The general situation that we shall describe, and of which collision and decay processes are just two particular cases, is the change in time of a nonstationary state $W(t)$. We shall use the basic assumptions of quantum mechanics, in particular the axiom V. We shall make the further simplifying assumption that the generator H of time translation can be split into two parts:

$$H = K + V, \tag{1.1}$$

where K is the energy operator of the isolated physical system with stationary states and is assumed to be well defined. This means that it is assumed to make sense to consider an approximate description of the physical system by a stationary physical system. There are many examples for which this is possible. Thus K may be the energy operator of an atom or of a molecule. In the approximate description the states belonging to an energy level are stationary, but in reality they are only quasistationary and decay into the ground state. This transition is then caused by $V = H - K$. Another example is the combination of two physical systems that can be spatially separated far apart from each other, a situation which one usually encounters in collision processes. If they are far apart, the energy operator of the combined system is K; and $V = H - K$, which describes the interaction between the two subsystems, has a finite range.[1]

The observables that are measured in these processes are assumed to commute with the operator K. For example, a detector may register all the ground states of atoms that were initially in excited states (by registering, for example, all emitted light of a particular frequency). The observable measured is thus the projection operator on that subspace of the space of physical states which contains the ground states of the atoms, i.e., the observable measured is the property of being in the atomic ground state (cf. Section II.4). In the other example of collision experiments the detector is placed far away from the target and therefore detects eigenstates or mixtures of eigenstates of the operator K. The property B measured by the detector is described by a projection operator Λ_B, which projects onto a (in general continuous) direct sum of energy eigenspaces of K. Usually B is a more specific property. For example, the detectors may not be placed all around the target, but only at a particular angle. Λ_B is then the projection operator onto that particular subspace of the above direct sum of energy subspaces that contains states whose momentum vectors are directed into the particular solid angle.

Thus the problem that we shall discuss (in the Schrödinger picture) is the following: The state $W(t)$ changes in time according to

$$W(t) = e^{-iHt/\hbar} W e^{+iHt/\hbar} \qquad (W \equiv W(0)). \tag{1.2}$$

[1] It should be remarked that the assumption (1.1) is not really necessary for the description of collision processes; it suffices to assume only that asymptotic direct-product states do exist.

numbers l and l_3 together with other, internal quantum numbers η; or $\hat{a}$ may consist of the direction $\mathbf{p}/|\mathbf{p}|$ of the momentum $\mathbf{p}$ together with other quantum numbers η. If the operators A_i $(i = 1, 2, \ldots, k)$ all commute with H, then the $|E_a \hat{a}^\pm\rangle$ are generalized eigenvectors of the c.s.c.o. $\{H, A_1, \ldots, A_k\}$; otherwise the additional labels $\hat{a}$ just serve to indicate *that* $|E_a \hat{a}^\pm\rangle$ *is obtained from* $|E_a \hat{a}\rangle$ *by way of the Lippmann-Schwinger equation*, and do not indicate that $|E_a \hat{a}^\pm\rangle$ is an eigenvector of all the A_i's (for more details cf. Section XV.1).

We shall assume that the K eigenvectors $|a\rangle$ are normalized according to

$$\langle a|a'\rangle = \langle E_a \hat{a}|E_{a'} \hat{a}'\rangle = \rho(E_a)^{-1}\delta_{\hat{a}\hat{a}'}\delta(E_a - E_{a'}). \tag{2.9a}$$

[$\rho(E_a)$ is a weight function that is arbitrary but fixed[3]; a convenient choice for $\rho(E_a)$ will be made later.] With such a normalization the summation $\sum_a$ is really an abbreviation for the more explicit

$$\sum_a = \int \rho(E_a)\, dE_a \sum_{\hat{a}}. \tag{2.10}$$

The generalized eigenvectors $|a^\pm\rangle$ of H have the same normalization

$$\langle a^\pm|a'^\pm\rangle = \langle E_a \hat{a}^\pm|E_{a'} \hat{a}'^\pm\rangle = \rho(E_a)^{-1}\delta_{\hat{a}\hat{a}'}\delta(E_a - E_{a'}) \tag{2.9b}$$

as the corresponding generalized eigenvectors $|a\rangle$ of K. The derivation of (2.9b) from (2.9a) is given in Appendix XV.A.

In general the spectrum of K and the spectrum of H are not the same. Usually K has only a continuous spectrum $\{E_a\}$ starting at a particular value, say $E = 0$, and going to infinity. But the spectrum $\{E_s\}$ of H is the combination of a continuous spectrum $\{E_a\}$, which is usually the same as the continuous spectrum of K, and a discrete spectrum $\{E_n\}$, which may be negative but is bounded from below. Physically the continuous spectrum corresponds to the scattering states and the discrete spectrum to the bound states of the projectile and target. There may be discrete eigenvalues in the continuous spectrum, but for physical reasons the energy eigenvalues cannot be arbitrarily large—in particular, not arbitrarily large and negative. We may thus choose the generalized eigenvectors of K:

$$\{|a\rangle = |E_a \hat{a}\rangle\}$$

as a basis for the space of physical states, or we may choose the set

$$\{|\alpha\rangle\} = \{|a^\pm\rangle = |E_a \hat{a}^\pm\rangle\} \cup \{|\alpha_n\rangle\}$$

of discrete eigenvectors $|\alpha_n\rangle$ and generalized eigenvectors $|a^\pm\rangle$ of H as a basis. The completeness property of these bases may be expressed as

$$I = \sum_a |a\rangle\langle a| = \int \rho(E_a)\, dE_a \sum_{\hat{a}} |E_a \hat{a}\rangle\langle E_a \hat{a}| \tag{2.11a}$$

[3] It is for the generalized energy eigenvectors $|E\rangle$ what $\rho(x)^{-1}$ is for the generalized eigenvectors $|x\rangle$, in (1.4.14) and (1.4.7c_ρ).