

Assessed Problem Sheet 3

Please hand this problem sheet in by Monday, 29 Nov, at the beginning of the lecture (12:10). As usual, you can work in groups. Each sheet can have up to 4 names associated with it. Please write everyone's name and student ID clearly onto the sheet. **Note the competition in Q1.**

- 1) What is the (classical, time-averaged) electric field caused by an H atom in the ground state, on distance scales of $\sim 10^{-11}m$? Sketch the field.

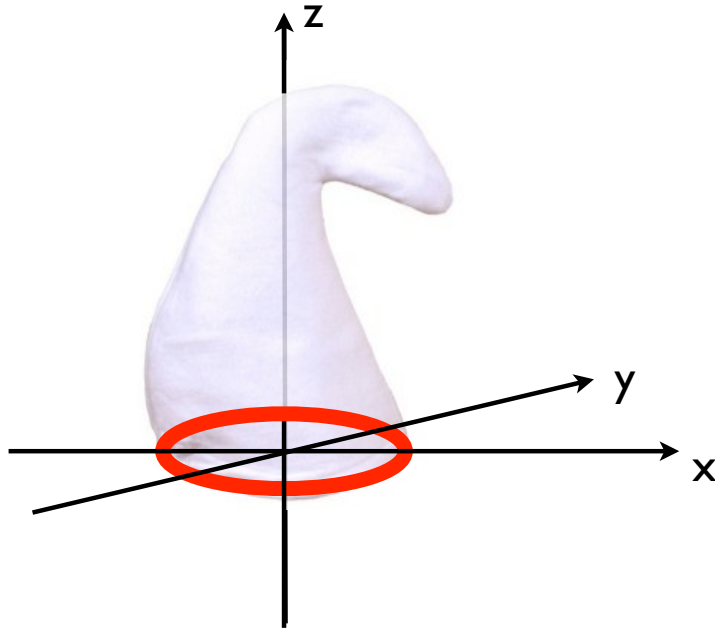
Competition: The first fully correct answer to this question wins a prize (one for each group member if handed in as a group). Over the course of the week, I will publish successively comprehensive hints how to solve this. Hint:

Derive the field from the charge distribution inside the atom. The size of an atom is $\sim 10^{-10}m$ while the size of the nucleus is $\sim 10^{-14}m$. Therefore, “on distance scales of $\sim 10^{-11}m$ ” means you can treat the nucleus as a point-particle (you know the field of that, no need to derive it), but you need to take into account the spread-out nature of the “electron cloud” described by the ground-state wave-function. Classical means: while the electron's wavefunction might be a consequence of QM, we treat calculating its field as a classical problem. And time-averaged: don't worry about any effects of the electron whizzing around, just use the “electron cloud” picture to derive a charge density.

- 2) Consider the following fields:

$$\vec{G} = \begin{pmatrix} x^3 + yz \\ y^3 + xz \\ z^3 + xy \end{pmatrix} \quad \vec{H} = \begin{pmatrix} y^2 \\ x^2 \\ 0 \end{pmatrix}$$

- (a) Show that \vec{G} has sources, while \vec{H} does not.
- (b) Which field (if any) is conservative?
- (c) Find the potential for the conservative field
- (d) Calculate the flux of \vec{G} and of \vec{H} through a sphere of radius R centered at $0/0/0$.
- (e) Calculate the flux of \vec{H} through the gnome hat surface, shown in the figure below.



This surface is bound by a circle of radius R in the $x - y$ plane (shown as a red (grey) circle) and is otherwise wobbly. Calculate the flux of \vec{H} through that surface.

- 3) More flux, and a vector potential: For the following magnetic vector potential:

$$\vec{A} = \begin{pmatrix} x^2 z - 2 - e^z \cos(x) \\ x + y - z \\ -xyz + e^z \sin(x) \end{pmatrix}$$

- (a) Find the corresponding magnetic field \vec{B}
 - (b) Evaluate the flux of \vec{B} through the upper half of the unit-sphere, bound by the unit-circle in the $x - y$ plane.
 - (c) What is the flux of \vec{B} through the full sphere, and what is it through the lower hemisphere?
- 4) Show that if \vec{P} and \vec{Q} are both gradient fields, i.e. $\vec{P} = \text{grad}\phi$ and $\vec{Q} = \text{grad}\psi$, then

$$\vec{R} \equiv \vec{P} \times \vec{Q}$$

has no sources.