Appendix 1 Simple Presentations of Bell's Inequalities

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Bell's papers and various theorems were critical to the continued development of quantum mechanics. Before his first papers became known in the physics community (1964), the issue of the foundations of the theory had stagnated after the debate stimulated by the EPR paper in 1935 except for a small flurry of interest when Bohm reinvented the pilot wave theory of de Broglie in 1952.

Despite the importance of Bell's work, it was slow to be absorbed by the community because it was notoriously hard to explain his approach to debunking hidden variables, nonlocality, and other concepts intended to defend the foundations of quantum theory. Even the great Feynman had trouble digesting the ideas involved. Mermin recounts getting a letter from Feynman thanking him for writing a decipherable explanation in a journal for physics teachers.[1]

It has now been 60 years since Bell's philosophy emerged. Most of the 100s of books on physics or the philosophy of quantum mechanics now attempt to relate his famous inequality to everyday experience. All sorts of approaches have been tried.

- Colors of socks [2]
- Marriage counseling[3]
- Venn diagrams[4]
- Experiments at Venus and Mars[5]
- The husband, the wife, and seafood[6]
- Coins [7]
- Rock, paper, scissors [8]
- Twins at bars in Cambridge, England and Cambridge, Massachusetts[9]

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[1] Mermin p. xv
[2] Bertlmann et al p. 43 and https://spectrum.ieee.org/what-is-quantum-entanglement
[3] d 'Espagnat, Bernard ***The Quantum Theory and Reality*** 1979 Scientific American
[4] Maccone, L. American Journal of Physics. 81 (11), November 2013
[5] van Dommelen Section 8.2
[6] Greenstein Chapters 7 and 12
[7] Visualization of quantum entanglement and [4]
[8] https://aeon.co/essays/our-simple-magic-free-recipe-for-quantum-entanglement
[9] Kaiser p. 51

The most important thing to know is that you don't need to understand quantum mechanics to see the logic of Bell's work. It's just careful analysis and the willingness to accept descriptions of what happens in various real experiments.

My version, adapted from Gouesbet[10] and Brody[11]

Let's consider the entire population of the United States. It is about equally divided between men and women, and some of each are smokers. Let's further divide the population into those older than 40 (this is an arbitrary break, as you shall see). So, we have now divided the entire population into three categories: sex, smoking habits, and age. The following figure shows the assumptions we have made.

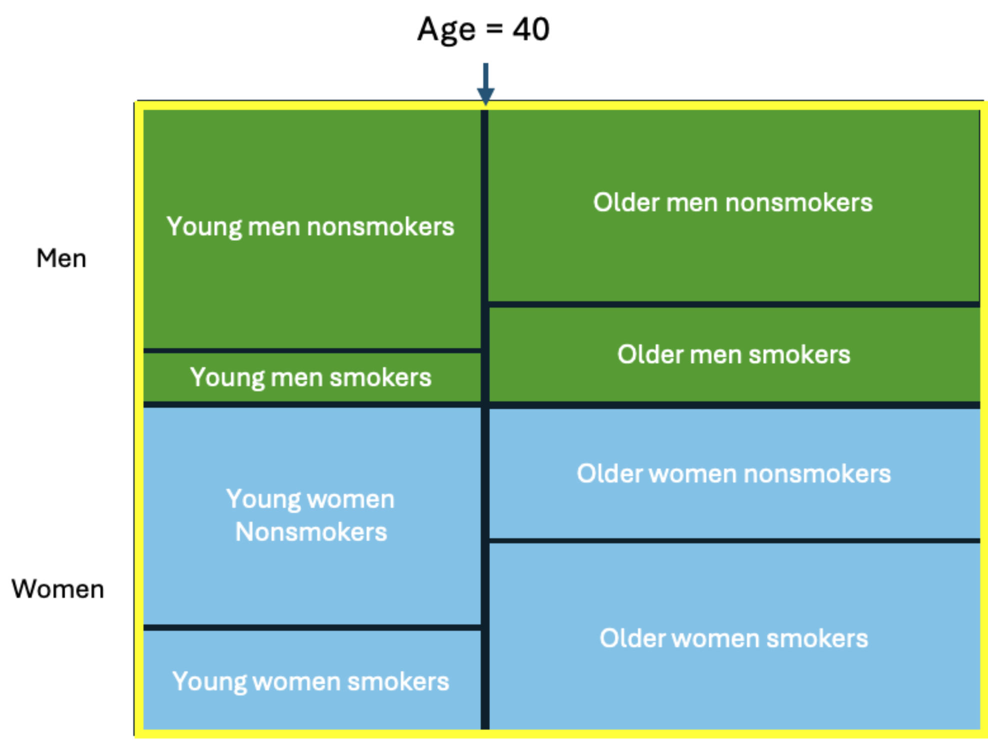


Figure 1 *This shows the population designated by age, sex, and smoking habits. The yellow box indicates the total population. I have made assumptions about the fraction of each group's smoking habits, but my choices do not impact the logic of what we are doing.*

Now consider the following theorem (which we will prove).[12]

The number of women younger than forty years of age is smaller than or equal to the number of smoking women augmented by the number of nonsmoking individuals younger than forty.

[10] Gouesbet p. 522
[11] Brody Entire book
[12] d'Espagnat p. 27

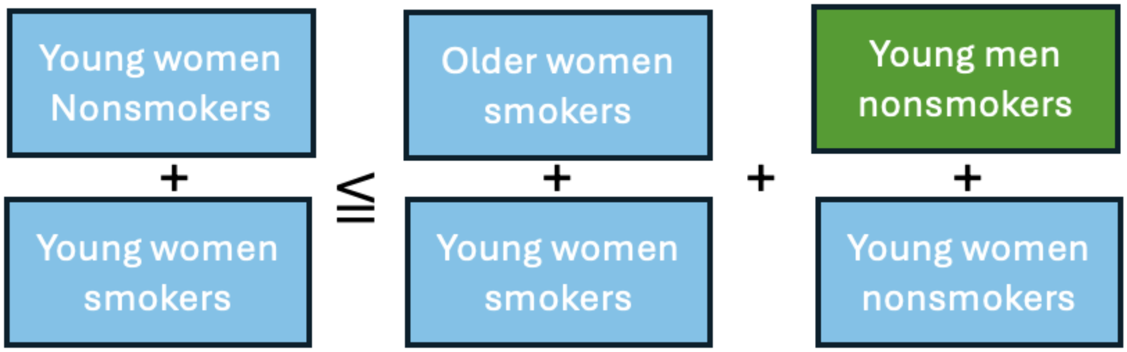
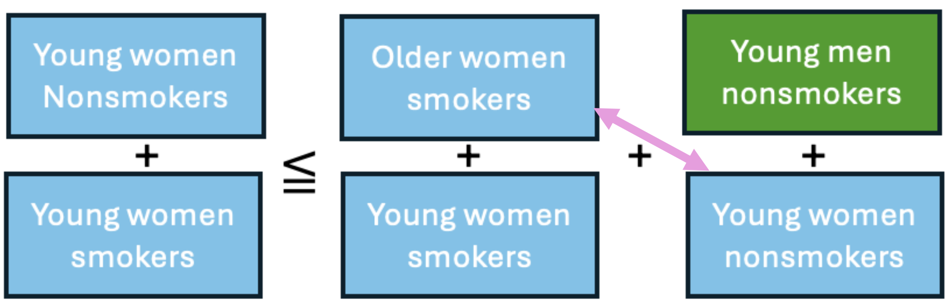


Figure 2 *The graphical form of the equation.*

We can prove the theorem graphically. Start with the graphical version of the statement.

Swap two boxes (young women nonsmokers and older women smokers).



And we get

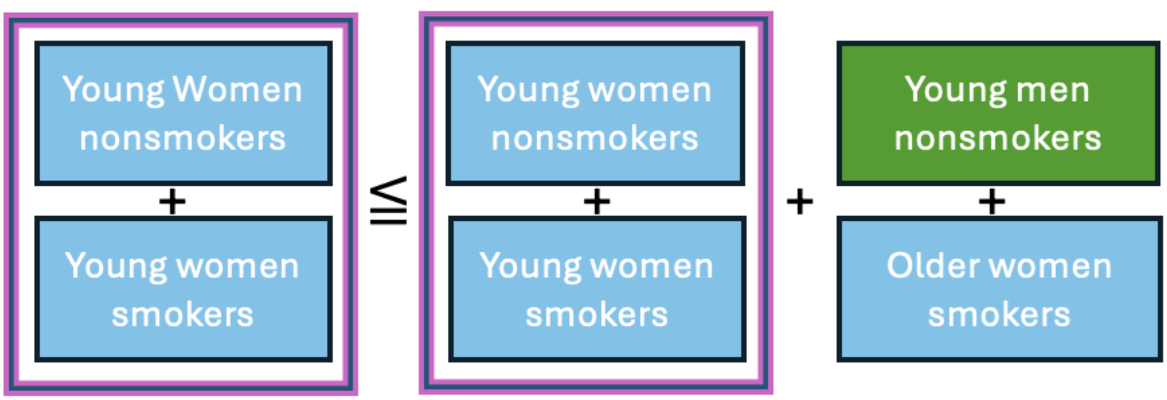


Figure 3 *A new form of the equation*

The groups in the magenta boxes are the same and we've added additional subgroups (Young men nonsmokers and older women smokers). The reasoning here is that the left side of the equation is the total number of women. It includes both smokers and nonsmokers. The logic we started with also includes people in other subgroups.

The graphical equation applies no matter what fraction of each population smokes. This is a version of a Bell inequality. Now we can do numerical experiments by dividing the total population of the US into three nearly equally sized groups (1, 2, and 3). Since the groups are large, each group will have (on average) the same number of women, women smokers, people over 40, etc.

Now that we have confirmed the theorem through the equation in Figure 2, we can create a mathematical form. We will use *P* to indicate the probability of each group being found in the three groups. The graphical equation can be written as follows. Note that we get different data (age, sex, smoking habits) from three groups (1, 2, or 3). We use the abbreviations YW for young women, WS (women smokers), and NSYP for nonsmoking young people.

$$P(\mathrm{YW})_1 \leqq P(\mathrm{WS})_2 + [P(\mathrm{NSYP})]_3$$

You might wonder if the equation is true since data is being extracted from three different groups but remember that these are large groups and will have the same statistics.

Let's rename the three categories of people as

A: old (+) or young (-)

B: male (+) or female (-)

C: smoker (+) or not(-)

Now the equation is in a general form.

$$P(\text{A- and B-})_1 \leqq P\,(\text{C+ and B-})_2 + P(\text{A- and C-})_3$$

This means P (all young women)$_1 \leqq P$ (women smokers)$_2$ + P (young nonsmokers)$_3$

Let's stop here and review what we've done. We have a general equation for three groups containing individuals with three characteristics (age, sex, and smoking habit) that can have only one of two possible values (old or young, male or female, smoking or not).

The logic of this equation has been reviewed for decades and is irrefutable. We can rest assured that it explains commonly experienced situations, even if the details are changed. We could have chosen chickens and categorized them by breed (one of two), living conditions (caged or free range, and country (US or Brazil). The equation would still be valid.

Now we need to put this into a quantum context. To do so, we will mimic the divisions used above. We chose three different ways to distinguish members of the groups: sex, smoking habits, and age.

For the quantum case, let us recall the focus of the EPR paper; it started the whole effort to resolve issues with the foundations of quantum mechanics and the nature of reality. The EPR paper stimulated Schrödinger to write an extensive paper on entanglement and the intersection of quantum mechanics and relativity, a topic currently addressed as *locality*[13].

[13] https://materias.df.uba.ar/f4Aa2012c2/files/2012/08/Schrod_cat.pdf

With those thoughts in mind, we are ready to use the analysis above to create a quantum version of the inequality related to the smoking and nonsmoking population. To convert the equations above into the context of quantum mechanics, we pick an experimental approach. The most convenient to think about is Bohm's version of the EPR experiment; Bohm's version would have us measure spin orientation in an experiment in which a central source *S* produces pairs of entangled photons (ν_1 and ν_2). The spins are correlated; in systems where the total spin is zero, if one is detected in the 'up' orientation, the other is 'down'. The particles fly away from each other. Each encounters a polarization sensor with variable orientation. The polarization sensors are so far apart (and from the source) that no signal from either the sensor or photon can signal its situation to its partner. The orientation of the polarization sensors is altered during the flight of the photons to further close an important loophole in the experiment.

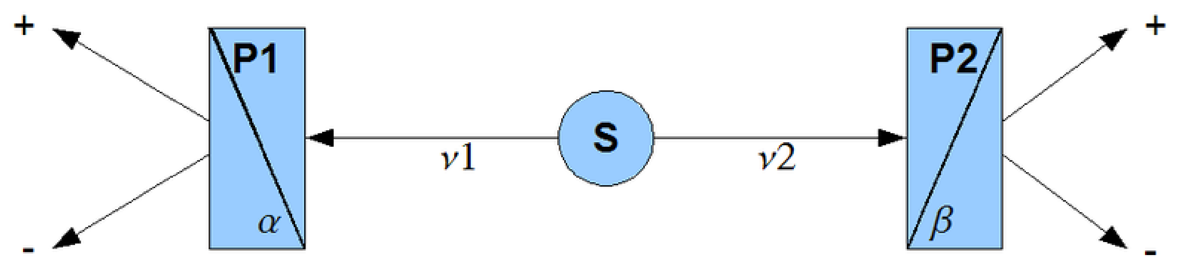


Figure 4 *Ideal EPR test with a particle source in the middle and detectors P_1 and P_2 oriented with angles α and β. A coincidence computer keeps track of the angles and the spin orientation as each pair of entangled photons pass through the experiment. To be sure that detectors P_1 and P_2 cannot communicate their orientation, their separation is so large that no signal can be conveyed between them from the time the first detection is made to the time the last detection is made. Perhaps P_1 is 1 meter from S and P_2 is 1 light-year from S.* *CC-BY-SA-2.5*[14]

Just as in the case of smokers, we can create a huge database of information as millions of pairs of entangled photons go through the experiment. Thus, we have the equivalent of groups 1, 2, and 3 in the smoking case. In the case of smokers, we chose three categories of description. In the quantum case, data is binned for three relative orientations of the polarization sensors: case A with parallel sensors, case B with sensors at 45°, and case C with sensors at right angles.

Before we start, let us repeat the equation from farther up the page so we can refer to it.

$$P(\text{A- and B-})_1 \leqq P\,(\text{C+ and B- })_2 + P(\text{A- and C-})_3$$

Bell's hypothesis can then be tested using the equation as stated below. Note the parallel of this wording with the original theorem.

The probability of a photon not passing at 0° (A-) and the other not at 45° (B-) is less than or equal to the probability of a photon passing at 90° (C+) and the other not at 45° (B-) plus the probability of a photon not passing at 90° (A-) and the other not at 90° (C-)

We can compare the results of experiments against this version of the inequality and we find that

[14] https://commons.wikimedia.org/wiki/File:Aspect_epr.png

Bell's inequality is violated by every quantum experiment.

The question is why? Brody provides an extensive discussion of the possibilities which I summarize in the following discussion.[15] The same is true of d'Espagnat.[16] The central issue is the assumptions we have made about reality and locality.

Realism says that objects have properties that exist regardless of whether anyone (or anything) is observing them. Realism also says that observation reveals properties that the objects had all along. Locality is a commonly accepted feature of physics. It says that the measurement of one object can't affect the measurement of another object that is arbitrarily far away (due to the influence of the noninfinite speed of light). The combination of these two ideas (local realism) is a tenant of the philosophy of the macro world; things have properties independent of our measurement of those properties and conducting such a measurement cannot alter the properties of anything that is not *local*, that is, not in causal contact with the measured object.

To explain the puzzling result that all quantum experiments violate the Bell inequality we derived from common experience, we have to consider our assumptions about local reality. Brody leads us through the choices we've made (without realizing them). Here is the key point, in one tight paragraph (with my minor edits).

*"When we rewrote the inequality to apply to entangled photons, we implicitly assumed that every photon all along has properties that predetermine whether it will pass through a polarizer at any chosen angle (just as …Indeed, the **photons would have to satisfy the inequality if they had fixed polarization properties all along** (realism), and if each photon was uninfluenced by the other photon's polarizer (locality). But in fact, the entangled photons violate the inequality."*

We can be confident that the inequality would not have been violated if the photons behaved like the age of the subjects in the smoking survey. If photons obtained and retained their specific polarization as soon as they entered the experiment (as demanded by realism) and were not induced to change when their partner was measured (locality), no violation would have occurred.

The *coup de grace* of this whole topic is that when detailed quantum computations are applied to the two-photon experiment, the theory and measurement agree that inequality must be violated.[17]

The resolution of this mystery might come from one of three places: locality is false, realism is false, or both are false.

Consider realism. It demands that every photon (or any other quantum particle) have measurable specific values of its properties (like sex or age) from the instant of its creation. The act of measurement takes place in an experiment in which the photon enters without knowing what it will encounter (the orientation of the polarization sensors). In a world with no realism, photons

[15] Brody Ch 4
[16] d'Espagnat Section 4.7
[17] Brody p. 69

must contain a mix of all possible orientations and let the interaction with the experiment force a specific value. That specific value would be determined by the orientation of both polarization sensors since the photons are entangled and each undergoes a measurement. Locality would be at risk since the coupling of the entangled photons would require communication of the situation of each to the other regardless of how far apart they are. What happens if they are both detected at the same time? Which has priority? Why would the orientation of a distant polarization filter impact the polarization properties of a photon in the experiment?

So, we have considered rejecting both realism and locality to explain why inequality is violated. What happens if both are rejected? Brody and many others have chosen this conception of quantum mechanics. This requires accepting 'spooky action at a distance' with the immediate impact of one measurement on the other no matter how far apart.

Again, quoting from Brody[18]

"My claim is that both photons are transformed by the first observation of either photon. Thus this transformation can never be observed; we can't perform any observation prior to the first observation. So we can never watch one particle change in response to the measurement of its twin. The innermost workings of nature remain forever out of reach. The quest for complete understanding is always an unscratchable itch. The only fact that's (almost) certain is that local realism cannot account for measured results."

All this questioning has generated a continuing debate among physicists who care about the foundations of quantum mechanics. The *gedankenexperiments* of the 1930s have now been implemented and they have proven that the quantum world is stranger than the authors of EPR imagined. And, as we have mentioned before, of all the possible explanations proposed in the EPR paper, the **one that Einstein would have liked the least is the one we get from experiments**.

It is a curious twist of fate that the EPR paradox, which assumed locality in order to prove realism, led finally to the demise of locality and left the issue of realism undecided—the outcome (as Bell put it) Einstein would have liked least. Most physicists today consider that if they can't have local realism, there's not much point in realism at all..[19]

By the way, if you wonder if there are some systematic errors in the entangled photon experiments that produce the results discussed ab, consider this. GHZ experiments[20] (named for Greenberger, Horne, and Zeilinger) involve the entanglement of three photons. Each photon behaves as if it knows the details of the experiments the other two photons will encounter. Moreover, unlike the Bohm EPR two-photon experiments which require the assembly of large bodies of experimental data, each run of the GHZ experiment (involving just three quantum particles) creates definitive

[18] Brody p. 121
[19] Griffiths p. 571 and footnote 14.
[20] https://en.wikipedia.org/wiki/GHZ_experiment

confirmation of the correctness of quantum mechanics and violation of the associated Bell inequality.

Finally. why would we immediately attack locality and reality as we try to explain violations of Bell's inequality? One need only go back and read the EPR paper. It is filled with references to reality and relativity/locality. Then see the reaction to EPR by Schrödinger in which he first explains the concept of entanglement.

There are additional discussions of Bell's developments in Chapter 4.

Locality tells us that instantaneous communication influence between entangled particles is the only reality we can count on, regardless of the principles of relativity.

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