# Reentry Dynamics

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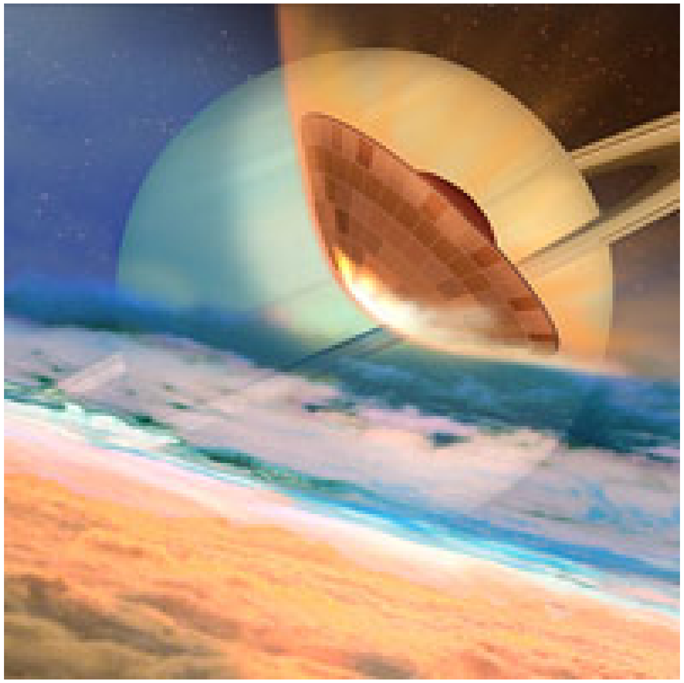
- The Reentry problem
- Reentry Dynamics
- Ballistic reentry
- Skip reentry
- Control over the lift

# Reentry



The problem for surviving reentry does not arise in every mission.

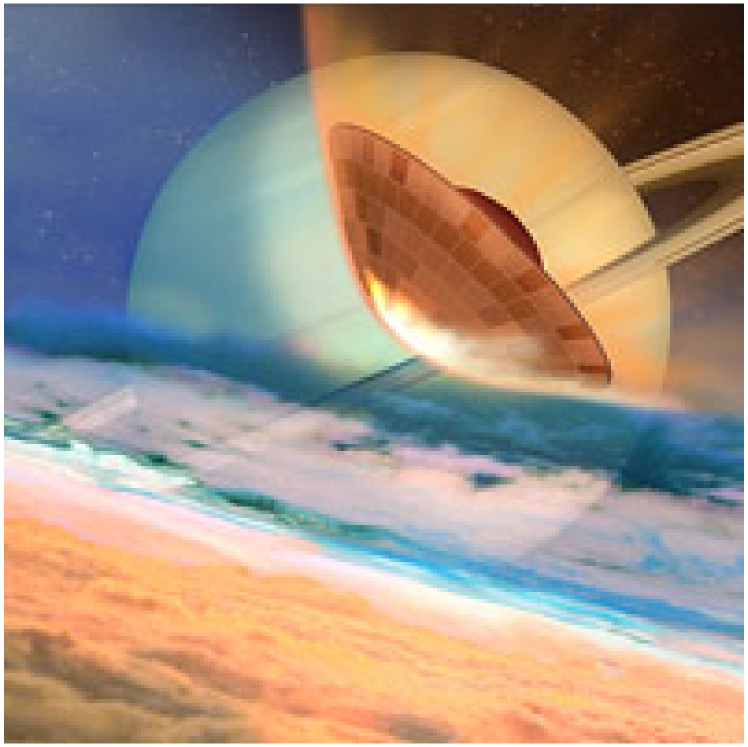
Most satellites are on one-way ride out of the Earth's atmosphere.

For LEO satellites, reentry will occur long after the useful lifetime.

For very high orbits, reentry might never occur.

Satellites are not designed to survive reentry forces and heat loading.

# Reentry

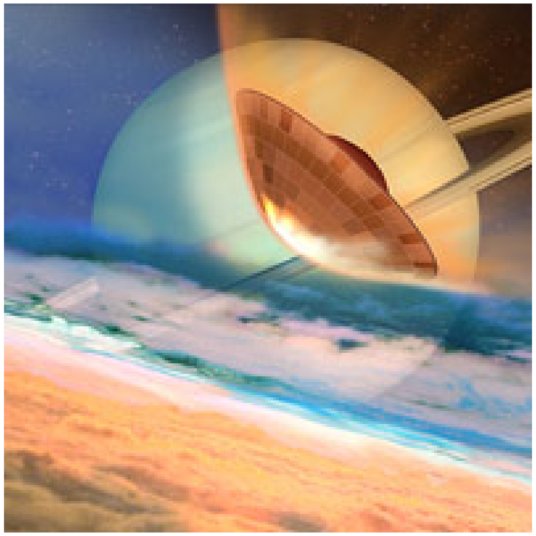
Surviving reentry is of a great importance for missions such as:

- ballistic missile (first to be tested)

- planetary-entry probes

- manned space missions

# Reentry



The Problem: how to dissipate the enormous amount of kinetic energy that is concentrated in the spacecraft!

The kinetic energy per unit mass, $E = \frac{1}{2}v_c^2$, where $v_c$ is the orbital velocity.

Huygens entered Titan's atmosphere with $v_c = 6.1$ km/s.

The entry phase lasted 3 minutes!

The velocity was reduced to $v_c$ =0.4 km/s ! This is 93.7% $\Delta v$ change or 99.6% kinetic energy per unit mass (J/kg) change (=dissipation).

# Reentry

A significant fraction of the energy of the launch vehicle has been concentrated in the spacecraft!

This energy must be dissipated during reentry! We cannot use a booster to dissipate this energy as this would require a booster with the size of the original one used for launch!

# Reentry

Atmospheric drag is used to slow the vehicle !

The rate at which drag dissipates energy is

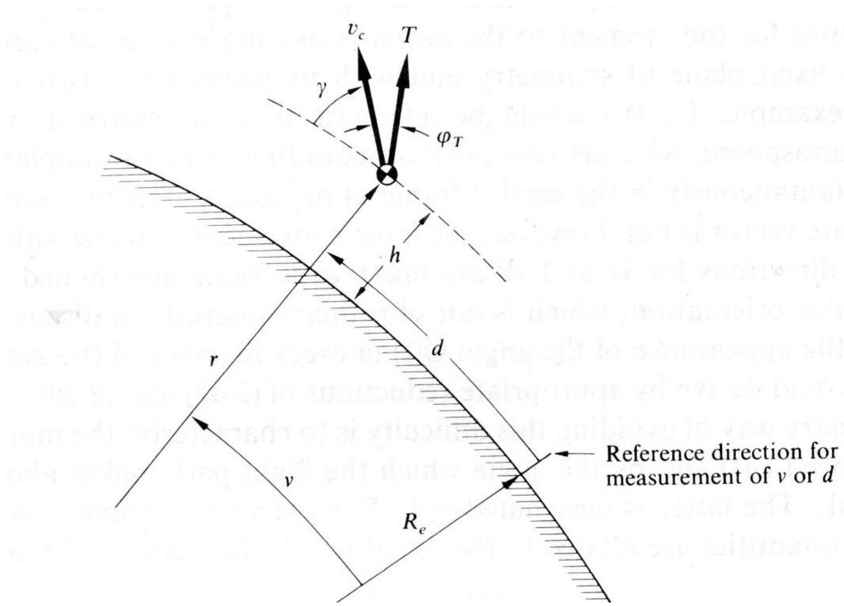
$dE/dt = \mathbf{D}.\mathbf{v}$, since $D \sim v^2$,

the rate of energy dissipation is then proportional to $\sim v^3$.

To survive the high temperature during reentry we have to design the reentry trajectory so that it is stretch over a long period time. This will reduce the amount of energy dissipated per unit time, [J/s].

For Huygens, we have $\Delta v_c = 6.3 - 0.4 = 5.9$ km/s, or 19.8 MJ/kg total kinetic energy dissipated over 3 min. This corresponds to 110kJ/s/kg dissipated power. The ceramic head shield on the probe will heat to up to 1800 degrees Celsius.

# Launch Dynamics



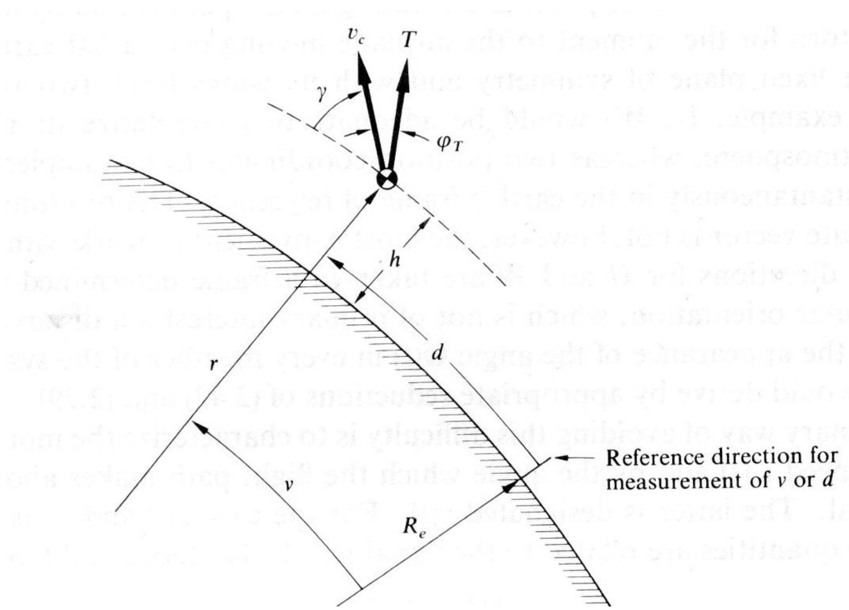

$r, \nu$ are the polar coordinates of the CM

$h$ is the altitude

$v_c$ is the velocity

$\gamma$ is the flight path angle (the angle from the local horizon to $v_c$)

# Launch Dynamics

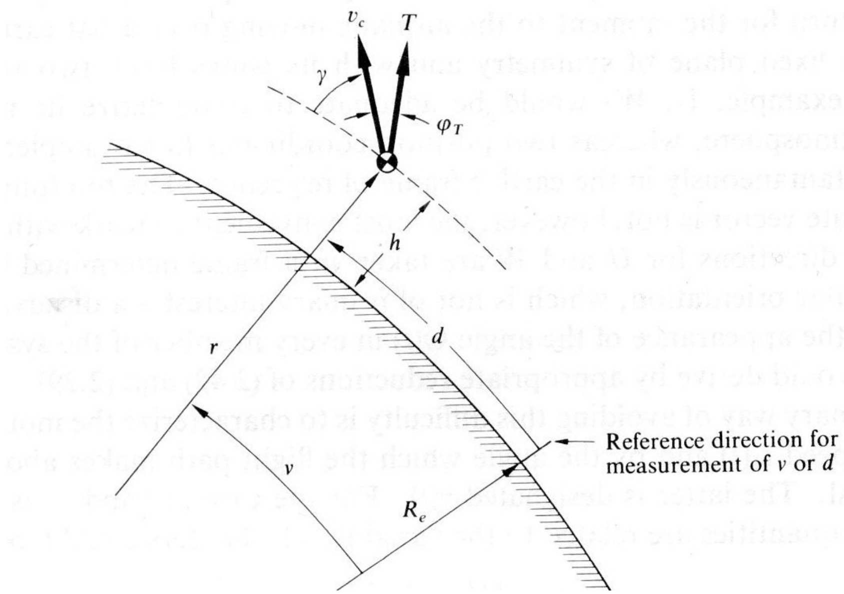



$R_e$ radius of the planet where $g$ has a constant value $g_e$

The local acceleration due to gravity towards the center is

$g = \frac{g_e R_e^2}{r^2} = \frac{k}{r^2}$

# Launch Dynamics




$$\dot{v}_c = -\frac{k sin(\gamma)}{r^2} - \frac{D}{m} + \frac{1}{m} T cos(\varphi),\ D = \frac{1}{2} C_d A \rho v_c^2$$

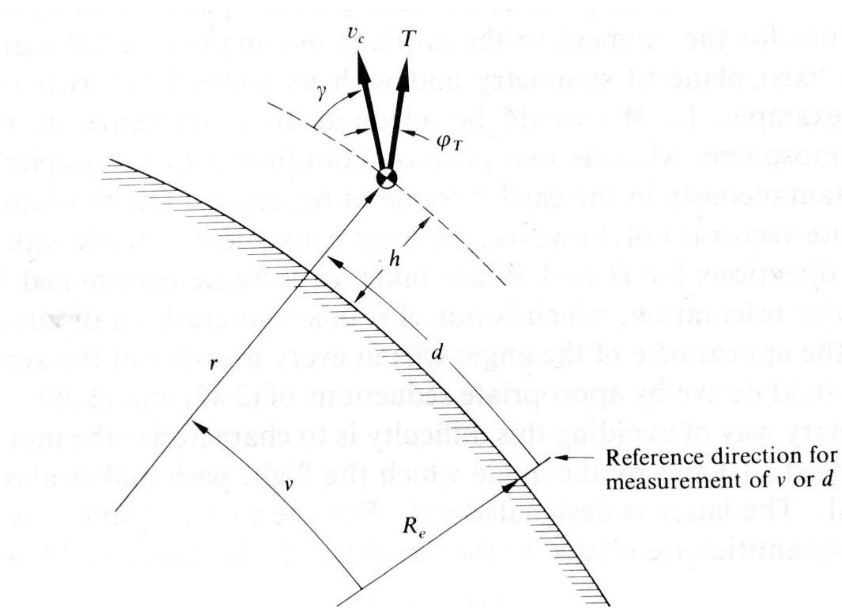
$$\dot{\gamma} = -\frac{k cos(\gamma)}{v_c r^2} + \frac{L}{v_c m} + \frac{v_c cos(\gamma)}{r} + \frac{1}{v_c m} T sin(\varphi)$$

$$\dot{\nu} = v_c \frac{cos(\gamma)}{r}$$

$$\dot{r} \equiv \dot{h} = v_c sin(\gamma)$$

$$\dot{m} = -\frac{T}{I_{sp}}$$

# Reentry Dynamics



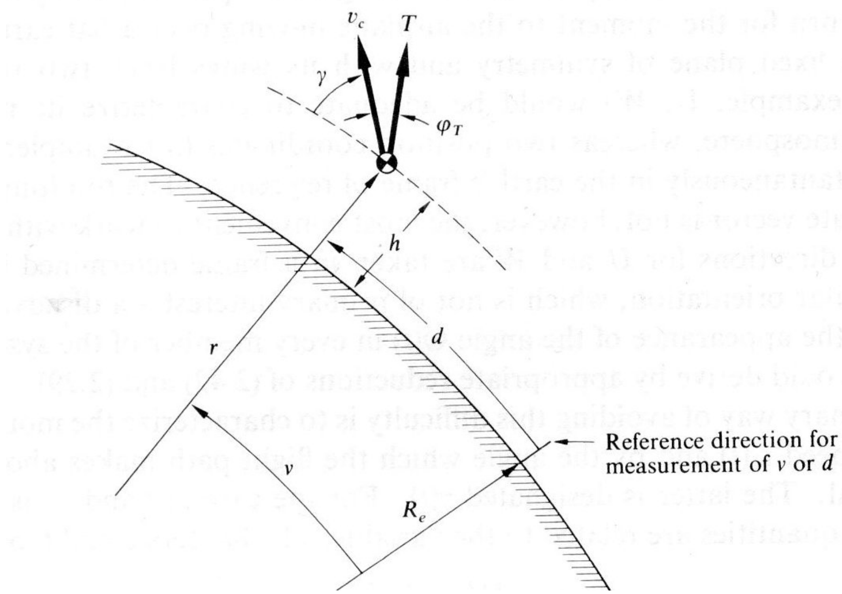

To derive the motion of the vehicle during reentry, we can adopt the development from last week (launch dynamics) to study the motion of a vehicle entering a planetary atmosphere from orbital flight. For athmospheric reentry,

1) $\gamma$ is negative!

2) The thrust is zero, $T = 0$

3) The mass is constant, $m = const$

# Reentry Dynamics



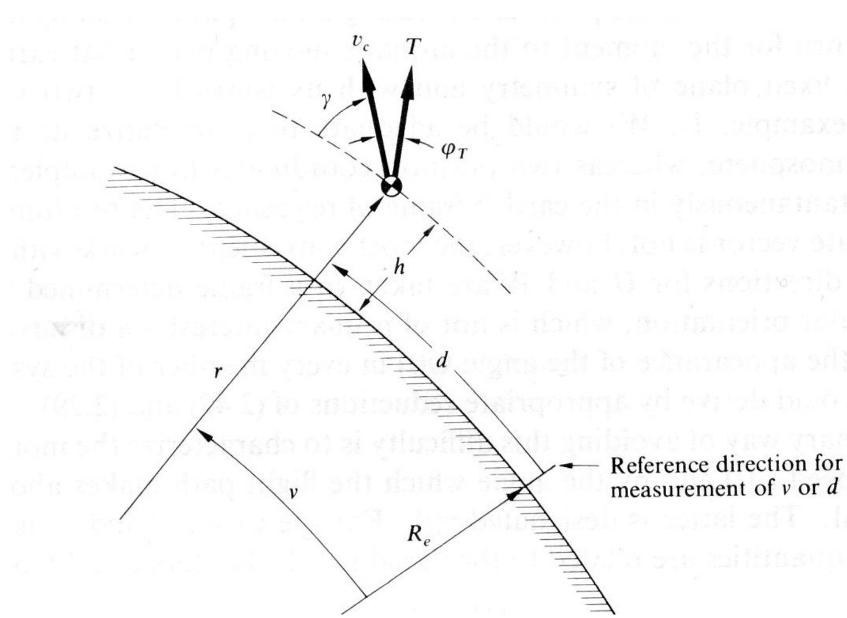

$$m\dot{v}_c = -mgsin(\gamma) - D, \; D = \frac{1}{2}C_d A \rho v_c^2$$

$$mv_c\dot{\gamma} = -mgcos(\gamma) + \frac{mv_c^2 cos(\gamma)}{r} + L$$

$$\dot{\nu} = v_c\frac{cos(\gamma)}{r}$$

$$\dot{r} \equiv \dot{h} = v_c sin(\gamma)$$

# Ballistic Reentry




For ballistic reentry $L = 0$, and the equations for the motions become

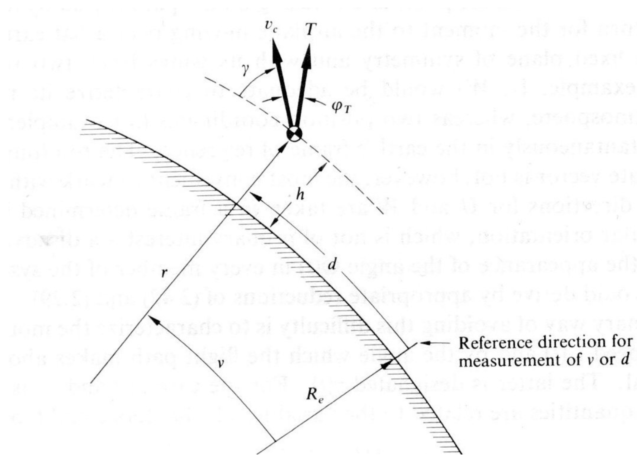
$$m\dot{v}_c = -mgsin(\gamma) - D, \; D = \frac{1}{2}C_d A \rho v_c^2$$

$$mv_c\dot{\gamma} = -mgcos(\gamma)$$

$$\dot{h} = v_c sin(\gamma)$$

$$\dot{d} = v_c cos(\gamma)$$

# Ballistic Reentry




To analyse the shape of the trajectory as a function of the altitude, we can use $h$ as an independent variable instead of $t$

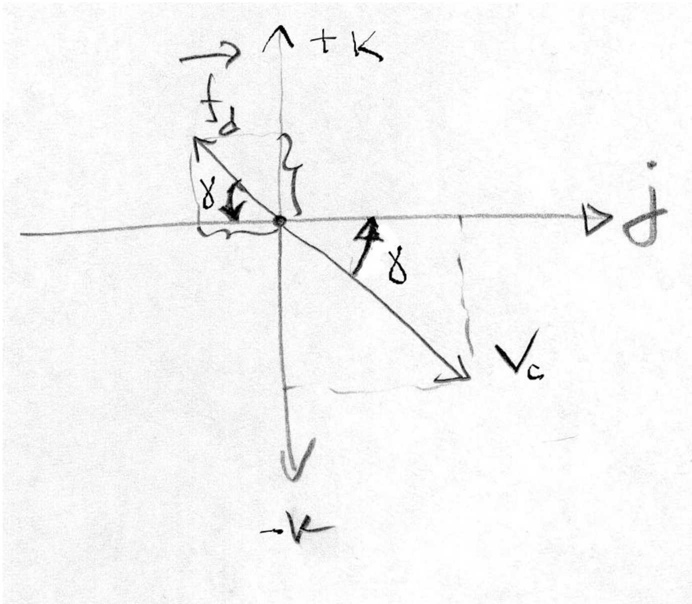
$$mv_c \frac{\dot{v}_c}{\dot{h}} = -mg - \frac{D}{sin(\gamma)}$$

$$\frac{\dot{\gamma}}{\dot{h}} = -\frac{gcot(\gamma)}{v_c^2}$$

$$\frac{\dot{d}}{\dot{h}} = cot(\gamma)$$

For steep entry angles, $-\gamma \approx 90^0$, $cot(\gamma)$ is small and the velocity is large. $\gamma$ would change very little and the downrange distance $\dot{d}/\dot{h}$ is a straight line.

# Ballistic Reentry – vehicle deceleration



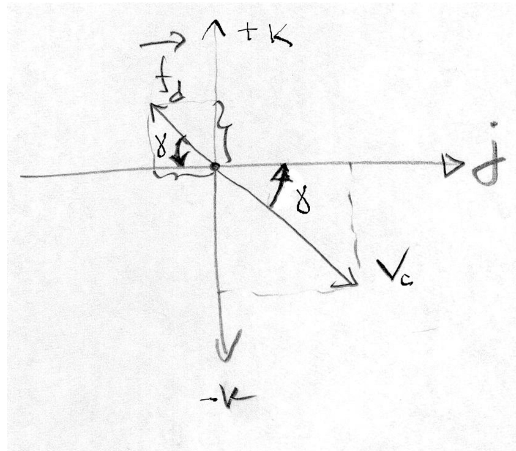

To find the de-acceleration (analytically) experienced from the vehicle (very important for the construction of the vehicle and the contents), we have to calculate the right hand side of

$$\frac{dv_c}{dt} = -\frac{D}{m}$$

In the figure above, the magnitude of the drag force is

$$|\overrightarrow{f_d}| = D = \frac{1}{2} C_d A v_c^2 \rho$$

# Ballistic Reentry – vehicle deceleration




We can write the components of this force along $\{k\}$ and $\{j\}$ as

$$\overrightarrow{f_d} = \{-Dcos(\gamma)\}j + \{Dsin(\gamma)\}k$$

The corresponding acceleration using the spacecraft's mass is

$$\overrightarrow{a} = \frac{\overrightarrow{f_d}}{m} = \{-\frac{D}{m}cos(\gamma)\}j + \{\frac{D}{m}sin(\gamma)\}k =$$

$$= \{-\frac{C_d A v_c^2 \rho}{2m}cos(\gamma)\}j + \{\frac{C_d A v_c^2 \rho}{2m}sin(\gamma)\}k$$

$\frac{m}{C_d A} = \beta =$ ballistic coefficient of the vehicle.

# Ballistic Reentry – vehicle deceleration

It can be shown that the maximum deceleration is

$$a_{max} = \frac{v_{c0}^2 sin(\gamma)}{2H}\frac{1}{e}$$

where :

$H$ is the scale height

$v_{c0}$ is the initial velocity

$\gamma$ is the flight path angle

# Vehicle deceleration and atmospheric density

We calculated previously that a vehicle subject to drag force only experiences deceleration

$$\overrightarrow{a} = \frac{\overrightarrow{f_d}}{m} = \{-\frac{D}{m}cos(\gamma)\}j + \{\frac{D}{m}sin(\gamma)\}k =$$

$$= \{-\frac{C_d A v_c^2 \rho}{2m}cos(\gamma)\}j + \{\frac{C_d A v_c^2 \rho}{2m}sin(\gamma)\}k$$

For steep entry with $-\gamma = 90^0$ we have

$$\overrightarrow{a} = \{\frac{C_d A v_c^2 \rho}{2m}sin(-90^0)\}k = -\frac{C_d A v_c^2 \rho}{2m}k$$

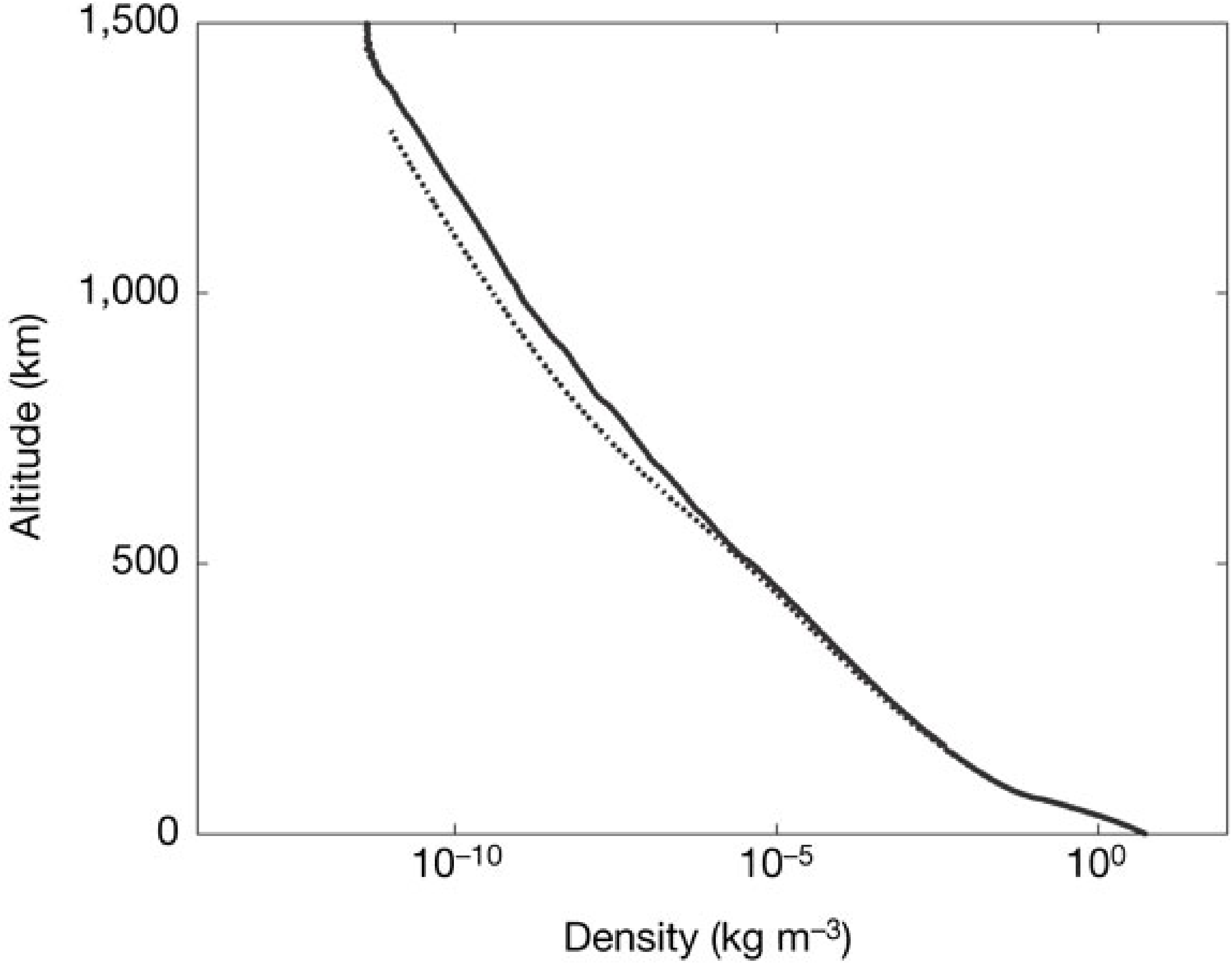
# Vehicle deceleration and atmospheric density

Knowing the instantaneous value for the acceleration from an on-board accelerometer and the velocity of the vehicle (by integrating the acceleration signal), we can calculate the profile of the atmospheric density of the planet

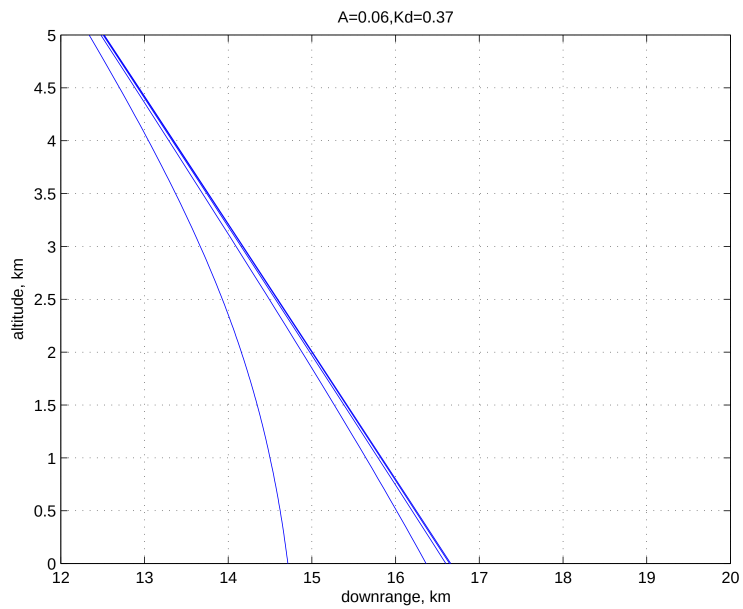
$\rho = \frac{m}{C_d A} \frac{2a}{v_c^2}$, in [kg/m3]

Example:$A = 10m^2, C_d = 1, v_c = 1km/s, m = 100kg, a = 20g$.

# Vehicle deceleration and atmospheric density



# Low drag reentry

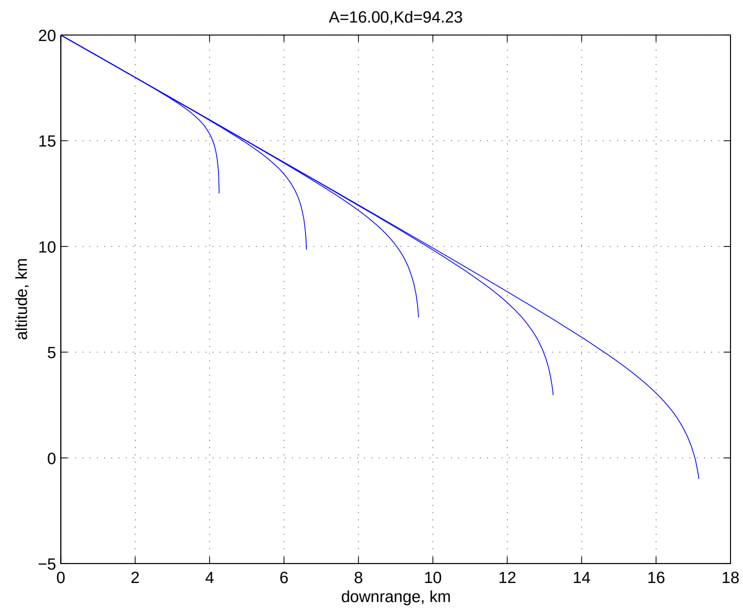



Low drag reentry $K_D = -\frac{C_D A H}{m sin(\gamma)}$ is important for ballistic missile applications.

Low drag reentry minimises the trajectory curvature and hence disturbance effects on vehicle, i.e. low targeting error.

Low drag reentry minimises atmospheric heating

# High drag reentry




Planetary entry probes are designed to have high values for $K_D = -\frac{C_D A H}{m sin(\gamma)}$

Since

$$a_{max} = \frac{v_{c0}^2 sin(\gamma)}{2H} \frac{1}{e}$$

small $\gamma$ leads to small deceleration !

# Ballistic Reentry

For ballistic reentry, the lift was set to $L = 0$, and the equations for the motions were reduced to

$$m\dot{v}_c = -mgsin(\gamma) - D$$

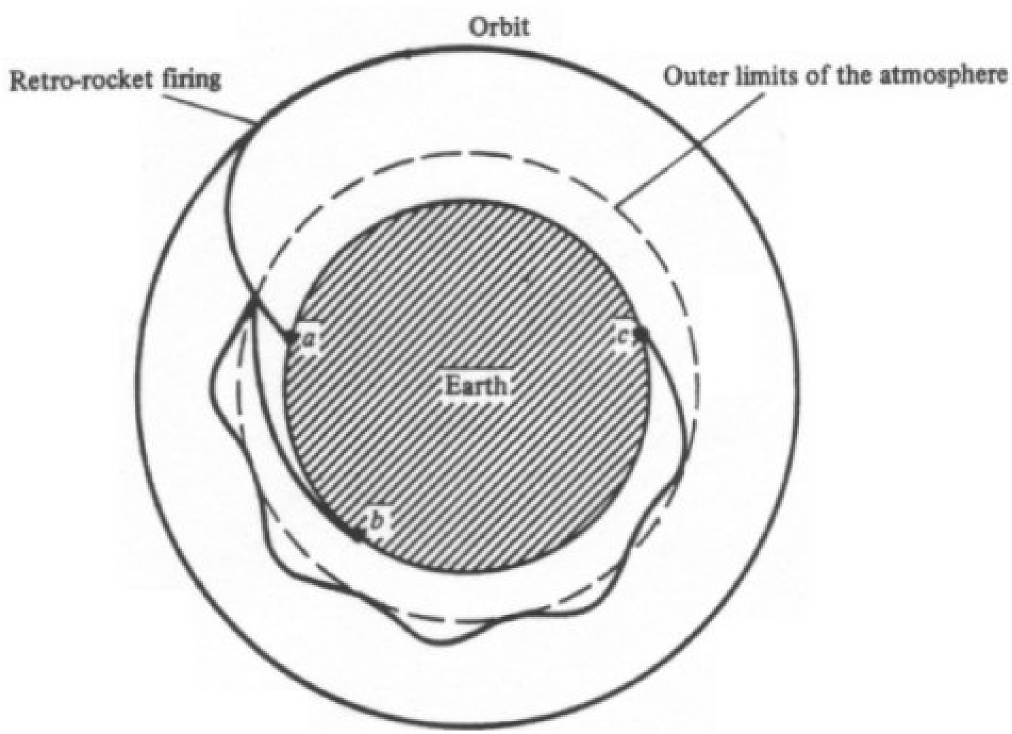
$$mv_c\dot{\gamma} = -mgcos(\gamma)$$

$$\dot{h} = v_c sin(\gamma)$$

$$\dot{d} = v_c cos(\gamma)$$

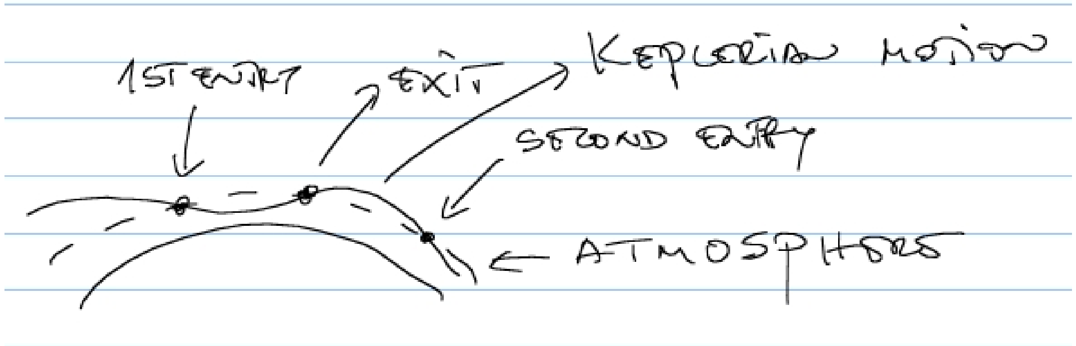
Although we can calculate and predict the trajectory – we do not have control over the trajectory ! Still Ballistic reentry remains the simplest and possibly the safest method. The limitation is that it works only for small-size probes.

# Skip Reentry




The idea dates from the World War II; Euger Sanger, a Swiss engineer, proposed a vehicle that after launch from Germany would be able to bomb the US and then return to the launch site. This would have been possible with efficient aerodynamic design.

# Skip Reentry




Adding a nonzero value for the lift, $L > 0$, change the equations to

$$m\dot{v}_c = -mgsin(\gamma) - D$$

$$mv_c\dot{\gamma} = -mgcos(\gamma) + L$$

$$\dot{h} = v_c sin(\gamma)$$

$$\dot{d} = v_c cos(\gamma)$$

When the Lift component is dominant over gravity, the flight path will be turned upward with $\dot{\gamma} > 0$. The vehicle enters the atmosphere, reaches a minimum altitude and then pulls up and eventually exists the atmosphere at a reduced speed.

# Skip Reentry

The lift and the drag forces are:

$D = \frac{1}{2} C_D A \rho v_c^2,$

$L = \frac{1}{2} C_L A \rho v_c^2$

And for exponential atmosphere flight, these become

$D = \frac{1}{2} C_D A v_c^2 \rho_0 e^{-h/H},$

$L = \frac{1}{2} C_L A v_c^2 \rho_0 e^{-h/H}$

$C_D$ and $C_L$ are the drag and lift coefficients,

$\rho_0 = 1.2366 kg/m^3$ is the base atmospheric density (Earth) and

$H = 7.1628 km$ is the height scale (Earth)

# Skip Reentry – velocity change

To analyse the velocity, we change the independent variables.

$$m\dot{v}_c = -\tfrac{1}{2}C_D A v_c^2 \rho_0 e^{-h/H}$$

$$m v_c \dot{\gamma} = \tfrac{1}{2}C_L A v_c^2 \rho_0 e^{-h/H}$$

$$m\dot{\gamma} = \tfrac{1}{2}C_L A v_c \rho_0 e^{-h/H}$$

$$m\dot{v}_c \dot{(m\gamma)}^{-1} = \frac{-\tfrac{1}{2}C_D A v_c^2 \rho_0 e^{-h/H}}{\tfrac{1}{2}C_L A v_c \rho_0 e^{-h/H}} = -\frac{C_D}{C_L} v_c$$

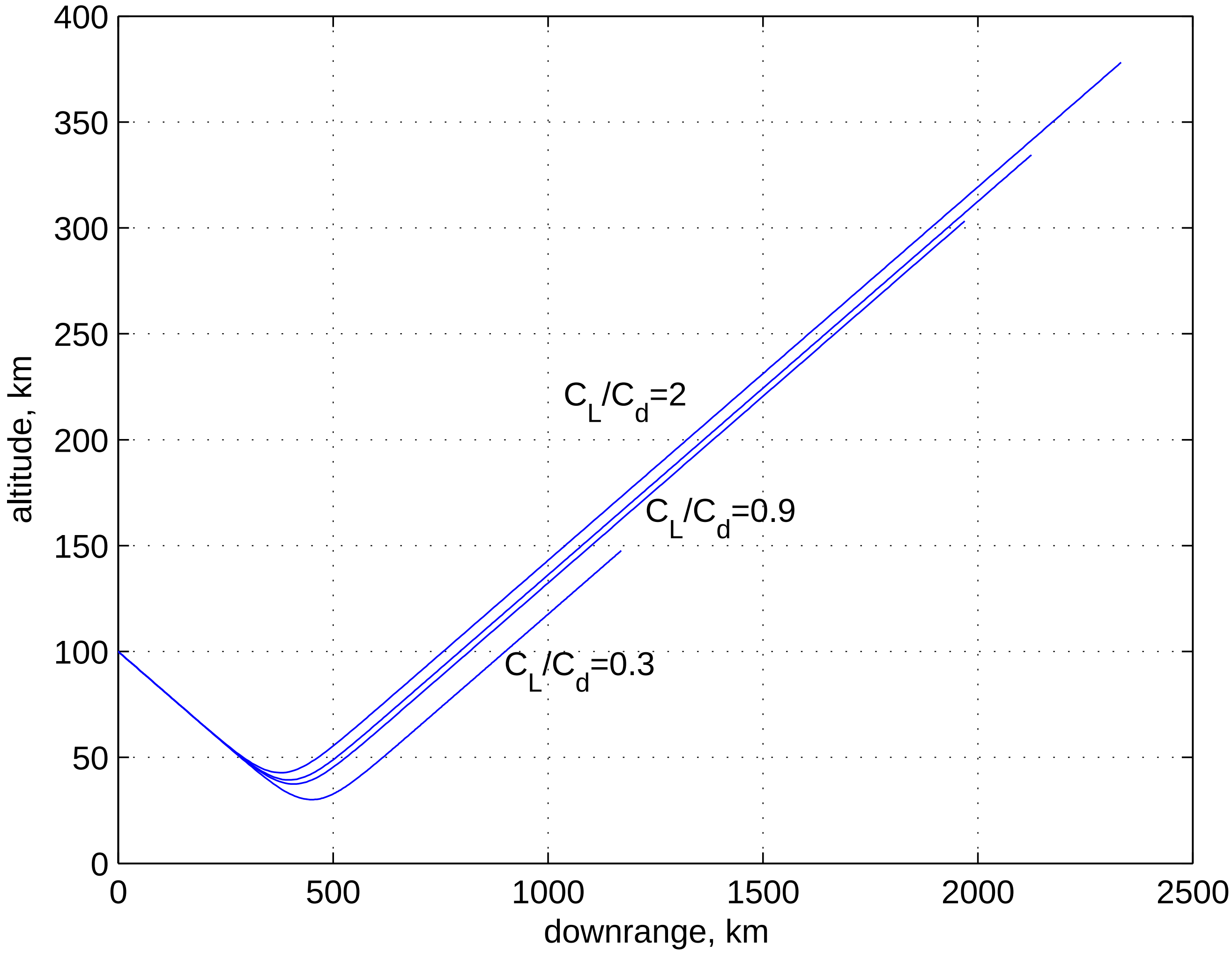
$$\frac{\dot{v}_c}{\dot{\gamma}} = -\frac{C_D}{C_L} v_c$$

$$\int_{v_0}^{v} \frac{1}{v_c} dv_c = \int_{\gamma_0}^{\gamma} -\frac{C_D}{C_L} d\gamma$$

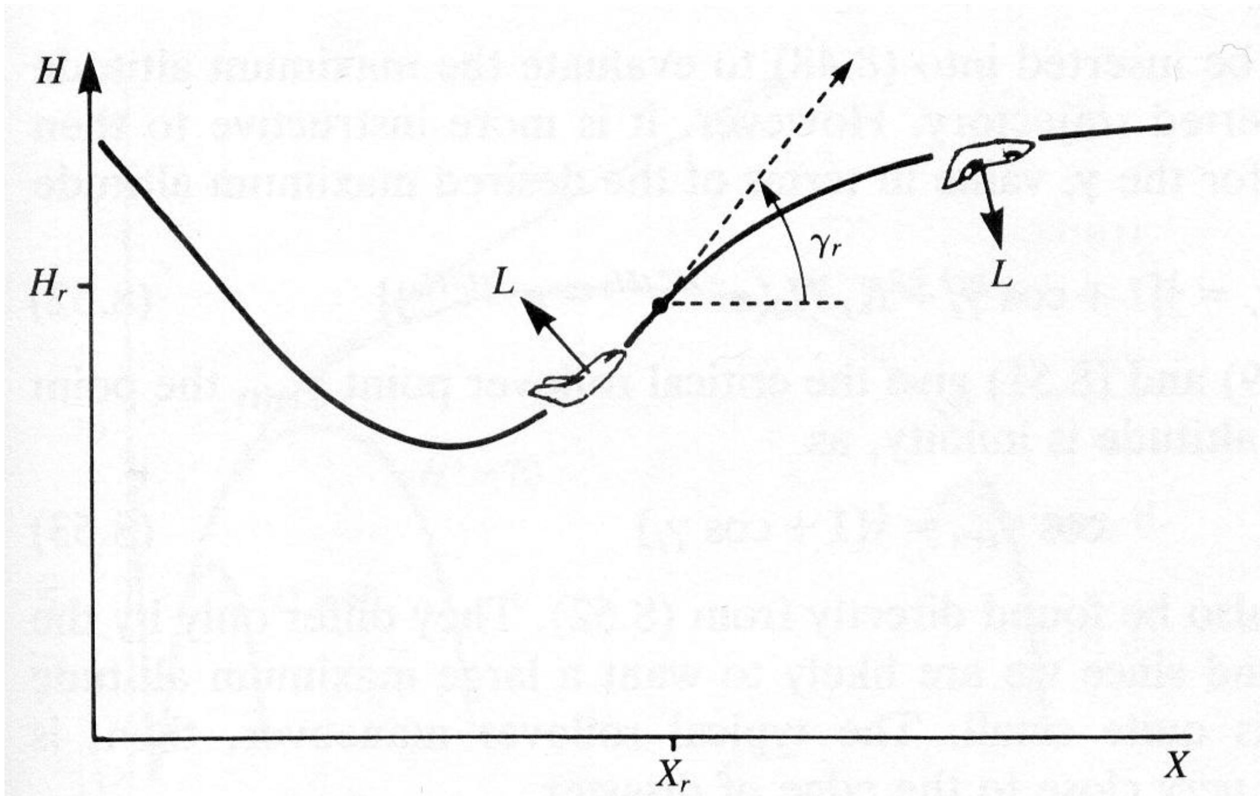
$$v_c = v_0 e^{-\frac{C_D}{C_L}(\gamma-\gamma_0)}, \gamma_0 < 0$$

When the flight path angle is monotonically increasing, the lift-to-drag ratio $C_L/C_D$ determines how fast the velocity would decrease with $\gamma$.

# Skip Reentry – velocity change



# Skip Reentry – Control over lift



# Skip Reentry – Guidance

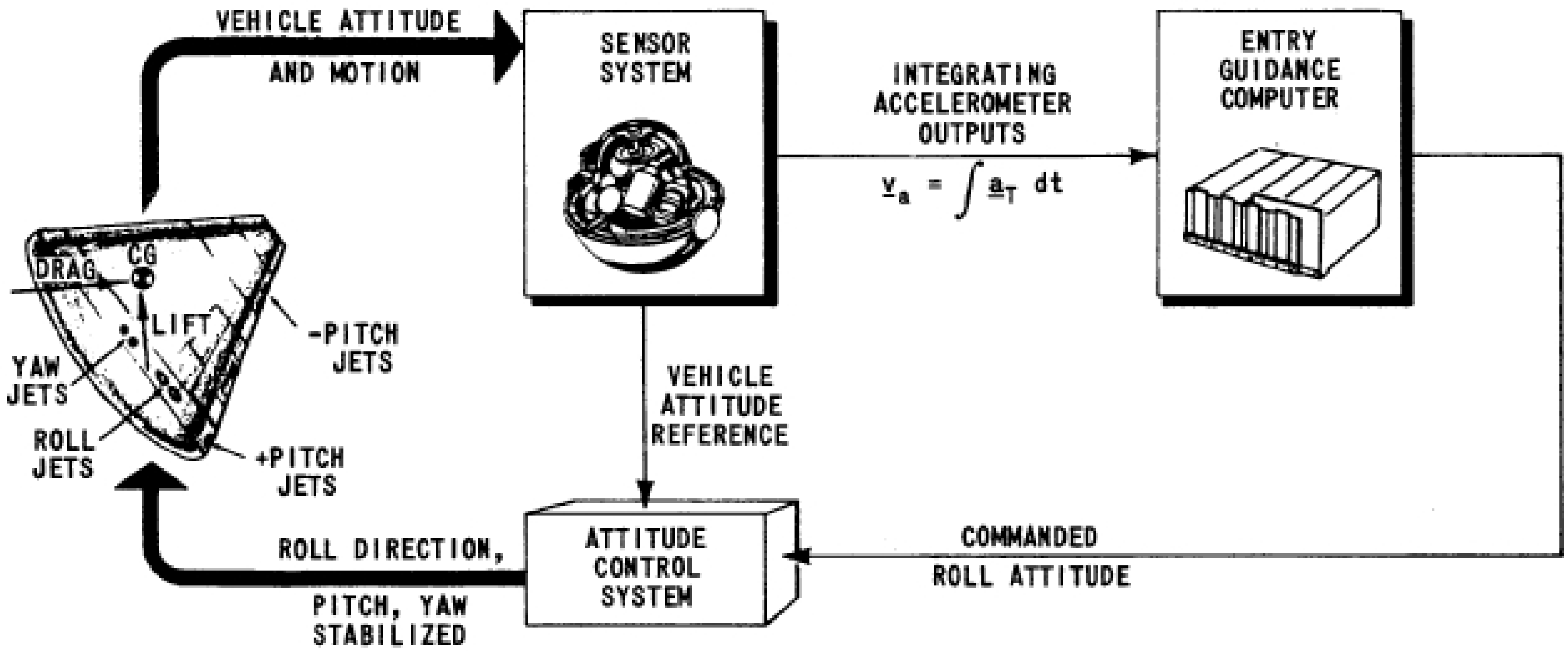



FIGURE 1. ENTRY GUIDANCE SYSTEM

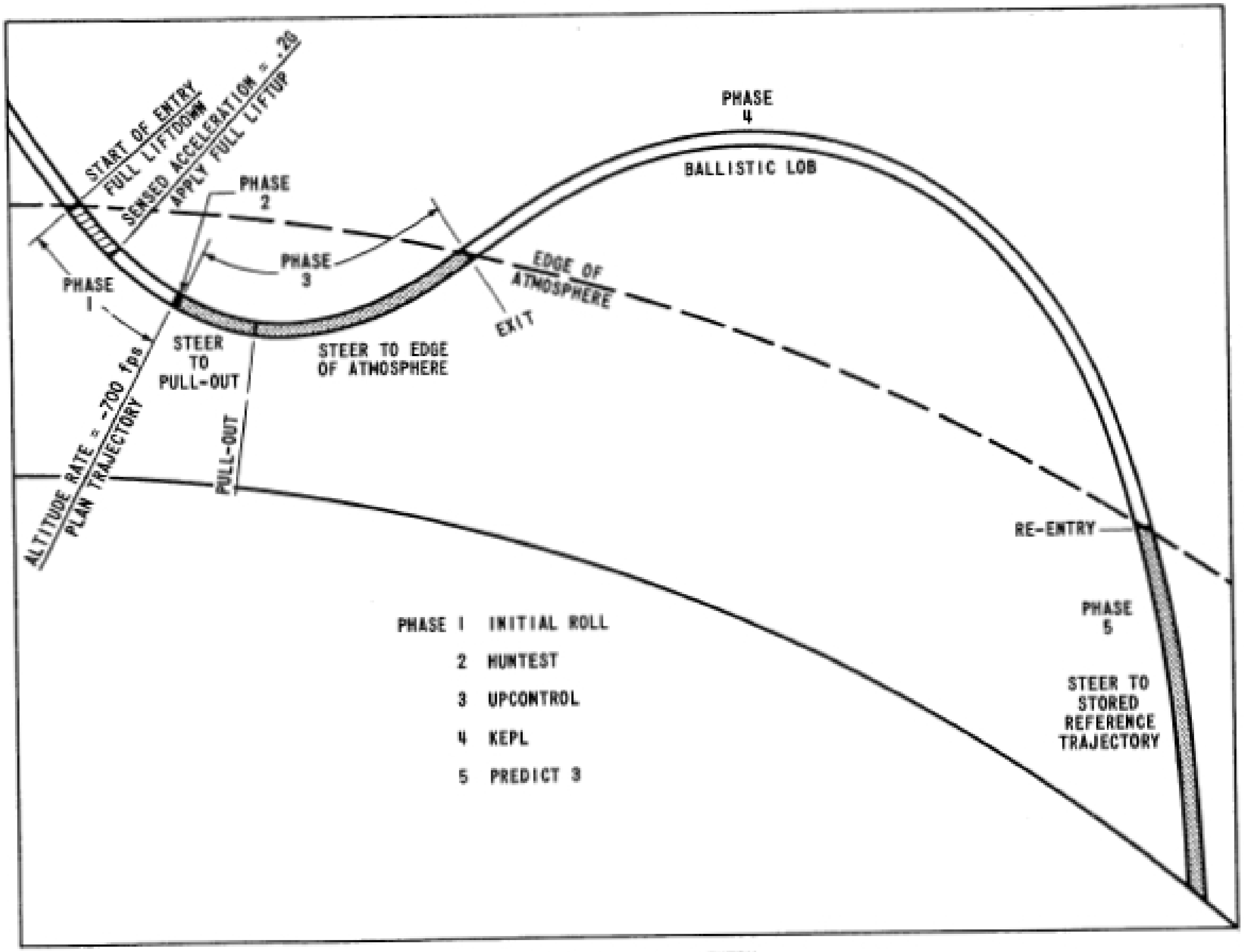



FIGURE 2. TYPICAL ENTRY

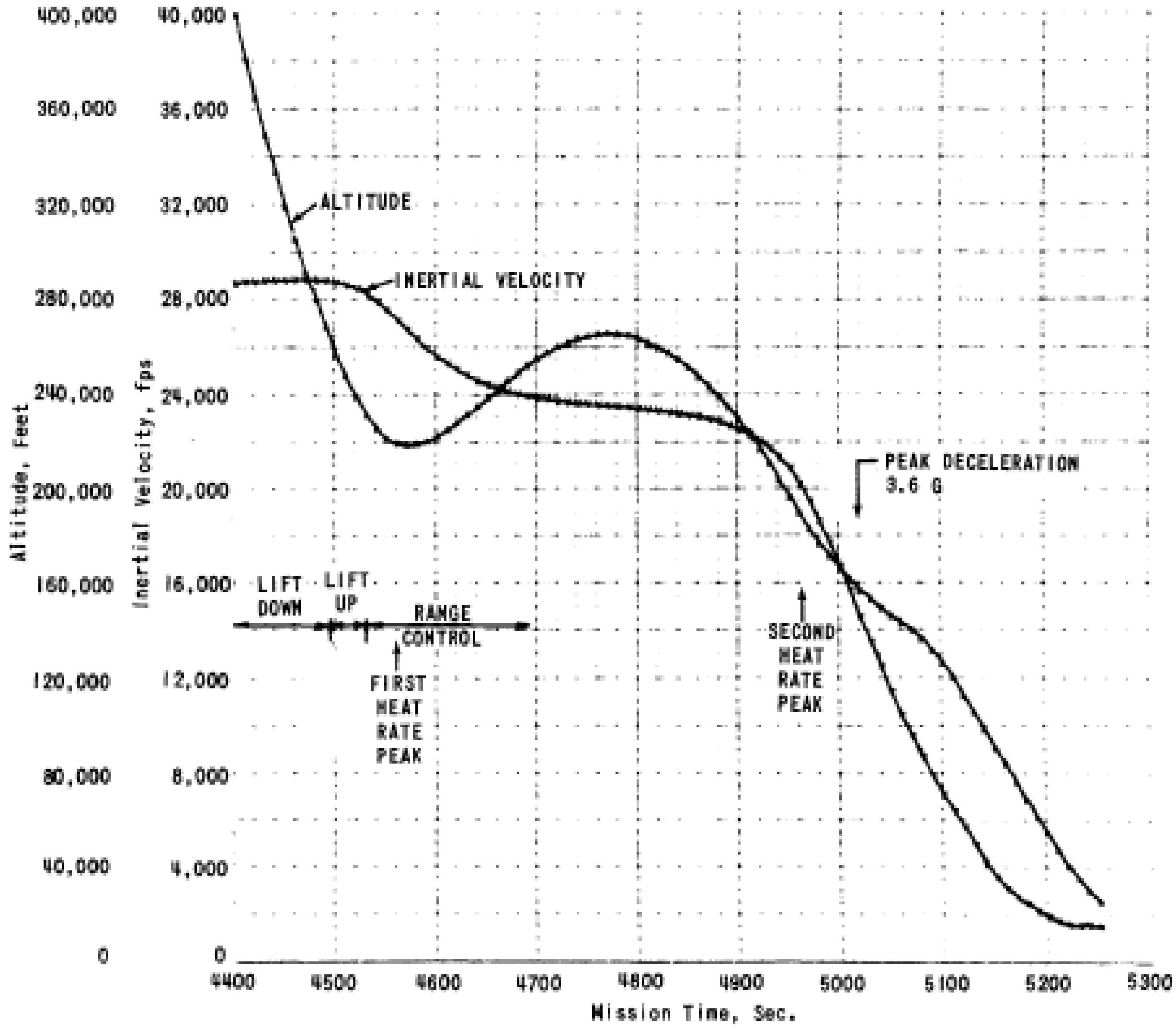



FIGURE 19. ALTITUDE AND INERTIAL VELOCITY FOR THE NOMINAL TRAJECTORY (10 SEC BETWEEN MARKS ON PLOTS)

# Skip Reentry

Ballistic reentry is always available for backup and Soyuz 33 (1979) used steep ballistic reentry due to failure in the engine with deceleration of up to 10g (rather than typical for this type of mission 3-4g). Recently the Soyuz TMA-1 (2002) used ballistic reentry with decelerations of up to 8g due to failure in the control system.

Lift to drag ration determines the downrange distance and hence landing site. This was a big issue during the Cold War as a proper trajectory design and control was needed to ensure that the capsule will not land in "enemy territory". The Soviet Zond4 probe, for example, was deliberately destroyed in 1968 because it was falling into American hands.

# Problem

A Moon reentry vehicle uses a powered descent. The total thrust is 150 N and the thrusters (Isp=320 sec) are rigidly mounted on the body; At a given moment of time, it is known that the velocity of the vehicle is 150 m/s at an altitude of 15 km. For this vehicle:

a) Write down the equations of motions for the center of mass.

b) If the vehicle's dry mass is 250kg and the fuel is 80kg, how long would it take to burn the fuel in full?

c) Derive the equations of motion for the case when the flight-path angle is approximately -90 degrees.

d) Using the assumptions made in c), write down the altitude and the velocity as a function of time.