Consider we the following onediemnsional example:

Bird A has motion law $A(t)$ in inertial system S of the ground. Assume we that the bird starts it's flying at moment 0 of system S. Let t_0 be a fixed moment of system S (in other words, t_0 is positive constant) and assume we that the bird meets fixed point P of system S at moment t_0 of system S. For example, P is a rock. The elapsed time from starting flying to meeting the rock from the view of the bird is

$$\tau_0 = \int_0^{t_0} \sqrt{1-(A'(t)/c)^2}dt. \tag{1}$$

In the moment of the bird and the rock meeting the bird has velocity $A'(t_0)$. Consider we inertial system S' related to system S by Lorentz transformations

$$x' = \frac{x - A'(t_0)t}{\sqrt{1-(A'(t_0)/c)^2}}, \quad t' = \frac{t - A'(t_0)x/c^2}{\sqrt{1-(A'(t_0)/c)^2}}.$$

Note that $A'(t_0)$ is constant because t_0 is constant. When meets the rock, the bird has velocity 0 in system S' and acceleration $A''(t_0)$. We consider this acceleration as relativistic irelevant. In other words, we consider system S' as bird's system at the moment of meeting the rock. Of course, at the moment of meeting the rock, the bird is not in the origin of spacetime in system S'.

At the moment of meeting the rock, the bird has coordinates $(A(t_0), t_0)$ in system S and corresponded coordinates (x'_0, t'_0) in system S', where

$$x'_0 = \frac{A(t_0) - A'(t_0)t_0}{\sqrt{1-(A'(t_0)/c)^2}}, \quad t'_0 = \frac{t_0 - A'(t_0)A(t_0)/c^2}{\sqrt{1-(A'(t_0)/c)^2}}.$$

Consider we the fixed point Q of system S, which has constant coordinate x_1 in system S. For example, Q is a tree. Let t_1 is a moment of system S. At this moment the tree has coordinates (x_1, t_1) in system S and consequently corresponded coordinates (x'_1, t'_1) in system S', where

$$x'_1 = \frac{x_1 - A'(t_0)t_1}{\sqrt{1-(A'(t_0)/c)^2}}, \quad t'_1 = \frac{t_1 - A'(t_0)x_1/c^2}{\sqrt{1-(A'(t_0)/c)^2}}.$$

We are interested for the position of the tree from the bird's view at the moment of the bird an the rock meeting. By this reason, we compute t_1 using condition $t'_1 = t'_0$ and obtain

$$t_1 = t_0 + \frac{A'(t_0)}{c^2}(x_1 - A(t_0)).$$

Now, we compute x'_1 using this value for t_1 and obtain

$$x'_1 = \frac{x_1 - A'(t_0)t_0 - (A'(t_0)/c)^2(x_1 - A(t_0))}{\sqrt{1-(A'(t_0)/c)^2}} = (x_1 - A(t_0))\sqrt{1-(A'(t_0)/c)^2} + \frac{A(t_0) - A'(t_0)t_0}{\sqrt{1-(A'(t_0)/c)^2}}.$$

We conclude that in the moment (1) of the bird's clock, the space coordinate of the tree in the bird's coordinate system is

$$\chi(t_0) = x'_1 - x'_0 = (x_1 - A(t_0))\sqrt{1-(A(t_0)/c)^2}.$$

Finally, we have the wordline of the tree in the bird's system expressed by formulas

$$\chi(\lambda) = (x_1 - A(\lambda))\sqrt{1-(A'(\lambda)/c)^2}, \quad \tau(\lambda) = \int_0^{\lambda} \sqrt{1-(A'(t)/c)^2}dt,$$

with parameter λ.