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Acceleration Mechanism for Neutron Production in Plasma Focus and z-Pinch Discharges

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A model has been developed for the acceleration of deuterons in the tightly compressed column of a z-pinch discharge, in particular, that of a plasma focus discharge. It was assumed that an annular distribution of current undergoes a rapidly contracting transition to a distribution peaked on axis and the resulting crossed fields, B_θ and E_z, were used to calculate deuteron trajectories in the plasma. These results show that a significant number of ions can be accelerated to energies of up to 600 keV and that they have a large value for their time-averaged axial velocities. The orbiting motion in the magnetic field produces an anisotropic distribution of collision velocities and a neutron-production model based upon nuclear collisions by these deuterons can account for the measured characteristics of neutron energies, neutron fluxes, and rates of production.

I. INTRODUCTION

To date no satisfactory model has been proposed to explain the neutron production in z-pinch discharges. Deuterium-filled linear z pinches have produced up to 10^8 neutrons per discharge and the axially emitted neutrons showed an average energy shift of 200–250 keV with respect to the center-of-mass energy.[1–3] The neutron yields from plasma focus types of z pinches are even higher (10^9–10^{11} per discharge) and average energy shifts of up to 500 keV were exhibited.[4,5] To explain these energy shifts, two modes of neutron production have been considered previously: a beam–target model in which linearly accelerated deuterons strike ions at rest and a "moving boiler" model, in which the neutron source is an axially moving thermal plasma. The measured characteristics of the emitted neutrons from various plasma focus devices indicate that neither model adequately explains the observed behavior.[5–10] In this paper a model for the acceleration of deuterons in a z-pinch discharge is presented, for which the calculated results are in good agreement with experimental observations.

Investigations of a long linear z pinch showed that the neutron production was uniform along the plasma column and to account for the observed energy shift in this device, a beam–target model was proposed in which the deuterons were axially accelerated by the electric fields generated by $m = 0$ sausage instabilities.[1] In a plasma focus discharge the neutron source has been shown to be quite small ($\sim$3 cm long)[5] and neutron flux measurements provide evidence that the flux anisotropy is much smaller than that expected for a simple beam–target model.[5–7] (The magnitude of the flux anisotropy is defined as the ratio of axial to radial neutron fluxes, henceforth referred to as the flux ratio.) Thus the currently popular "moving boiler" model was proposed, where a hot plasma has a high axial velocity.[6] The neutrons emitted from our devices exhibited energy shifts of up to 1000 keV with an average shift of over 400 keV in the forward direction. This shift required the reacting deuterons to have axial center-of-mass velocities averaging 2×10^8 cm/sec, which is an order of magnitude larger than the maximum radial velocity observed for the collapsing plasma column. Therefore it is difficult to understand how the plasma attains such a high velocity on the basis of a fluid model.[11] Recently we performed an analysis of the neutron energy spectra, which provides additional evidence that the neutron source is not thermonuclear.[10] To account for the observed neutron energy spectra by a beam–target process, the accelerated deuterons require axial velocities of up to 6×10^8 cm/sec or energies of up to 360 keV.

The operation and general characteristics of a typical plasma focus device have previously been described[5]; at a bank voltage of 18 kV (27 kJ) neutron yields averaging over 10^{10} were obtained. In this device the discharge current reached a peak value of over 800 kA which dropped to about 600 kA at the time of neutron emission. Other devices have discharge currents at the time of neutron production ranging from 0.3 to 1.5 MA. We reported previously that the neutron pulse had a risetime of 20–25 nsec, and more recent measurements made with detectors closer to the source revealed a risetime as short as 10 nsec on some shots. The radial velocity of the visible plasma boundary averaged about 15 cm/μsec with a peak value of about 30 cm/μsec just before peak compression. Near the anode the minimum radius of the visible plasma column was about 1 mm and the column attained a length of about 3 cm. Plasma densities in the range of 10^{19}–10^{20} cm^{-3} have been indicated by various measurements.[12–14]

In addition to neutrons, the discharge produces a broad spectrum of x rays with energies up to at least 300 keV. These x rays have an energy spectra, which is not Maxwellian but is approximated by a power law.[4,5,15] In addition, the hard x rays were usually emitted at a time corresponding to the beginning of the most intense neutron production. On a macroscopic basis it is easy to show that electric fields of over 200 kV/cm are generated at the boundary of the rapidly collapsing plasma, but it is difficult to see how electrons can be accelerated to high energies by this field since they must move across the strong magnetic field surrounding the discharge current.

Recently, elaborate computer calculations have been employed to examine the features of the plasma dynamics in a plasma focus.[14] However, they all assume an isotropic fluid plasma and neglect anisotropic motion of ions. The acceleration mechanism presented here is based on computer calculations of ion trajectories in the crossed electric and magnetic fields generated by a rapidly collapsing current distribution. The chief purpose is to show how ions can be accelerated to high energies in the discharge and it will be shown that the results are consistent with the observed neutron and x-ray emissions. Section II gives a description of the model and assumptions, Sec. III gives the assumed current distributions and resulting energies, Sec. IV gives the justifications for the assumptions, and Sec. V gives estimates on the neutron production.

II. ANALYTIC MODEL

This model assumes an axisymmetric, cylindrical current distribution $j(r, t)$ which is finite in thickness and initially annular in shape. The distribution is then assumed to contract rapidly to the axis in some manner. Such a time variation in the current density gives rise to both an azimuthal magnetic field $B_\theta(r, t)$ and an axial electric field $E_z(r, t)$ whose values are derived from Maxwell's equations

$$\frac{\partial(rB_\theta)}{\partial r} = rj(r, t), \qquad \frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}.$$

Since details of the actual current distribution are not known, the inferred distribution with forms of $j(r, t)$ has been approximated, yielding analytic expressions for $B_\theta(r, t)$ and $E_z(r, t)$. Using these resulting expressions for B_θ and E_z, the equation of motion was solved to obtain the trajectories of deuterons having different initial parameters. To a first approximation, the effect of collisions can be neglected during the acceleration of many of the ions. This permitted me to assume, for ease of calculation, that the motion of the accelerated deuterons takes place only in the two-dimensional r-z plane. Then the equation of motion in two dimensions is

$$m_i\ddot{r} = -e\dot{z}B_\theta(r, t),$$

$$m_i\ddot{z} = e\dot{r}B_\theta(r, t) + eE_z(r, t),$$

where m_i is the ion mass, e is the ionic charge, and $\dot{r}$ and $\dot{z}$ are the velocity components.

In these calculations, the following assumptions were introduced; where necessary, they will be justified later in the paper. First of all, the current density is assumed to drop to zero at some outer boundary R and ion trajectories are assumed to be confined within this boundary. As already mentioned, the effects of ion–ion collisions are neglected in the calculations. This does not mean that such collisions are completely neglected, for it is recognized that collisions establish a random distribution of ion velocities at the beginning of the acceleration. Also, since the collision frequency is energy dependent, only a small fraction of the accelerated ions will avoid losing their energy via collisions and escape to high energies.

In this analysis electron–ion collisions are not neglected and the plasma conductivity is assumed to be finite. In fact a key assumption is that the plasma resistivity or resistance increases in the tightly compressed plasma column, leading to a rapid transition of the current to an axial concentration. Such a rise in plasma resistivity is probably associated with axial loss of plasma. During the pinching action radial electric fields are present in the plasma sheath, but they are ignored in the present analysis. This assumption appears to be reasonable after the plasma column reaches maximum compression.

The adoption of cylindrical coordinates for these calculations appears acceptable, even though high-speed photography has revealed that the radial collapse of the plasma column is not simultaneous along the axis. Over distances of a few millimeters the observed curvature should be negligible. The shortness of the pinch and its noncylindrical nature do play a prominent role in the discharge behavior, though. First of all, axial ejection of plasma results, and this appears to be linked to the acceleration process. In a future paper we will present experimental results correlating the neutron emission with

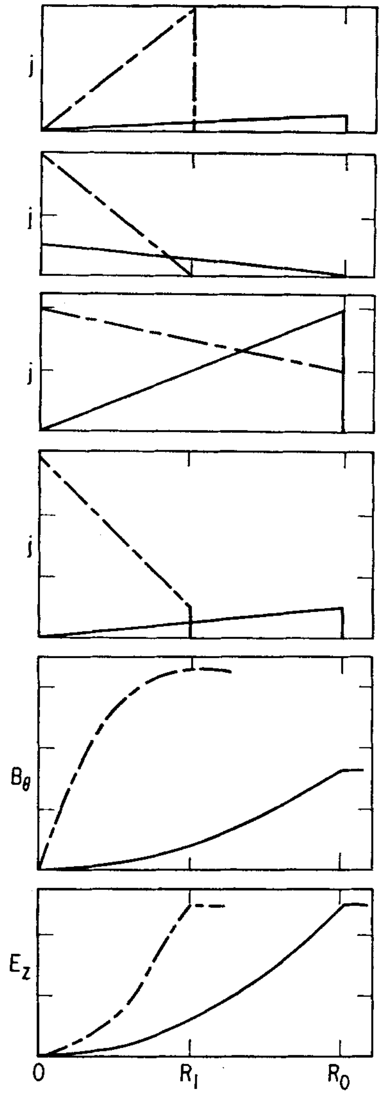

FIG. 1. Assumed current distributions $j(r, R, t)$: (a) contracting, peak density on boundary; (b) contracting, peak density on axis; (c) fixed boundary, transition to axial concentration; (d) contracting with transition to axial concentration. Solid curves show initial distributions and broken curves show distributions at later times. All scales arbitrary and linear. Curves (e) and (f) give radial dependences of magnetic and electric fields for distribution (d).

the presumed rapid current transition. Second, since the pinching is nonsimultaneous, it is assumed that the accelerating fields are likewise not generated simultaneously along the column. Thus, a tandem acceleration mechanism is possible, where deuterons accelerated in the region of initial compression move up the column and are further accelerated by the fields generated at a slightly later time. Neutron-collimation measurements have shown that the neutron production is not simultaneous along the plasma column.[5]

Ideally, one would like to compute the plasma dynamics in a complete, self-consistent manner taking into account anisotropic particle motion and collisions as well as the interaction between plasma and magnetic field. But this appears to be far beyond the scope of present-day computer capabilities.

III. RESULTS OF CALCULATIONS

The current distributions $j(r, R, t)$ chosen had the time variation expressed in either of two ways depending on whether the current boundary R was fixed or constricting. Most of the distributions varied in the second way and the current density was expressed as $j(r, R)$ with the time dependence entering via the boundary velocity dR/dt. In the case of a constant value for the boundary radius, the density was expressed as $j(r, t)$ and the time dependence entered directly as a specified time period τ for the transition from a boundary-peaked distribution to an axially peaked one. The following expressions were found useful for approximating the assumed variations in the current distribution:

$$j(r, R) = j_b(r/R)^\beta, \quad (1)$$

$$j(r, R) = j_a[1 - (r/R)^\alpha], \quad (2)$$

$$j(r, t) = j_a[1 - (r/R)^\alpha] + j_b(r/R)^\beta, \quad (3)$$

$$j(r, R) = j_a + (j_b - j_a)(r/R)^\alpha, \quad (4)$$

where j_a is the current density on the axis, j_b is the current density at the boundary R and the constants α, $\beta > 0$. These four forms of the current distribution are illustrated in Fig. 1 for $\alpha = \beta = 1$; in addition, the radial dependences of B_θ and E_z for the last expression are also shown. In general, both j_a and j_b varied with time in a way consistent with the total discharge current, which was assumed to remain constant during the time of interest. For all three contracting distributions, the boundaries were assumed to contract at a uniform rate $dR/dt = -V_b$. Values for α and β ranged from 0.3 to 3.0. The first expression, Eq. (1), represented a radially contracting current distribution peaked at the boundary. It is typical of the earlier stages of the plasma collapse when collisions are probably dominant. Thus, the results arising from use of this expression are not discussed although considerable energy gains were found using it. The second expression, Eq. (2), represents a similar contracting current distribution which is peaked on the axis. The third expression, Eq. (3), represents a transition from a density peaked on the boundary to one peaked on axis while the boundary remained fixed. In this case

I assumed that j_a rose linearly from a value of zero at $t = 0$ to a maximum value at $t = \tau$ when j_b had fallen to zero. Finally, the last expression, Eq. (4), combines the contraction and transition of current density represented separately in Eqs. (2) and (3). This expression, with j_b constant and j_a rising from zero in accord with the contracting boundary, was the one used in the previously given brief account of my calculations.[16] It must be kept in mind that in this axially symmetric system, the values of both the electric and magnetic fields are always zero on the axis and they rise to their maximum values at the boundary. At any given time, the ratio of the fields, E_z/B_θ, varied as r/R to a first approximation.

Recent observations of ours show a significant correspondence between the fading of the luminous plasma boundary and the onset of intense neutron emission. On this basis I assume that the acceleration really begins in earnest when the current density undergoes a transition to an axial concentration. In addition, it appears reasonable to assume that this transition is accompanied by a contraction such as represented simply in Eq. (4). Using the fields for this distribution, the orbits of two typical deuterons were computed, representing two classes of trajectories as shown in Fig. 2. In these plots of the spatial trajectories, both ions started out at the same off-axis position with the same initial energy, but their initial velocities were directed differently. The ions making up the first class of trajectories (type I) never reach the axis and gradually drift against the electric field in the $\mathbf{B} \times \nabla \mathbf{B}$ direction. Ions of the second class (type II) oscillate through the axis and exhibit a pronounced movement in the direction of the electric field. We shall see shortly that such ions are the ones gaining the most energy and that they acquire the large axial velocities responsible for the neutron-energy shifts.

This energy gain of the ions in the crossed fields of the discharge can be compared to ion acceleration in a cyclotron. For the first class, this analogy requires a cyclotron having an electric field between the Dees which does not change sign, but whose amplitude varies on alternate half-cycles; this variation in amplitude corresponds to the gradient in the plasma electric field. As an analogy to the second case, the electric field in the cyclotron would reverse sign on alternate half-cycles. It must be pointed out that this distinction between the two classes of ions is clear cut in our two-dimensional calculations, but it still holds for three-dimensional motion where trajectories only rarely pass exactly through the axis.

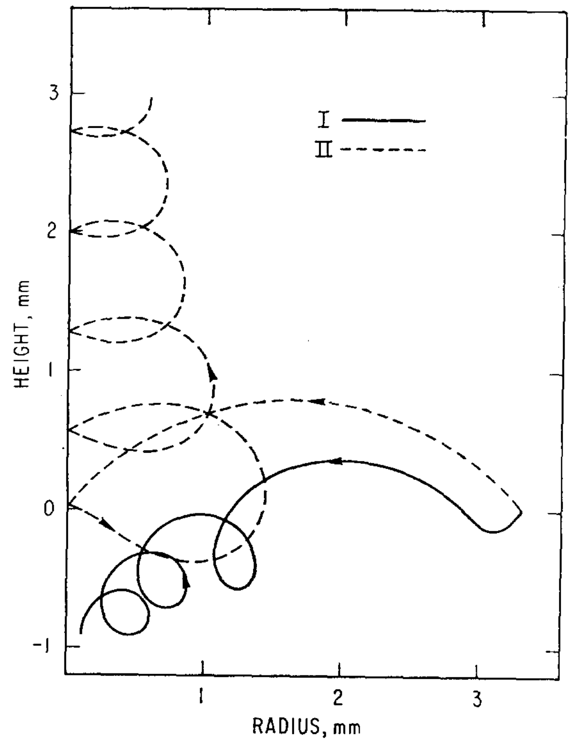

FIG. 2. Deuteron trajectories for two types of ion acceleration. Reflections from $r = 0$ in type II represent passage through the axis.

For the representative pair of examples shown in Fig. 2, the time histories of the ion positions and velocities are shown in Fig. 3. Both ions had the same initial energy of 2 keV and their radial velocities were identical, but their axial velocities were initially oppositely directed. In type I the deuteron gains considerable energy within a fraction of a gyro-orbit and then gradually gains more energy during subsequent orbits. Type II is seen to be the more interesting case, though, for then the energy gain on succeeding orbits is much greater. Even more important is the fact that these ions not only reach very high energies, but their average axial velocities attain very high values. The apparent discontinuities in the radial velocity occur when the trajectory passes through the axis. As already mentioned, consideration of the azimuthal motion of the ions would give a similar motion, but without the sharp discontinuities in the trajectory plots.

Trajectory calculations were made for the different current distributions using a wide range of initial values for the ion energy, different orientations of the initial velocity vector, and several initial values for both the boundary radius and the ion position. Rather than trying to give many plots and tables of all the results, only the salient features will be

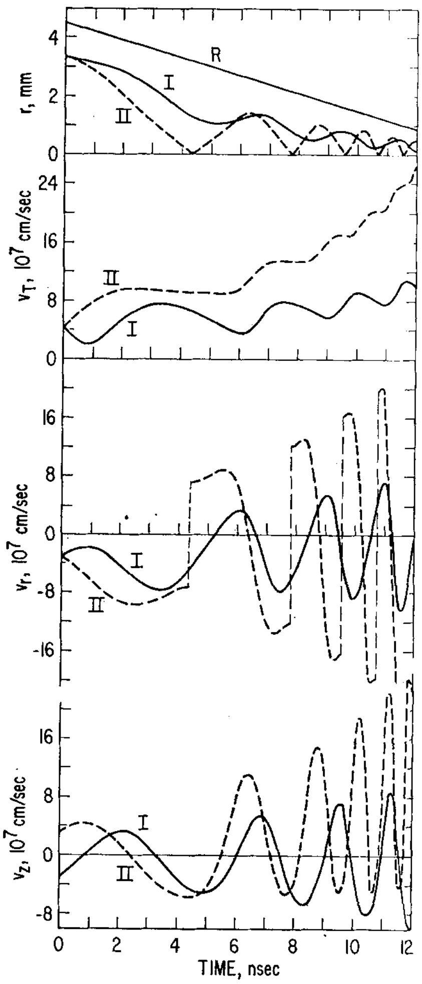

FIG. 3. Time dependence of deuteron radius r, speed v_T, radial velocity v_r, and axial velocity v_z for two types of ion acceleration. Energy proportional to v_T^2 (10×10^7 cm/sec $\simeq$ 10 keV. Discharge current I is 700kA.

given. Although there is uncertainty about the actual shape of the current distribution, the form expressed through Eq. (4) appears to be quite reasonable for use in explaining the neutron production. But before discussing the results using this expression, it is instructive to examine the types of energy gains resulting from use of Eq. (3), which defines a redistribution of the current density and those resulting from use of Eq. (2), which represents a simple contraction of the current distribution.

Calculations for the redistribution case with a fixed boundary R, Eq. (3), show that 2.5 keV deuterons increase their energy by up to six times (to 15 keV) for the parameters $I = 700$ kA, $R =$ 2 mm, and $\tau = 20$ nsec. Similar calculations show that ions with an initial energy of 10 keV gain up to a factor of 5 in energy (to 50 keV), when the same parameters are used. The energy gain was found to be essentially independent of τ and R. As might be expected the energy gain was slightly higher for smaller values of α and larger values of β, corresponding to initial and final distributions more sharply peaked on the boundary and axis, respectively. Only the discharge current played a significant role in the energy gain: an increase in the current by a factor of 2 resulted in about a 40% increase in the final energy.

When we consider the constricting, axially peaked distribution, Eq. (2), then the energy gain is somewhat larger. Deuterons with an initial energy of 2.5 keV will have their energies increased by factors of 2, 7, and 16 (to about 5, 17, and 40 keV) as the boundary radius shrinks down to 0.5, 0.2, and 0.1 of its initial value, respectively. Ions having a higher initial energy will increase in energy by slightly smaller factors. It is important to note that this energy gain depends only on the ratio of initial to final boundary radii. In contrast to the previously discussed case the energy gain here is pretty much independent of the discharge current. Compared to the previous case, it was surprising to find that the energy gain was slightly higher when the distribution was less sharply peaked on axis (larger α). Again in this case the energy gain is essentially independent of the boundary velocity which corresponds to a transition time. As already mentioned, calculations were also made using Eq. (1) and the energy gains were significant. However, this form does not correspond to the assumed discharge behavior, although it may contribute to somewhat higher initial deuteron energies.

We are now ready to discuss the energies resulting from use of the distribution defined by Eq. (4). The results given in Fig. 3 show that the fields generated by a time-varying current distribution can accelerate the deuterons to very high energies and that their time-averaged axial velocities attain high values. For example, the type II deuteron illustrated in Figs. 2 and 3 had an initial energy of 2 keV which increased to energies of 18, 60, 150, and 260 keV as the boundary shrank down from 4.5 mm to values of 2.0, 1.0, 0.5, and 0.3 mm, respectively. As found previously for the simple contracting distribution, the gain in energy depends only on the ratio of initial boundary radius to final radius. In fact, the calculated energy gain is limited only by the minimum radius of the contracting current boundary. An estimate for this value can be obtained by assuming the electrons have an average axial velocity of only 1×10^9 cm/sec and an average density of 10^{19} cm^{-3}. Then the boundary could shrink to a radius of less than 0.2 mm and still carry a current of 1 MA. Since this distribution represents a combination of the distributions given by Eqs. (2) and (3), we would expect that the energy gain depends somewhat on the magnitude of the discharge current. The calculations bear this out; a factor of 2 increase in the current results in about a 20–30% larger energy gain. A factor-of-2 increase in the boundary velocity also adds about 25% to the energy gain. The effect of the velocity orientation is illustrated in Fig. 4 for four typical ions which start out at the same radial position with the same energy. Here the energy is plotted as a function of time and, in addition, the axial position of these four ions is also shown as a function of time. This figure illustrates that the energy gain is not proportional to the magnitude of displacement in the direction of the electric field. It is an interesting result that the final energy of an accelerated ion is

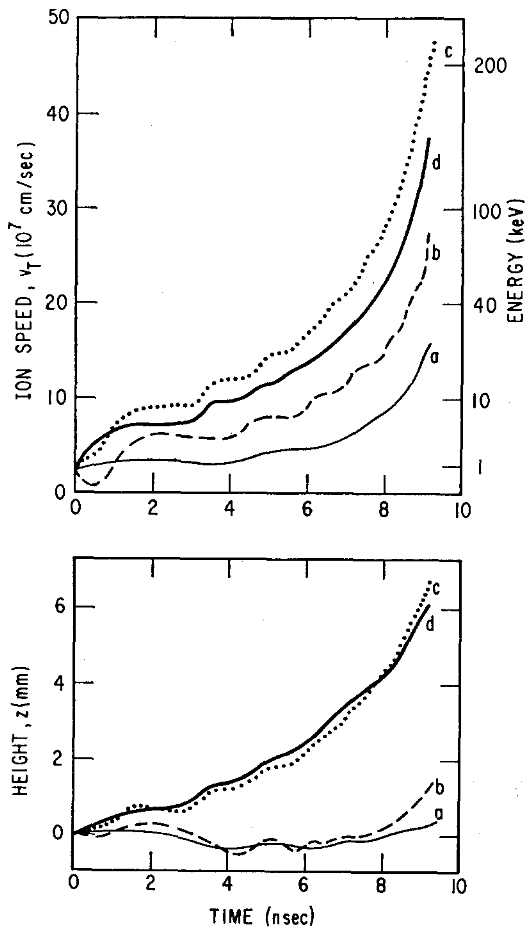

FIG. 4. Time dependences of deuteron speeds v_T and axial displacements z for ions having initially the same energy and radial position but starting with different velocity orientations. Initially $v = 2.5 \times 10^7$ cm/sec, $r = 1$ mm, and $R = 2$ mm with $V_b = 2 \times 10^7$ cm/sec and $I = 700$ kA.

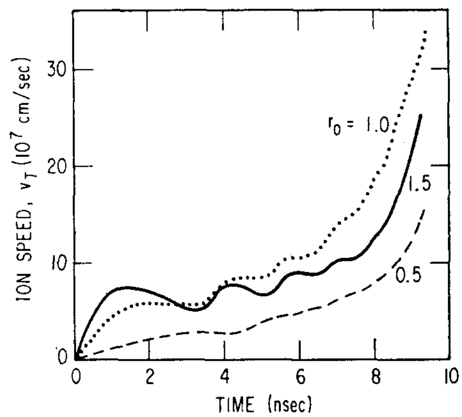

FIG. 5. Time dependence of deuteron speeds v_T for ions starting from rest at different initial radial positions r_0. Initially $R = 2$ mm with $V_b = 2 \times 10^7$ cm/sec and $I = 700$kA.

only weakly dependent on its initial energy. To drive this point home, the expected energy gains for ions starting from rest have been calculated, as illustrated in Fig. 5. One finds that the energy gain for an ion initially at rest is approximately the average of the energies achieved by a group of ions starting from the same point with a random orientation of initial velocities. Of course, the highest energies will be attained by energetic ions which start off in the right direction, but these calculations show that even ions with small velocities in the r-z plane can be accelerated to high energies. On the basis of these various calculations it is estimated that around 25% of all the ions could be accelerated in a type II trajectory. It was also found that some ions start out in a type I trajectory for a few orbits and then reach the axis to continue their acceleration in a type II mode. Values for the initial boundary radius and boundary velocity were chosen to be consistent with visible-light photography and rates of neutron and x-ray emission. Also the calculated axial displacement was found to be proportional to the initial boundary radius and these displacements had to be consistent with the length of the neutron source.

The limitation on the actual number of ions which will be accelerated to high axial velocities is limited by the collision probability. As is well known the collision frequency decreases rapidly with an increase in the ion energy, so it is largely a question of what fraction of the ions can run away in the fields. The collision frequency will be discussed later in the paper. However, for ions with a random energy distribution averaging about 1 keV, it appears that at least 1% of the ions could be accelerated unimpeded by collisions.

The magnitude of the ion velocities is not limited to values obtained in the previous calculations. As already mentioned in Sec. II, the energies and axial velocities of the deuterons can be boosted by a sort of tandem acceleration. The accelerated ions move up the plasma column and enter a region where the current distribution undergoes a transition and collapse at a somewhat later time. Various neutron-energy measurements indicate in a rough manner that the neutrons emitted in the middle of the pulse farther up the column do exhibit the largest energy shifts. Yet another phenomenon which could aid some ions in achieving still higher energies is the effect of collisions. Small-angle Coulomb collisions could redirect some ions in a favorable way so that they could gain more energy from the electric field than if they orbited unimpeded.

IV. JUSTIFICATION OF ASSUMPTIONS

A. Collisions

The effects of ion–ion collisions were neglected in the calculations and we examine the justification of this assumption. For a distribution of ions the usual thermalization time is given by [17]

$$t_c \simeq 2 \times 10^7 T_d^{3/2}/n \ln \Lambda \quad \text{sec},$$

where T_d is the deuteron temperature in electron volts, n is the density in cm^{-3}, and $\ln \Lambda$ is the usual logarithmic factor, of order 10. For ions with energies much greater than the random energy of the background an appropriate collision time is[17]

$$t_s \simeq 1 \times 10^7 E_d^{3/2}/n \ln \Lambda \quad \text{sec},$$

where E_d is the deuteron energy in electron volts. As the worst case, we assume a density of 10^{20}. Then for ions with energies of 1 and 10 keV the corresponding collision times are 0.6 and 10 nsec, respectively. Therefore, within a few nanoseconds, a significant number of ions could run away.

B. Plasma Resistivity

It was assumed that a large increase in the plasma resistance occurred as the plasma column became tightly constricted and that this increase was associated with an axial loss of plasma. The usual expression for the plasma resistivity η is given by[17]

$$\eta = 5 \times 10^{-3} \ln \Lambda / T_e^{3/2} \ \Omega\text{-cm},$$

where T_e is in electron volts and the resistivity is very insensitive to the density. An extension of this analysis showed that the resistivity transverse to a strong magnetic field obeys the same expression, with a factor of 2 increase in the coefficient because of a change in the electron-energy distribution. However, this expression does not take into account the situation when the electron cyclotron frequency ω_e is much larger than the electron–ion collision frequency t_{ei} given by [18]

$$t_{ei} = 7 \times 10^5 T_e^{3/2}/n \ln \Lambda.$$

In this case the resistivity is expected to be increased by the factor $[1 + (\omega_e t_{ei})^2]$. Now for densities approaching 10^{20} and for temperatures in the vicinity of 100 eV, values of t_{ei} will be of order 10^{-12} sec. Thus for $\omega_e t_{ei}$ to be greater than 1, a magnetic field greater than 10^5 G is required. Such fields will occur only when the discharge current constricts to a radius of less than a centimeter. At this stage, electrons are forced to run away on axis to maintain the discharge current, resulting in the assumed current transition. And as current begins to flow on axis the effective resistivity near the axis rises sharply.

Another way of looking at this enhancement in the plasma resistance is to consider the rapid decrease in the cross-sectional area of the plasma column. Since the resistivity is insensitive to density, the resistance will be greatly enhanced if the plasma compression occurs more rapidly than the electron temperature can rise to offset the diminished area. In the same way the discharge current can be maintained only by high-energy runaway electrons on axis.

C. Ion Trajectories

For ease of computation, it was assumed that the ion trajectories were confined within the current boundary. Certainly the real current distribution has no sharp boundary and ion trajectories are likewise not limited, but calculations for many different ion trajectories showed this assumption was not restrictive. In fact the trajectories of very high energy deuterons tended to overstep the boundary only when the current distribution had shrunk down to a very small radius. Yet in no way does this affect the conclusions that were reached.

V. NEUTRON PRODUCTION

For the two simple models of neutron production, moving boiler and beam–target, the neutron yields are given by the following expressions[19]:

$$Y_{\text{mb}} = \tfrac{1}{2} n_d^2 \langle \sigma_v \rangle A L t_n,$$

$$Y_{\text{bt}} = F_d \sigma n_d A L t_n,$$

where n_d is the ion density, $\sigma_{\text{D-D}}$ is half the total energy-dependent D-D cross section, A is the cross-sectional area of the hot plasma, L is the source length, t_n is the production time, and F_d is the flux of high-energy deuterons. It has already been established that the neutron source is about 3 cm long and the production time at a given point is around 40 nsec.[4] For the sake of comparison it is useful to first consider the neutron yield from a thermonuclear plasma. If we assume an average ion density of 10^{20} across a radius of 1 mm, then a neutron yield of 10^{10} requires a deuteron temperature of 1.4 keV. Such a temperature could be realized from thermalization of the total energy in the imploding sheath, if this energy is assumed to be divided initially between directed kinetic energy and the thermal energy within the collapsing sheath. However, this does not allow for the energy needed to explain the high-velocity axial motion. In addition, the average density is probably lower, so that a yet higher temperature is required to achieve the observed neutron yield. In any case, other observations provide strong evidence that the neutron source is not thermonuclear.[10]

Let us now consider the linear beam–target model. A stream of deuterons with energies averaging 160 keV are assumed to be axially directed through a plasma column having a density of 3×10^{19}. Then to account for a neutron yield of 10^{10}, an ion current of about 20 kA is required, which represents about 3% of the discharge current in our device. As already discussed earlier, such a model gives a calculated flux anisotropy much larger than what is measured. Also one would expect very little neutron production near the anode where the acceleration would presumably begin, which is contrary to the experimental results. Yet another point of conflict in this model, is that the above estimate for the ion current is too large by two orders of magnitude to be consistent with conservation of momentum with the electron current. Of course, this question of momentum balance could also apply to the moving boiler model.

The model presented in this paper appears to satisfy the problems inherent in these other models. We have seen how a large number of deuterons can be accelerated to high energies in the crossed fields and how the trajectories of these ions loop around in the magnetic field. It must be emphasized again that the energy gain of an ion does not necessarily correspond to a net displacement along the axis. In fact ions can gain appreciable energy even when they suffer a net shift against the electric field. The curved trajectories of the accelerated deuterons help explain several facets of the experimental observations. First of all this orbiting motion produces an anisotropic distribution of velocity vectors, which can account for the neutron-flux anisotropies observed on most devices. Second, many of these high-energy ions can now make fusion collisions near the anode. And third, the average path length of an ion in the plasma is much greater than its average axial displacement, so that even fewer accelerated ions are necessary to produce a given neutron yield.

To estimate the neutron yield on the basis of the present model requires calculations very similar to those for the linear beam–target model. Consistent with the observed energy shifts, the accelerated ions are assumed to have energies ranging up to 600 keV. Of course, collisions between two such ions could result in even larger center-of-mass energies, but such collisions will be rare. The net result of these considerations is that the ion current necessary to give the same neutron yield under the above conditions is reduced by about a factor of 5 below that required by the linear beam-target model (to less than 1% of the total current). These high-energy deuterons represent less than 10% of the actual ion density even if one assumes they are confined to a radius of only 0.1 mm. Thus it appears that most fusion collisions take place with low-energy ions. This still leaves the sticky point of momentum conservation with the electron current. It is believed that this is accounted for by the retrograde motion of many low-energy ions. Certainly the axial jetting of plasma accounts for more ion momentum than do these few high-energy ions.

The apparent discrepancy between the assumed transition time for diffusion of the magnetic field (5–20 nsec) and the observed duration of neutron production in a plasma focus (70–150 nsec) can be accounted for by two phenomena. First, the accelerating electric fields are not generated simultaneously along the axis, because there is a time spread of over 30 nsec, for the nonsimultaneous collapse of the plasma column to the axis. Second, after the deuterons have been accelerated to high energies, they will continue to orbit in the azimuthal magnetic field, thereby extending the neutron production time.

VI. CONCLUSION

The assumed rise in the plasma resistance and a consequent transition in the current distribution appears consistent with the sharp pulse of hard x rays which are generated by the discharge. As we have seen, deuteron velocities of 6×10^8 cm/sec and even more can result from acceleration in the fields generated by this rapid current transition. The trajectories of these high-energy ions appear to satisfy all aspects of neutron emission from plasma focus discharges. In addition, it seems that the neutron production in long, linear pinches arises in a similar manner even though axial ejection of plasma is limited by the length of the plasma column. Direct verification of the model presented here would require detailed measurements of the radial distribution of the current density, plasma density, and neutron production with submillimeter accuracy. However, such measurements appear beyond the present state of the art. Attempts to calculate the acceleration of electrons to very high energies in a manner similar to that for the ion acceleration have so far been unsuccessful.

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[1] O. A. Anderson, W. R. Baker, S. A. Colgate, J. Ise, Jr., and R. V. Pyle, Phys. Rev. **110**, 1375 (1958).

[2] A. M. Andrianov, O. A. Bazilevskaia, S. I. Braginskii, B. G. Brezhnev, S. Khvaschevski, V. A. Khrabrov, N. G. Kovalski, N. V. Filippov, T. I. Filippova, V. E. Palchikov, I. M. Podgorny, Yu. G. Prokhorov, and M. M. Sulkovskaya, in *Proceedings of the Second International Conference on Peaceful Uses of Atomic Energy* (United Nations, Geneva, 1958), Vol. 31, p. 348.

[3] J. W. Mather and A. H. Williams, in *Proceedings of the Second International Conference on Peaceful Uses of Atomic Energy* (United Nations, Geneva, 1958), Vol. 32, p. 26.

[4] D. A. Meskan, H. L. L. van Paassen, and G. G. Comisar, presented at the Conference on Pulsed High Density Plasmas of the American Physical Society, Los Alamos, New Mexico (1967).

[5] M. J. Bernstein, D. A. Meskan, and H. L. L. van Paassen, Phys. Fluids, **12**, 2193 (1969).

[6] N. V. Filippov and T. I. Filippova, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1966), Vol. II, p. 405.

[7] P. J. Bottoms, J. P. Carpenter, J. W. Mather, K. D. Ware, and A. H. Williams, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969), Vol. II, p. 67.

[8] J. H. Lee, H. Conrads, M. D. Williams, L. P. Shomo, H. Hermansdorfer, and K. Kim, Bull. Am. Phys. Soc. **13**, 1543 (1968).

[9] C. Patou, A. Simonnet, and J. P. Watteau, Phys. Letters **29A**, 1 (1969).

[10] M. J. Bernstein and F. Hai, Phys. Letters **31A**, 317 (1970).

[11] G. G. Comisar, Bull. Am. Phys. Soc. **14**, 1014 (1969).

[12] J. W. Mather, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969), Vol. II, p. 389.

[13] N. J. Peacock, P. D. Wilcock, R. J. Speer, and P. D. Morgan, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1969), Vol. II, p. 51.

[14] P. D. Morgan, N. J. Peacock, and D. E. Potter, presented at the *Third European Conference on Plasma Physics and Controlled Fusion*, Utrecht (1969).

[15] H. L. L. van Paassen, R. H. Vandre, and R. S. White, Bull. Am. Phys. Soc. **14**, 1013 (1969).

[16] M. J. Bernstein, Phys. Rev. Letters **24**, 724 (1970).

[17] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience, New York, 1962), Chap. 5.

[18] M. J. Bernstein and G. G. Comisar, J. Appl. Phys. **41**, 729 (1970).

[19] S. Glasstone and R. H. Lovberg, *Controlled Thermonuclear Reactions* (Van Nostrand, Princeton, N. J., 1960), Chap. II.