

Equation for the electric field in the vertical due to a point charge:

$$E_y = \frac{kq}{r^2} \sin(\theta)$$
$$dE = \frac{k dq}{r^2} \sin(\theta)$$

From a diagram we can get  $\sin(\theta)$  into terms we can use:

$$\sin(\theta) = \frac{z}{\sqrt{x^2 + z^2}}$$

We can find the change in charge by creating a ratio between the circumference at any point “x” away from the origin divided by the surface area of the disk:

$$dq = \frac{2\pi r}{\pi r^2} Q = \frac{2Q}{r}$$

From the diagram we can determine the distance from the point on the disk to the point on the z-axis:

$$r = \sqrt{x^2 + z^2}$$

Substituting in the all the value gives:

$$dE = \frac{k \left(\frac{2Q}{r}\right)}{x^2 + z^2} \frac{z}{\sqrt{x^2 + z^2}}$$

Integrating both sides gives:

$$E = \int_0^a \frac{k \left(\frac{2Q}{r}\right)}{x^2 + z^2} \frac{z}{\sqrt{x^2 + z^2}} dx = \frac{2kQ}{z\sqrt{a^2 + z^2}}$$