Let $S = \{v_i \mid i \in I\}$. Let $B = \{u_1\}$ be a 1-element linearly independent subset of $\text{span}(S)$. Then $u_1 = \Sigma_{j \in J_1} a_j v_j$ for some $a_j \in F$ (all nonzero), where J_1 is some finite subset of *I* (with at least one element since $u_1 \neq 0$). Choose any v_{j_1}, where $j_1 \in J_1$. Then

$$\text{span}\left((S - \{v_{j_1}\} \cup \{u_1\}\right) = \text{span}(S).$$

Now let $B = \{u_1, u_2\}$ be a 2-element linearly independent subset of $\text{span}(S)$. Then $u_2 = \Sigma_{j \in J_2} a_j v_j$ for some $a_j \in F$ (all nonzero), where J_2 is some finite subset of *I*. Since $B = \{u_1, u_2\}$ is linearly independent, then $J_1 \cup J_2$ has at least two elements, which means that $J_2 - \{j_1\}$ has at least one element. Thus we choose any v_{j_2}, where $j_2 \in J_2 - \{j_1\}$. Then

$$\text{span}\left((S - \{v_{j_1}, v_{j_2}\} \cup \{u_1, u_2\}\right) = \text{span}(S).$$