined Free Mass is only useful for calculation purposed and is not directly measurable.

Also we have for the current value of the mass of the proton:

$$M = \Re \frac{M_{pl}}{3r_{pl} N}, \quad (15)$$

An interesting point is the fact that if we designate a Compton radius, and Compton Mass for the universe:

$$M_u = \frac{\hbar}{\Re_c} \quad (16)$$

We can then write Eq. (12) as:

$$(Nm_p) M_u = \frac{m_{pl}^2}{3}. \quad (17)$$

Which is simply that the product of the mass in the universe, times the Compton mass of the universe is a constant equal to 1/3 the square of the Planck mass. That is the product of the increasing mass in the universe and the decreasing Compton mass has a constant proportionality.

This also shows that the switch from redshift resulting from expansion in the Milne model, to redshift resulting from increased, mass requires only a relatively simple change in the Milne Model. The change being the assertion that, a photon emitted from an atom at an earlier time, arrives at a location at a later time with the same energy as it had on emission, the expansion having no effect on the photon energy. These fits with our earlier determinations that gravitation does not effect on the energy of the photon [2].

Using Eq. (10) and the (WMAP) value of the Current value of the radius of the universe we can calculate mass and the equivalent number of protons in the universe.

$$N = 3.375e79 \text{ protons}$$

This compares favorably with the values of  $1.793e79$  cited from studies of Cosmic microwave background radiation, super-clusters, Big Bang nucleo-synthesis, and NASA luminosity studies.

The above calculations would suggest a variation in Alpha of  $7.4e-11$  per year in contrast to the current astronomically measured value of the variation in Alpha of less than one part in  $6.e-17$  [4], per year. Since our fundamental equation asserts that Alpha and mass are inverse, the changes in a particle spectra are only proportional to the mass and thus ratios of changes in alpha cannot be distinguished from changes in mass ratios. From this it can be asserted that alpha changes cannot be detected in cosmological spectra. A direct measurement via the Gyromagnetic ratio however is without this problem and should accurately reflect any cosmological change.

### Initial Universe

It is pushing the envelope a bit far to extrapolate Eq. (13) to the initial universe but it is nevertheless interesting. When the value of radius is at minimum, or  $\mathfrak{R}_0 = 9r_{pl}$  the value of alpha is negative one, meaning the charge of the electron and proton are reversed and gravitation is repulsive. As the radius expands to  $\mathfrak{R}_0 = e \times 9r_{pl}$ , alpha goes infinite, and gravitation goes to zero, then reverses to attractive, as alpha goes positive. Though this is heavy on speculation, it does have the feel of plausibility.

### Testing the Theory

From Eq. (13) it is seen that there is a time dependent variation in alpha. Taking the differential gives:

$$\frac{d\alpha^{-1}}{d\mathfrak{R}} = \frac{1}{\mathfrak{R}}, \quad (18)$$

or:

$$\Delta\alpha^{-1} = \frac{\Delta\mathfrak{R}}{\mathfrak{R}} = 7.298e-11 \text{ per year} \quad (19)$$

This is about one part in 13.7 billion per year.

The most recent and best test of the value of  $\alpha^{-1}$  by Gabrielse et al [4], is to an accuracy of 0.7 ppb [5] which, if the theory is correct, in a few years the value should shift out of the error bars, and the variation become detectable.

The schedule for this should be:

137.035999710	Harvard 2006 value of $1/\alpha$ [5]
+/- .000000096	Current error bars .7 in one billion
+ .000000010002	Change in one year. One part in 13.7 billion
+ .00000010002	Change in 10 years
+ .00000020004	Change in 20 years

From this we can see that in about 9.6 years the value of  $\alpha^{-1}$  should shift out of the error bars of the current value, and thus the test should be valid.

The Gabrielse measurements which determine the spin g factor by using the difference in electron's cyclotron frequency, and its spin precession frequency in a Penning trap, is a direct proportional measurement of g-2. Since the g factor is only a function of alpha, the question of co-temporal variables that could mask a change in alpha does not exist.

It has to be acknowledged that there are both astronomical and geological measurements that tend to show that the variation in alpha is considerably lower than that shown in the above development and credibility requires an explanation of the fault in those measurements.

#### *Geological measurements*

As for the geological of measurements of isotope ratios in the Oklo site in Gabon, by Damour and Dyson [9], we will use the argument of Varshalovich et al. [6 ]. This argument reasons that these tests are dependence on the model of the phenomenon is fairly complex, involving many physical effects, and rely on assumptions that are not justified.

#### *Astronomical measurements*

The astronomical Measurements are a bit more straightforward, but may also contain a flaw. The measurements are based on the fact that the doublet splitting in the absorption spectra is proportional to  $\alpha^2$  [6 ], and thus a change in the spectra can indicate a change

in  $\alpha$ . Arriving at the proportionality of alpha with spectral lines involves a degree of analysis using other physical constants that may or may not have temporal variability. In the following we will attempt to show that if stripped down to essentials the proposed proportionality may not be valid.

A Bohr model representation of the split energy of a doublet in an atomic system is essentially:

$$\Delta E_s = g_s \mu_B B = g_s \frac{Q\hbar}{2M_e} B = g_s \frac{Q\hbar}{2M_e} \frac{Q}{r^2} \frac{v}{c} = g_s \frac{\alpha}{M_e} \left( \frac{\hbar^2 v}{2r^2 c} \right) = g_s \frac{\alpha^2}{M_e} \left( \frac{\hbar^2}{2nr^2} \right) \quad (20)$$

And the Rydberg formula for the energy of the central line is:

$$\Delta E_B = E_i - E_f = \alpha^2 \frac{M_e c^2}{2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (21)$$

Ignoring the particular lines, The ratio of these energies is:

$$\frac{\Delta E_s}{\Delta E_B} = \frac{g_s \frac{\alpha^2}{M_e} \left( \frac{\hbar^2}{2r^2} \right)}{\alpha^2 \frac{M_e c^2}{2}} = g_s \left( \frac{\hbar}{M_e c r} \right)^2 \quad (22)$$

The quantity in parentheses, the angular momentum, is quantized in an atomic system, and can not be temporal variable. We would argue that only g “can” be a temporal variable in this relation.

For the astrophysical measurements, the primary relation used in determining the temporal variation of alpha was originally by Varshalovich et al [6], and later used by Murphy et al [7] and then by Chand et al [8]. It can be expressed in the same energy ratios as:

$$\Delta \alpha_z / \alpha \approx \frac{c_r}{2} \left[ \frac{(\delta \lambda / \lambda)_z}{(\delta \lambda / \lambda)_0} - 1 \right] \approx \left[ \frac{(\Delta E_s / \Delta E_B)_z}{(\Delta E_s / \Delta E_B)_0} - 1 \right] \quad (23)$$

Where z refers to the redshifted values, and  $c_r$  is a constant that is approximately equal to one.

Substituting the values from Eq. (22):



$$\Delta\alpha_z/\alpha = \frac{c_r}{2} \left[ \frac{(g_s)_z}{(g_s)_0} - 1 \right], \quad (24)$$

Which shows that the measurable change is only the change in the spin g factor, and thus much less sensitive to a change in alpha than  $\alpha^2$ . Note that this result does not depend on the conjecture put fourth in this paper, but only the constancy of the angular momentum ratio in Eq. (22).

If we put in our conjectured change in  $\alpha$  from Eq.(13), and use the relation between the radius of the universe an z to be:

$$1+z = \frac{\lambda_c}{\lambda_z} \sim \frac{\mathfrak{R}_c}{\mathfrak{R}_z} \quad \mathfrak{R}_z = \mathfrak{R}_c \left( \frac{1}{1+z} \right) \quad (25)$$

Then we have:

$$\frac{1}{\alpha_z} = \ln \left[ \frac{\mathfrak{R}}{\mathfrak{R}_0} \left( \frac{1}{1+z} \right) \right] = \frac{1}{\alpha} - \ln(1+z) \quad (26)$$

Since the spin g factor is only a function of  $\alpha$ , the values of  $\alpha_z$  from Eq. (26) can be put into Eq. (24) for finding the value as a function of z. Using the constants for the alpha terms in  $g_s$  found in Gabrielse [4], the values calculated are shown in Figure 1 and Table 1

Fig 1

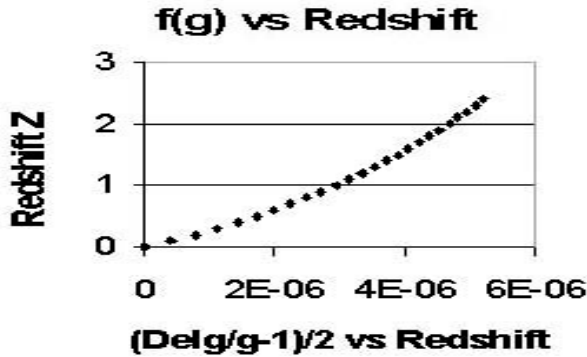
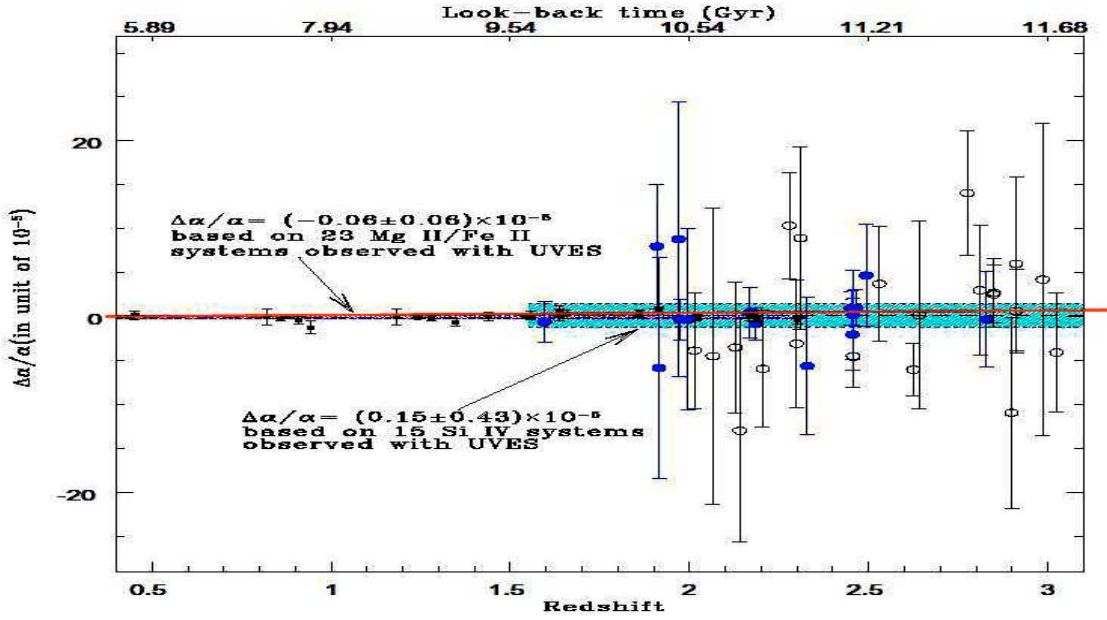


Table1

Z	f(g)
0.5	1.72E-06
1.0	2.94E-06
1.15	3.25E-06
1.5	3.89E-06
2.0	4.67E-06

These levels can be contrasted with the measured data from Murphy and Chand by use of the data plotted in Chand [8] in Figure 2.

Figure 2



The red line is the change in  $\Delta\alpha_z/\alpha$  as per the foregoing development

The most accurate astrophysical data published to date shows that the maximum temporal variation of the value of  $\Delta\alpha_z/\alpha$  for a  $z$  value of 1.1508, to be less than about  $(0.05 \pm 0.24) \times 10^{-5}$ . [8]. This is fairly close to the calculated value shown in the table for that redshift, suggesting that the precision in the astrophysical measurements are not yet of sufficient to accuracy test the conjecture.

## CONCLUSION

A value of the Fine Structure Constant has been developed, and calculated with a minimum of selectable constants, namely the value of the initial universe being nine times the Planck length. The precision of the calculation being within one part in 5000 gives a degree of confidence that the procedure has merit. Since the derived value has a temporal dependence, and can soon be testable using current unambiguous experimental methods, the merit will be discernable with certainty. It could be argued that it has already been proven false by the astronomical measurements, but we have shown that these measurements may be subject to some false assumptions that could lead to a false understanding of the results.

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