**Can anybody help with group velocity simulations?**
http://www.physicsforums.com/showthread.php?t=762019

A beat superposition waveform can be '*created*' by adding the amplitudes of 2 plane waves at all values of [x], which then change as a function of time [t]. As a basic permutation, these waves might be travelling in the same or opposite directions., i.e.

[1]
$$A = A_0\left[\cos\left(\omega_1 t - k_1 x\right) + \cos\left(\omega_2 t - k_2 x\right)\right];\ \ same\ \ direction$$
$$A = A_0\left[\cos\left(\omega_1 t - k_1 x\right) + \cos\left(\omega_2 t + k_2 x\right)\right];\ \ opposite\ \ direction$$

It was assumed that the [ωt±kx] form, rather than [kx±ωt], was more appropriate where time [t] is only incrementing in the positive sense. As such, the direction of the underlying plane waves [$\omega_1$,$k_1$] and [$\omega_2$,$k_2$] would be defined by [±k]. In a non-dispersive media, where both waves propagate at [v=±1], the relationship between [ω] and [k] would be defined by:

[2] $$\pm v = \frac{\omega}{k} = \pm 1$$

So based of the arguments above, it was assumed that the implied sign [±] associated with the direction of the wave should be transferred to [k] not [ω], i.e.

[3] $$\pm k = \frac{\omega}{\pm v} \quad not \quad \pm\omega = \pm vk$$

The additive solution of both equations in [1] leads to the same composite beat waveform as expressed by:

[4] $$A = 2A_0\cos\left(\omega_p t - k_p x\right)\cos\left(\omega_g t - k_g x\right)$$

However, the sign change implied in the second equation in [1] is now hidden in the definition of the variables [$\omega_p$,$k_p$] and [$\omega_g$,$k_g$], which are reflected in the two variant solutions shown below:

| $\cos\left(\omega_1 t - k_1 x\right) + \cos\left(\omega_2 t - k_2 x\right)$ | $\cos\left(\omega_1 t - k_1 x\right) + \cos\left(\omega_2 t + k_2 x\right)$ |
|---|---|
| $\omega_p = \frac{\omega_1+\omega_2}{2};\quad k_p = \frac{k_1+k_2}{2};\ v_p = \frac{\omega_p}{k_p}$ | $\omega_p = \frac{\omega_1+\omega_2}{2};\quad k_p = \frac{k_1-k_2}{2};\ v_p = \frac{\omega_p}{k_p}$ |
| $\omega_g = \frac{\omega_1-\omega_2}{2};\quad k_g = \frac{k_1-k_2}{2};\ v_g = \frac{\omega_g}{k_g}$ | $\omega_g = \frac{\omega_1-\omega_2}{2};\quad k_g = \frac{k_1+k_2}{2};\ v_g = \frac{\omega_g}{k_g}$ |

In an attempt to simulate the resulting beat waveforms, it was assumed that either set of equations might be used, if the [±] sign was accounted for by [±k]. By way of example, in a non-dispersive case, the velocities of the 2 waves can be defined as [$v_1$=+1, $v_2$=-1], such that the values of [ω,k] could be calculated around some central frequency [ω] using [±Δω], such that the following 2 waves might be used as the basis of the simulation:

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| Wave-1 | Wave-2 |
|---|---|
| $\omega=0.2;\ \Delta\omega=0.02$<br>$\omega_1=\omega+\Delta\omega=0.22$<br>$v_1=+1$<br>$k_1=w_1/v_1=0.22$ | $\omega=0.2;\ \Delta\omega=0.02$<br>$\omega_2=\omega-\Delta\omega=0.18$<br>$v_2=-1$<br>$k_2=w_2/v_2=-0.18$ |

While the values of $[\omega_1,k_1]$ and $[\omega_2,k_2]$ do not change, applying these values to the 2 sets of equations above appears to lead to different values of $[v_p]$ and $[v_g]$. However, in both cases, both sets of values appear to be wrong.

| $\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$ |
|---|
| $\omega_p = \frac{0.22+0.18}{2} = 0.2; \quad k_p = \frac{0.22+(-0.18)}{2} = 0.02; \quad v_p = \frac{0.2}{0.02} = 10$<br>$\omega_g = \frac{0.22-0.18}{2} = 0.02; \quad k_g = \frac{0.22-(-0.18)}{2} = 0.2; \quad v_g = \frac{0.02}{0.2} = 0.1$ |

| $\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t + k_2 x)$ |
|---|
| $\omega_p = \frac{0.22+0.18}{2} = 0.2; \quad k_p = \frac{0.22-(-0.18)}{2} = 0.2; \quad v_p = \frac{0.4}{0.2} = 1$<br>$\omega_g = \frac{0.22-0.18}{2} = 0.02; \quad k_g = \frac{0.22+(-0.18)}{2} = 0.02; \quad v_g = \frac{0.02}{0.02} = 1$ |

While it has been argued that the direction of the waves should be reflected in [k] not [ω], the reversal of the position in [3] was tried, i.e.

[5] $\quad \pm\omega = \pm vk$

As such, the table of wave-1 and wave-2 must be changed so the direction of the wave-2 is now reflected in the sign of $[\omega_2]$ not $[k_2]$. Note $[\omega=k]$ and $[\Delta\omega=\Delta k]$ when $[v=1]$:

| Wave-1 | Wave-2 |
|---|---|
| $k=0.2;\ \Delta k=0.02$<br>$k_1=k+\Delta k=0.22$<br>$v_1=+1$<br>$\omega_1=v_1k_1=0.22$ | $k=0.2;\ \Delta k=0.02$<br>$k_2=\omega-\Delta k=0.18$<br>$v_2=-1$<br>$\omega_2=v_2k_2=-0.18$ |

Again, if we substitute the revised values of $[\omega_1,k_1]$ and $[\omega_2,k_2]$ to the 2 sets of equations defining $[\omega_p,k_p]$ and $[\omega_g,k_g]$:

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| $\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$ |
|---|
| $\omega_p = \frac{0.22+(-0.18)}{2} = 0.02; \quad k_p = \frac{0.22+0.18}{2} = 0.2; \quad v_p = \frac{0.2}{0.02} = 0.1$ <br> $\omega_g = \frac{0.22-(-0.18)}{2} = 0.2; \quad k_g = \frac{0.22-0.18}{2} = 0.02; \quad v_g = \frac{0.2}{0.02} = 10$ |

| $\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t + k_2 x)$ |
|---|
| $\omega_p = \frac{0.22+(-0.18)}{2} = 0.02; \quad k_p = \frac{0.22-0.18}{2} = 0.02; \quad v_p = \frac{0.2}{0.02} = 1$ <br> $\omega_g = \frac{0.22-(-0.18)}{2} = 0.2; \quad k_g = \frac{0.22+0.18}{2} = 0.2; \quad v_g = \frac{0.2}{0.02} = 1$ |

Again, we get two set of results for [$v_p$] and [$v_g$], which cannot both be correct. Out of all the permutations shown, the only one that seems to make sense and work consistently in all simulations tested is:

| $\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$ |
|---|
| $\omega_p = \frac{0.22+(-0.18)}{2} = 0.02; \quad k_p = \frac{0.22+0.18}{2} = 0.2; \quad v_p = \frac{0.2}{0.02} = 0.1$ <br> $\omega_g = \frac{0.22-(-0.18)}{2} = 0.2; \quad k_g = \frac{0.22-0.18}{2} = 0.02; \quad v_g = \frac{0.2}{0.02} = 10$ |

In this specific non-dispersive case, where [$v_1$=+1, $v_2$=-1], the rationale for [$v_p$=0.1] can be explained as being analogous to a standing superposition waves. For as [Δω] approaches zero, the phase velocity of the superposition wave, i.e. the higher frequency component of the beat waveform must also approach zero as suggested by the result above, and none other. While a group velocity [vg>1] appears anomalous, it is argued that the group wave envelope is not actually propagating through space-time, as it more accurately reflects the phase shift between waves-1 and wave-2 occurring at all value of [x] simultaneously in time [t]. This phase shift effect, leading to the perception of [vg>1] is higher when wave-1 and wave-1 propagate in opposite directions, as reflected in the result above, and none other. However, while now having to argue for the results above, it is still unclear why these equations require the directional [±] sign to be assigned to [ω] not [k].

While it is realised that nobody may be interested in working through all this detail, but should anybody be able to resolve the apparent anomaly, it would be appreciated if they could let me know via the PF forum. Thanks.